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Robust stabilization of nonlinear sampled-data systems

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Abstract— **A new approach is proposed in this paper to study robust stabilization of a class of sampled-data systems with non-linear uncertainties. It is assumed that the systems are closed via a network channel which may be subject to data packet dropout and communication delays. Such networked sampled-data systems are modelled as continuous-time nonlinear systems with delayed input. Sufficient conditions on the existence of stabilizing state feedback controllers are established in terms of linear matrix inequalities (LMIs). The maximum bound on the nonlinearity can be computed by solving a constrained convex optimization problem. Moreover, the upper bound on the delayed input can be obtained by solving a quasi-convex optimization problem. Finally, a simulation example is presented to illustrate the efficiency and feasibility of our proposed approach.**

Keywords: LMIs, sampled-data systems, non-linear perturbations, NCSs, stabilization.

I. INTRODUCTION

Because of the advances in digital systems and communication networks, more and more control engineers would like to use a real-time communication channel interfaced to a digital system to exchange information and to complete the control task. Compared with conventional point-to point control systems, the advantages of networked control systems (NCSs) are less wiring, lower install cost as well as greater agility in diagnosis and maintenance. Examples include industrial automation, intelligent vehicle systems and advanced aircraft and spacecraft, etc. However, the insertion of communication channel to sampled-data systems makes the analysis and design of the closed-loop system complex. Conventional sampled-data systems' theories with such assumptions as non-delayed sensing and determinism must be revaluated before they can be applied to NCS [17], [13].

Networked-induced delays and Packet losses typically have negative effects on the NCSs' stability and performance. So far, different methodologies have been formulated to deal with the problem of network delays. An augmented state vector method is proposed in [13] to control a linear system over a periodic delay network. Queuing mechanisms are developed in [4], [10], which utilize some deterministic or probabilistic information of NCSs for control purpose. Random delays are discussed in [12] via an optimal stochastic control methodology. See also [3], [8] and the references therein for related works. However, no method has been given in the references above on how to estimate the maximum allowable value of the network-induced delays that preserves the stability of NCSs.

Packet losses often happen due to link failure or packets are purposefully dropped in order to avoid congestion or guarantee the most recent data to receiver. An augmented state space method is developed to deal with the problem of data packet dropout [18], which has not considered the effect of networked-induced delay. The performance of real-time NCSs with data dropout is discussed in [9]. Using an uncertainty threshold principle, [1] presents a general framework for stability analysis of NCSs in the presence of packet losses. [17] models NCSs with data packet dropout and delays as switched linear systems and presents sufficient conditions on the stabilisation of the NCSs, whose results can only be applied to the NCSs with constant transmission delays. However, their approaches do not work in the case with nonlinear perturbations.

Motivated by recent research on NCSs and nonlinear perturbations [14], [15], [19], we will investigate the robust stabilization problem for a class of sampled-data systems with nonlinear uncertainties. The only information about the nonlinearity is that it satisfies a quadratic constraint. We are particularly interested in the case that the control loops are realized via a real-time communication channel which may subject to data packet dropout and communication delays. For simplicity, we consider the setup with a clock-driven sensor, and both the controller and the actuator are combined into one event-driven node.

Because data packet dropout, transmission delays and nonlinear uncertainties might be potential sources to the instability and poor performance of NCSs, this paper considers stabilization of such NCSs. The uncertain sampled-data systems with data packet dropout and transmission delays are modelled as nonlinear systems with time-varying input delay, which might be subject to fast time-varying. There are two main methods to deal with linear systems with time-varying delays: one is the Krasovskii-based method and the other is the Razumikhin-based method. Lyapunov-Razumikhin function method is the main approach to dealing with the stability issue without any restrictions on the derivative of the delay, which usually lead to conservative results, see e.g. [7]. Recently, for the first time [5] has used Lyapunov-Krasovskii technique to deal with the case that the delay part violates the condition $\dot{\tau}(t) < 1$. [6] has applied this technique to ordinary sampled-data systems which has been modelled as linear continuous-time systems with time-varying input delays under the constraint $\dot{\tau}(t) = 1$. However, to obtain the stabilizing controller in terms of linear matrix inequalities (LMIs) the method presented in [6] must first fix

