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# Decentralized Control across Bit-Limited Communication Channels: An Example

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***Abstract*— We formulate a simple discrete time two-agent optimal formation control problem with limited communication capacity. Two cases are studied: without communication constraints and with limited communication capacity. In the first case, we show how to approach the globally optimal solution with local decisions. In the second case, a performance metric is introduced and we show that the performance can be impaired by as much as 20% if we limit the amount of information exchanged in real time. We also introduce a noisy channel into our problem and present some preliminary results.**

## I. Introduction

Since Shannon's 1948 landmark paper [13] much progress (both theoretical and practical) has been made in information theory. The success of this classical information theory can be seen (albeit implicitly) through the ubiquitous presence of electronic data transmission and storage devices ranging from mobile phones, satellite radio to wireless local area networks (WLAN). At the same time, advances in fabrication technology and computer architecture have led to the rapid growth of computation capabilities while simultaneously decreasing chip size and power consumption. This in turn, has made the implementation of distributed cooperative agents both a practical reality and an active research area.

While optimization of coordinated actions by distributed agents has been widely studied by the control community (see, e.g., [1], [5], [14]); until recently, little attention has been paid to the "information" aspect of the problem. However, in the context of real time multi-agent coordination, classical information theory faces several fundamental problems, such as:

- Infinite delays: Classical information theory allows infinite delays. Various asymptotic analysis (*i.e.*, letting codeword length approach infinity) are inappropriate for the real time setting where the delayed information is often useless.
- Limited concept of feedback: Classical information theory assumes a one-way channel. In an interactive setting (such as the one studied in [11], [12]), where agents communicate with each other, we must consider a two-way noisy channel.
- Quality of information: Classical information theory maintains no sense of the "quality" or the "importance" of information. Often, an agent that faces severe communication constraints must decide which part of the information is the most important to send and traditional information theory tools are not easily applied.

Many researchers have already seen the limitations of traditional information theory and progressed toward a more real-time and interactive version of information theory. Schulman in [11], [12] pioneered the coding theorem for interactive computation that is analogous to Shannon's work in one-way communication. Mitter in [8] described the need for a unified approach to control, communication, and computation. Tatikonda [15] and Sahai [10] have also pioneered work in the area of control under communication constraints, where the aforementioned issues are of critical importance. Important contributions have also been made by Elia and Mitter [4], Brockett and Liberzon [3], Hespanha et al. [7], Ishii and Francis [6], Nair and Evans [9], and many others.

Following in the spirit of these and other researchers, we attempt to gain insight by studying an example problem. We carry out this study under a two-agent formation optimal control framework.

As an initial step, we investigate the problem of controlling the formation of two agents that move around on the integer line. In each "round" (unit of time), each agent can move one step in either the positive or negative direction or remain in place. The goal of the agents is to reach their respective final destinations from a arbitrary starting point while maintaining a desired separation (1-dimensional "formation") using a fixed amount of communication capacity that is measured in bits per round. To quantify system performance, we define an associated metric and use tools from optimal control theory to study the performance of this system under various communication constraints. It turns out that allowing any positive communication rate between the agents can improve performance by 20%. Moreover, any amount of communication is asymptotically equivalent to unlimited communication.

This paper is organized as follows. In section 2, we formulate the basic problem. In section 3, optimal control with full and partial information feedback is discussed where we assume the channel is error free. In section 4, the case where the communication channel introduces error is studied. Finally, conclusions and directions for future work are given in section 5.

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## II. Problem Set Up

Two scalar agents move along the integer line (which we shall refer to as the x-axis), with the position of the $i^{th}$ agent ($i \in \{1, 2\}$) at time step k given by $x_i(k) \in \mathbf{Z}$. The position of each agent is governed by the relation:

$$x_i(k+1) = x_i(k) + u_i(k) \tag{1}$$

where $k \in \mathbf{Z}$ is the time index and $u_i \in \{-1, 0, 1\}$.