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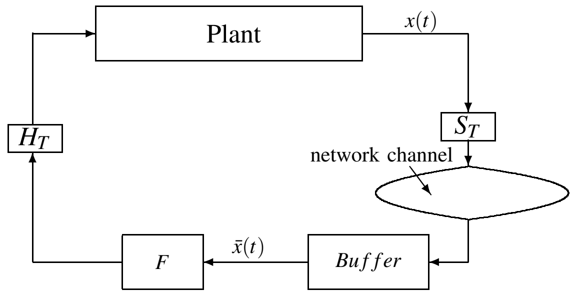

Fig. 1. A sampled-data system closed via an network channel

a tuning parameter or use an iterative algorithm. Moreover, their approach does not work in the case with nonlinear uncertainties. In this paper, we propose an approach to construct a stabilizing linear constant feedback law in terms of LMIs where there is no tuning of parameters or iterative algorithm involved. An estimation of the upper bound on the nonlinearity can be obtained with the computation of a convex optimization problem. Also, the upper bound of the delay function which is related to data packet dropout and communication delays can be calculated directly by solving a generalized eigenvalue problem using the efficient LMI toolbox.

The paper is organized as follows. Section II models an uncertain sampled-data system with data packet dropout and delays as a nonlinear continuous-time system with time-varying input delays. Section III develops sufficient conditions on the stabilization of such systems; linear constant feedback laws can be formulated in terms of LMI. Section IV presents a simulation example to illustrate the efficiency and feasibility of our proposed approach. Section V concludes this paper.

Notation: In this paper, $\mathbf{R}$ is the set of all real numbers, $\mathbf{R}^n$ is the set of all n-tuples of real numbers. A^T and A^{-1} denote the transpose and the inverse of a matrix A, respectively. $A > 0$ $(A < 0)$ means that A is positive definite (negative definite). * represents blocks that are readily inferred by symmetry and I_m denotes the unit matrix with m rows and m columns. Z_+ denotes the set of non-negative integer and R_+ is the set of non-negative real number.

II. Problem formulation

The state-space model of nonlinear sampled-data system shown in Fig. 1 consists of a continuous-time plant

$$\dot{x}(t) = Ax(t) + Bu(t) + h(t, x(t)), \tag{1}$$

and a piecewise constant controller

$$u(t) = F\bar{x}(t_k), \qquad t \in [t_k, t_{k+1}), \qquad k = 1, 2, \cdots, \tag{2}$$

which are connected via a network channel. $x(t) \in \mathbf{R}^n$, $u(t) \in \mathbf{R}^m$ are the plant state and the plant input, respectively. F is the state feedback gain matrix to be designed, A, B are known real constant matrices with appropriate dimensions. It is assumed that the pair (A, B) is stabilizable, the sampling period is a positive scalar T and t_k is the sampling instant. We insert a sampler S_T and a zero order hold H_T in the loop as in Fig. 1. The sampled value will be transmitted through a network channel and the successfully transmitted value will be registered in a buffer. $\bar{x}(t_k) \in \mathbf{R}^n$ is the output information of the buffer which will be used to construct the controller. $h: R_+ \times \mathbf{R}^n \to \mathbf{R}^n$ is the nonlinear uncertainties of the plant. We assume that $h(t,x)$ is a piecewise-continuous nonlinear function in x, satisfying $h(0) = 0$ and the quadratic constraint condition

$$h^T(t, x(t))h(t, x(t)) \le \alpha^2 x^T(t) H^T H x(t), \tag{3}$$

where $\alpha > 0$ is the bounding parameter on the uncertain function h and H is a constant matrix. Also, we assume that $x = 0$ is the only equilibrium of the system (1)–(2). Note that constraint (3) is equivalent to

$$\begin{bmatrix} x \\ h \end{bmatrix}^T \begin{bmatrix} -\alpha^2 H^T H & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} x \\ h \end{bmatrix} \le 0. \tag{4}$$

For any given H, we define the set

$$\begin{aligned} H_\alpha = \{h:\ &\mathbf{R}^{n+1} \to \mathbf{R}^n | h^T(t, x(t))h(t, x(t)) \le \\ &\alpha^2 x^T(t) H^T H x(t) \text{for all } (t, x) \in R_+ \times \mathbf{R}^n\}. \end{aligned} \tag{5}$$

We first consider the effect of data packet dropout on the plant. Packet losses often happen due to link failure or packets are purposefully dropped in order to avoid congestion or guarantee the most recent data to receiver. The output of the buffer can be described as follows:

The plant (1) with no packet dropout at time t_k: $\bar{x}(t_k) = x(t_k)$;

The plant (1) with one packet dropout at time t_k: $\bar{x}(t_k) = x(t_k - T)$;

$$\vdots$$

The plant (1) with $d(k)$ packets dropout at time t_k: $\bar{x}(t_k) = x(t_k - d(k)T)$.

The quantity of dropped packets is accumulated from the latest time when $\bar{x}(t_k)$ has been updated.

Then, we consider the delay effect. We denote the communication delays as $\tau_s(t)$. Because of the delays, It might happen that more than one sensor messages arrive at the buffer of the controller at the same sampling period. Then the most fresh message will be used to construct the controller, the rest will be discarded.

Combining the effects of data packet dropout and communication delays, the input of the controller can be given as

$$\bar{x}(t_k) = x(t_k - d(k)T - \tau_s(t)).$$

From the above analysis, the closed-loop system with the effects of packet loss and delays can be given by

$$\dot{x}(t) = Ax(t) + BFx(t_k - d(k)T - \tau_s(t)) + h(t, x(t)),$$

for $t \in [t_k, t_{k+1})$. Let $\tau(t) = t - t_k + d(k)T + \tau_s(t)$, then the system can be expressed as:

$$\dot{x}(t) = Ax(t) + BFx(t - \tau(t)) + h(t, x(t)), \quad t \in [t_k, t_{k+1}). \tag{6}$$

The quantity $d(k) \in Z_+$ may vary with time t and it is assumed that

$$0 \le \tau(t) = t - t_k + d(k)T + \tau_s(t) \le T + d(k)T + \tau_s(t) \le \bar{\tau}, \tag{7}$$

for $t \in [t_k, t_{k+1})$, where $\bar{\tau}$ is a positive scalar.

In this way, the closed-loop system (1)–(2) with plant uncertainty, data packet dropout and communication delays is modelled as the nonlinear continuous-time system (6) with time-varying delays, and this enables us to apply the theory of delay systems to the analysis and design of such sampled-data systems.

It can be seen that the time-varying delay function $\tau(t) = t - t_k + d(k)T + \tau_s(t)$ is piecewise-linear with the derivative of $\tau(t)$ maybe larger than one.

Remark 1: In this paper, we present the piecewise-constant control law as a continuous-time one and deal with sampled-data systems in the continuous-time domain, instead of discretizing the plant (1), thus the inter-sample behavior is taken into account.

Our major objective is to design linear constant feedback laws to stabilize this type of systems and, at the same time, maximize the nonlinear bound or maximize the upper bound of the delay function $\tau(t)$ (relate to data packet dropout and communication delays of the original system (1)–(2)) which does not violate the stability of the closed-loop system (1)–(2).

III. Sampled-data stabilization of an NCS

In this section, an LMI approach will be developed to solve the stabilization problem formulated in the previous section.

Now, we introduce the definition of robust stability.

Definition 1: The system (6) with a state feedback gain F is robustly stable with degree α if the equilibrium $x = 0$ is globally asymptotically stable for all $h(t,x(t)) \in H_\alpha$ and delay function $\tau(t)$ satisfies condition (7).

The following lemma will be used in the proof of our main results.

Lemma 1: [11] (Moon's inequality) For any $a \in \mathbf{R}^{n_a}$, $b \in \mathbf{R}^{n_b}$, $N \in \mathbf{R}^{n_b \times n_a}$, $R \in \mathbf{R}^{n_a \times n_b}$, $Y \in \mathbf{R}^{n_a \times n_b}$, $Z \in \mathbf{R}^{n_b \times n_b}$, the following holds:

$$-2b^T N a \le \begin{bmatrix} a \\ b \end{bmatrix}^T \begin{bmatrix} R & Y - N^T \\ Y^T - N & Z \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix},$$

where

$$\begin{bmatrix} R & Y \\ Y^T & Z \end{bmatrix} \ge 0.$$

From the previous section, the NCS (1)–(2) with data packet dropout and communication effects can be modelled as nonlinear delay system (6). To study robust stabilization problem of the NCS (1)–(2), we first study robust stability problem of the system (6).

Theorem 1: Given scalars $\bar{\tau} > 0$, $\alpha > 0$ and a gain matrix F, the system (6) is stable with degree α for all $\tau(t) \le \bar{\tau}$ if there exist matrices $\tilde{P}_1 > 0$, $\tilde{P}_2$, $\tilde{P}_3$, $\tilde{Z}_1$, $\tilde{Z}_2$, $\tilde{Z}_3$, and $\tilde{R} > 0$ satisfying

$$\begin{bmatrix} \tilde{\Lambda} + \bar{\tau}\tilde{Z} + \begin{bmatrix} \alpha^2 H^T H & 0 \\ 0 & \bar{\tau}\tilde{R} \end{bmatrix} & \tilde{P}^T \begin{bmatrix} 0 \\ I \end{bmatrix} \\ * & -I \end{bmatrix} < 0, \tag{8}$$

and

$$\begin{bmatrix} \tilde{R} & [0 \ \ F^T B^T]\tilde{P} \\ * & \tilde{Z} \end{bmatrix} \ge 0, \tag{9}$$

where

$$\tilde{\Lambda} = \tilde{P}^T \begin{bmatrix} 0 & I \\ A + BF & -I \end{bmatrix} + \begin{bmatrix} 0 & A^T + F^T B^T \\ I & -I \end{bmatrix} \tilde{P},$$

$$\tilde{P} = \begin{bmatrix} \tilde{P}_1 & 0 \\ \tilde{P}_2 & \tilde{P}_3 \end{bmatrix}, \ \tilde{Z} = \begin{bmatrix} \tilde{Z}_1 & \tilde{Z}_2 \\ * & \tilde{Z}_3 \end{bmatrix}.$$

Proof: We present (6) in an equivalent descriptor form [6]

$$\begin{aligned} &\dot{x}(t) = y(t), \\ &0 = -y(t) + Ax(t) + BFx(t - \tau(t)) + h(t,x(t)), \end{aligned} \tag{10}$$

or equivalently

$$\dot{x}(t) = y(t),$$
$$0 = \begin{cases} -y(t) + Ax(t) + Bu(t) + h(t,x(t)), & \text{if } t \in [0, \bar{\tau}), \\ -y(t) + (A + BF)x(t) - BF\int_{t-\tau(t)}^{t} y(s)ds \\ +h(t,x(t)), \text{if } t \ge \bar{\tau}, \end{cases} \tag{11}$$

which is valid in the case of piecewise-continuous delay $\tau(t)$ for $t \ge 0$. Given a matrix F and initial condition $x(t) = \phi(t)$ ($t \in [-\bar{\tau}, 0]$), where ϕ is a continuous function, $x(t)$ satisfies (6) for $t \ge 0$ iff it satisfies (11).

We use the following Lyapunov-Krasovskii functional

$$V(t) = s^T(t)EPs(t) + \int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} y^T(l)Ry(l)dld\theta, \tag{12}$$

where

$$s(t) = \begin{bmatrix} x(t) \\ y(t) \end{bmatrix}, \ E = \begin{bmatrix} I_n & 0 \\ 0 & 0 \end{bmatrix}, \ P = \begin{bmatrix} P_1 & 0 \\ P_2 & P_3 \end{bmatrix}, \ R > 0,$$

which satisfies the following inequalities:

$$a|x(t)|^2 \le V(t) \le b \sup_{s \in [-\bar{\tau}, 0]} |s(t+s)|^2, \ a > 0, \ b > 0.$$

Note that

$$s^T(t)EPs(t) = x(t)^T P_1 x(t).$$

Thus, differentiating the first term of (12) with respect to t, we have

$$\frac{d}{dt}\{s^T(t)EPs(t)\} = 2x^T(t)P_1 x(t) = 2s^T(t)P^T \begin{bmatrix} \dot{x}(t) \\ 0 \end{bmatrix}. \tag{13}$$

Substituting in the right-hand side of (13) by the expressions for $\dot{x}$ and 0 in (11), we obtain

$$\begin{aligned} \tfrac{dV(t)}{dt} &= s^T(t)\Lambda s(t) + \eta_1 + 2s^T(t)P^T \begin{bmatrix} 0 \\ I \end{bmatrix} h(t,x(t)) \\ &+ \bar{\tau}y(t)Ry(t) - \textstyle\int_{-\bar{\tau}}^{0} y^T(t+\theta)Ry(t+\theta)d\theta, \end{aligned} \tag{14}$$

where

$$\Lambda = P^T \begin{bmatrix} 0 & I \\ A+BF & -I \end{bmatrix} + \begin{bmatrix} 0 & A^T+F^TB^T \\ I & -I \end{bmatrix} P,$$

$$\eta_1 = -2s^T P^T \begin{bmatrix} 0 \\ BF \end{bmatrix} \int_{t-\tau(t)}^{t} y(s)ds.$$

Taking $N = P^T \begin{bmatrix} 0 \\ BF \end{bmatrix}$, $b = s(t)$, $a = y(s)$, we obtain from Lemma 1 that

$$\eta_1 \leq \int_{t-\tau(t)}^{t} \begin{bmatrix} y(s) \\ s(t) \end{bmatrix}^T \begin{bmatrix} R & Y-N^T \\ Y^T-N & Z \end{bmatrix} \begin{bmatrix} y(s) \\ s(t) \end{bmatrix} ds,$$

where R, Y, Z are constant matrices with appropriate dimensions satisfying

$$\begin{bmatrix} R & Y \\ Y^T & Z \end{bmatrix} \geq 0. \tag{15}$$

Choosing $Y = [0\ \ F^TB^T]P$, we find

$$\eta_1 \leq \int_{-\bar{\tau}}^{0} y^T(t+s)Ry(t+s)ds + \bar{\tau}s^T(t)Zs(t).$$

Then, we have

$$\begin{aligned} \frac{dV(t)}{dt} &\leq s^T(t)(\Lambda + \bar{\tau}Z + \begin{bmatrix} 0 & 0 \\ 0 & \bar{\tau}R \end{bmatrix})s(t) \\ &\quad +2s^T P^T \begin{bmatrix} 0 \\ I \end{bmatrix} h(t,x(t)) \\ &= W(t)^T \Omega W(t), \end{aligned}$$

where

$$W(t) = \begin{bmatrix} s(t) \\ h(t,x(t)) \end{bmatrix},$$

$$\Omega = \begin{bmatrix} \Lambda + \bar{\tau}Z + \begin{bmatrix} 0 & 0 \\ 0 & \bar{\tau}R \end{bmatrix} & P^T \begin{bmatrix} 0 \\ I \end{bmatrix} \\ * & 0 \end{bmatrix}.$$

Since a sufficient condition on the stability of the system (6) is $\frac{dV(t)}{dt} < 0$, we require

$$W(k)^T \Omega W(k) < 0. \tag{16}$$

By the well-known S-procedure [16], inequality (16) with constraint (4) is equivalent to the existence of matrices $P_1 > 0$, P_2, P_3, Z_1, Z_2, Z_3, $R > 0$ and a scalar $\varepsilon \geq 0$ such that

$$\begin{bmatrix} \Lambda + \bar{\tau}Z + \begin{bmatrix} \varepsilon\alpha^2 H^T H & 0 \\ 0 & \bar{\tau}R \end{bmatrix} & P^T \begin{bmatrix} 0 \\ I \end{bmatrix} \\ * & -\varepsilon I \end{bmatrix} < 0, \tag{17}$$

and

$$\begin{bmatrix} R & [0\ \ F^TB^T]P \\ * & Z \end{bmatrix} \geq 0. \tag{18}$$

It is well known that minimization under non-strict LMI constraints gives the same result as minimization under strict LMI constraints when both strict and non-strict LMI constraints are feasible [2]. This is true for (17)–(18) because if there is a solution for $\varepsilon = 0$ there is a solution for some $\varepsilon > 0$ and sufficiently small α. Thus we can substitute $\varepsilon > 0$ for $\varepsilon \geq 0$ in (17)–(18). Therefore (17)–(18) are equivalent to the existence of matrices $\tilde{P}_1 := P_1/\varepsilon > 0$, $\tilde{P}_2 := P_2/\varepsilon$, $\tilde{P}_3 := P_3/\varepsilon$, $\tilde{R} := R/\varepsilon > 0$, $\tilde{Z} := Z/\varepsilon$ such that (8)–(9) hold. ■

We introduce another important definition.

Definition 2: The control law of (2) robustly stabilize (1) with degree α if there exists a state feedback gain F such that the closed-loop system (1)–(2) is asymptotically stable for all $h(t,x(t)) \in H_\alpha$ and delay function $\tau(t)$ satisfies condition (7).

Then, we pay our attention to design a linear constant feedback law such that the NCS (1)–(2) with the effects of data packet dropout and communication delays is stable.

Theorem 2: Given a scalar $\bar{\tau} > 0$, the control law of (2) robustly stabilize (1) with degree $\alpha = 1/\sqrt{\gamma}$ for all $\tau(t) \leq \bar{\tau}$ if there exist matrices $\bar{R} > 0$, $Q_1 > 0$, Q_2, Q_3, $\bar{Z}_1$, $\bar{Z}_2$, $\bar{Z}_3$, $\bar{Y}$ and a scalar $\gamma > 0$ such that

$$\begin{bmatrix} Q_2+Q_2^T+\bar{\tau}\bar{Z}_1 & Q_3-Q_2^T+Q_1A^T+\bar{Y}^TB^T+\bar{\tau}\bar{Z}_2 & 0 & \bar{\tau}Q_2^T & Q_1H^T \\ * & -Q_3^T-Q_3+\bar{\tau}\bar{Z}_3 & I & \bar{\tau}Q_3^T & 0 \\ * & * & -I & 0 & 0 \\ * & * & * & -\bar{\tau}\bar{R} & 0 \\ * & * & * & * & -\gamma I \end{bmatrix} < 0, \tag{19}$$

and

$$\begin{bmatrix} Q_1+Q_1^T-\bar{R} & 0 & \bar{Y}^TB^T \\ * & \bar{Z}_1 & \bar{Z}_2 \\ * & * & \bar{Z}_3 \end{bmatrix} \geq 0, \tag{20}$$

where $\gamma = \alpha^{-2}$. Furthermore, the state feedback control law is given by

$$u(t) = \bar{Y}Q_1^{-1}\bar{x}(t).$$

Proof: It is noted that the inequalities of Theorem 1 are bilinear in $\tilde{P}$ and F. To get LMIs, we use $\tilde{P}^{-1}$. It follows from the requirement of $0 < \tilde{P}_1$, and the fact that in (8) $-\tilde{P}_3 - \tilde{P}_3^T$ must be negative definite, that $\tilde{P}$ is nonsingular. Define

$$Q := \tilde{P}^{-1} = \begin{bmatrix} Q_1 & 0 \\ Q_2 & Q_3 \end{bmatrix},\ \bar{R} := \tilde{R}^{-1},$$

$$\bar{Z} := Q^T \tilde{Z} Q = \begin{bmatrix} \bar{Z}_1 & \bar{Z}_2 \\ * & \bar{Z}_3 \end{bmatrix},\ \bar{Y} := FQ_1.$$

Multiplying diag$\{Q^T, I\}$ and its transpose on the left and on the right sides of (8), respectively, using Schur complement formula, we know that (8) is equivalent to (19).

Multiplying diag$\{Q_1, Q^T\}$ and its transpose on the left and on the right side of (9), respectively, we have

$$\begin{bmatrix} Q_1\bar{R}^{-1}Q_1 & 0 & \bar{Y}^TB^T \\ * & \bar{Z}_1 & \bar{Z}_2 \\ * & * & \bar{Z}_3 \end{bmatrix} \geq 0. \tag{21}$$

Since $\bar{R} = \tilde{R}^{-1} > 0$, and

$$[Q_1 - \bar{R}]\bar{R}^{-1}[Q_1 - \bar{R}]^T > 0, \tag{22}$$

we obtain that

$$Q_1\bar{R}^{-1}Q_1^T \geq Q_1 + Q_1^T - \bar{R}.$$

Then, (21) can be guaranteed by (20). This completes the proof. ■

To establish robust stabilization of the system (1) under constraint (3) with maximum α, we propose the convex optimization problem

$$\left.\begin{array}{ll} \text{minimize} & \gamma \\ \text{subject to} & Q_1 > 0,\ Q_2,\ Q_3,\ \bar{Z}_1,\ \bar{Z}_2,\ \bar{Z}_3,\ \bar{Y} \text{ and} \\ & (19)-(20) \text{ for a given scalar } \ \bar{\tau} > 0. \end{array}\right\} \quad (23)$$

Now, we conclude our result as follows.

Theorem 3: Given a scalar $\bar{\tau} > 0$, if the optimization problem (23) is feasible, the control law of (2) robustly stabilize (1) with maximum nonlinear bound $\alpha = 1/\sqrt{\gamma}$ with

$$F = \bar{Y}Q_1^{-1}.$$

Remark 2: Theorem 3 provides a delay-dependent condition on the robust stabilization of the plant (1) by the control law (2) with its maximum nonlinear bound in terms of the solvability of LMI. It is noted that finding the largest $\bar{\tau}$ (related to the upper bound of data packet dropout and communication delays of the system (1)) that preserves the stability of the closed-loop system can be computed by solving the following quasi-convex optimization problem (a generalized eigenvalue problem):

$$\left.\begin{array}{ll} \text{minimize} & \frac{1}{\bar{\tau}} \\ \text{subject to} & Q_1 > 0,\ Q_2,\ Q_3,\ \bar{Z}_1,\ \bar{Z}_2,\ \bar{Z}_3,\ \bar{Y} \\ & \text{and } (19)-(20). \end{array}\right\} \quad (24)$$

Remark 3: Using inequality (22), the robust stabilization problem can be solved directly in terms of a set of LMIs and no tuning of parameters is involved. Sampled-data stabilization of linear systems has been considered by Fridman et al. in [6], where they must first fix a tuning parameter or use an iterative algorithm to obtain the stabilizing controller in terms of LMIs. Our proposed approach has another advantage that an estimation of maximum delay bound $\bar{\tau}$ can be calculated directly by Remark 2.

IV. An illustrative example

Example 1: Let us consider the sampled-data system

$$\begin{bmatrix} \dot{x}_1(t) \\ \dot{x}_2(t) \end{bmatrix} = \begin{bmatrix} 1 & 1 \\ 0 & 0.99 \end{bmatrix} \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix} + \begin{bmatrix} 0 \\ 10 \end{bmatrix} u(t) + h(t,x(t)) \quad (25)$$

with $H = [1\ 0]$ and the state feedback gain F to be designed.

It is found using the software LMI toolbox in MATLAB that the corresponding quasi-convex optimization problem (24) is feasible and we find that the upper bound of the delay function $\tau(t)$ is $\bar{\tau} = 0.2509$. Given $\bar{\tau} = 0.2509$, solving the convex optimization problem (23), we obtain

$$Q_1 = 10^4 \times \begin{bmatrix} 3.4414 & -3.4491 \\ -3.4491 & 6.8925 \end{bmatrix},$$

$$Q_2 = 10^5 \times \begin{bmatrix} -0.9293 & 0.9114 \\ 0.5787 & -1.1841 \end{bmatrix},$$

$$Q_3 = 10^4 \times \begin{bmatrix} 0.9293 & -0.5784 \\ -0.9217 & 1.2244 \end{bmatrix},$$

$$\bar{Z}_1 = 10^5 \times \begin{bmatrix} 1.4939 & -1.2382 \\ -1.2382 & 2.2520 \end{bmatrix},$$

$$\bar{Z}_2 = 10^4 \times \begin{bmatrix} -1.4931 & 1.3676 \\ 1.2372 & -1.7462 \end{bmatrix},$$

$$\bar{Z}_3 = 10^5 \times \begin{bmatrix} 1.4928 & -1.3672 \\ -1.3671 & 3.6057 \end{bmatrix},$$

$$\bar{R} = 10^4 \times \begin{bmatrix} 4.1371 & -4.1495 \\ -4.1495 & 5.8811 \end{bmatrix},$$

$$\bar{Y} = 10^5 \times [-0.0012\ \ -1.0257],$$

$$\gamma = 6.3229 \times 10^5.$$

It can be easily calculated from Theorem 2 that

$$\alpha_{max} = 0.0013,\ \ F = [-0.2999\ \ -0.2989]$$

with the eigenvalues of matrix $A+BF$ located at $-0.4995 + 0.8664i$ and $-0.4995 - 0.8664i$.

It is noted that if the upper bound of the delay function $\bar{\tau}$ is increased, the nonlinear bound α has to be decreased. The relation between the upper bound of delay function $\bar{\tau}$ and the maximum nonlinear bound α for the system (25) is illustrated in Fig. 2.

For the purpose of simulation, we choose the non-linear function as

$$h(t,x(t)) = \begin{bmatrix} 0.3x_1(t)sin(x_1(t)) \\ 0 \end{bmatrix},$$

which satisfies the quadratic constraint (5). It is found that $x = 0$ is the only equilibrium of the closed-loop system. For $\bar{\tau} = 0.22$, solving the convex optimization problem (23), we obtain from Theorem 2 that

$$\alpha_{max} = 0.1365,\ \ F = [-0.3590\ \ -0.3170].$$

With the initial state $x(0) = [-15\ \ 10]^T$, the state trajectory of the system (25) with the delayed input is shown in Fig. 3, from which we know that the unstable system can be effectively stabilized.

V. Conclusion

This paper have investigated the robust stabilisation problem for a class of sampled-data systems with nonlinear uncertainties. The controller and the plant are connected via a network channel which may be subject to data packet dropout and communication delays. We have developed sufficient conditions on the robust stabilisation for such NCSs. The stabilizing feedback controllers produce a closed-loop system which is maximally tolerant to the uncertain nonlinear terms or is maximally tolerant to the delayed input. An illustrative example is worked out to show how an unstable system can be effectively stabilised via the controller designed in this paper.

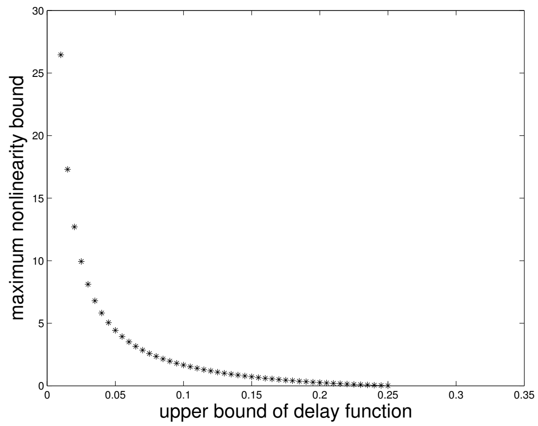

Fig. 2. Relation between $\bar{\tau}$ and α

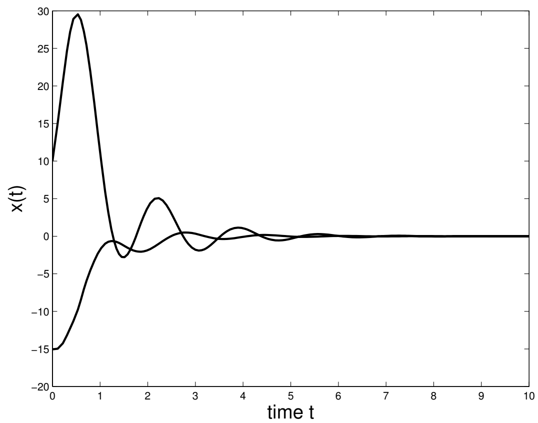

Fig. 3. The trajectory of system (25) with delayed input

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