The goal of each agent is to move to some final position $x_{i,f}$, while maintaining a desired a separation $d$ from the other agent. This problem is meant to approximate a very simple formation task. This goal is consistent with minimizing the cost function given by:

$$J = \sum_{k=0}^{\infty} [(x_1(k) - x_{1,f})^2 + (x_2(k) - x_{2,f})^2 + (x_2(k) - x_1(k) - d)^2]. \tag{2}$$

For our initial study of the problem we make the simplifying assumption that the initial configuration of the agents satisfies $x_1(0) < x_2(0)$ and that $d = (x_{2,f} - x_{1,f}) > 0$. In addition, because two physical agents cannot pass through one another, we constrain the problem such that $x_1(k) < x_2(k)$ for all $k$. Note that it is reasonable to expect that the solution to this problem would be the mirror image if we instead let $x_2(0) < x_1(0)$ and $x_2(k) < x_1(k)$ for all $k$.

The stationary point of (1), for each agent $i = 1, 2$, occurs when $x_i(k+1) = x_i(k)$ for all $k$, this point occurs when each agent reaches its respective final position $x_{i,f}$. In remainder of this paper we use the terms system equilibrium to describe the situation when both agents have reached their stationary points, (*i.e.*, when $x_i(k) = x_{i,f}$ for all $i$).

With the assumptions above, the problem of interest reduces to one where at any time $k$, the agents can only be in one of the 4 configurations which are described in Table I and shown graphically in Figure 1.

| Name | Conditions | Remarks |
|---|---|---|
| State 1 | $x_1(k) \leq x_{1,f}$ and $x_2(k) \leq x_{2,f}$ | bottom of figure |
| State 2 | $x_1(k) > x_{1,f}$ and $x_2(k) < x_{2,f}$ | |
| State 3 | $x_1(k) < x_{1,f}$ and $x_2(k) > x_{2,f}$ | |
| State 4 | $x_1(k) \geq x_{1,f}$ and $x_2(k) \geq x_{2,f}$ | top of figure |

TABLE I

Agent Initial Configurations

Based on the configurations depicted in Figure 1, it can be seen that the configurations labeled as states 2 and 3 will migrate into states 1 or 4 as time progresses; if they do not reach system equilibrium first. However once the agents reach either of the configurations labeled as states 1 or 4 they remain in that configuration for every time step until the system equilibrium is reached; (*i.e.*, both the agents stop forever at their destinations).

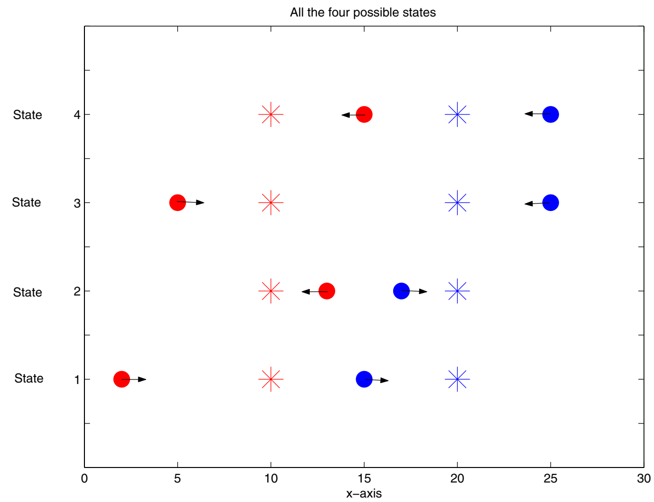


Fig. 1. Agent 1 is depicted in red, agent 2 is depicted in blue and * denotes their desired positions.

## III. Error Free Communication

### A. Perfect Communication

The case where each agent can pass an unlimited amount of information without information loss or errors is referred to as the case of 'Perfect Communication'. For this case, a decentralized steepest descent algorithm is proposed. This so called greedy algorithm is then shown to be globally optimal.

We use the following notation:

1) $\rightarrow$ indicates that agent $i$ travels one step to the right (corresponding to application of $u_i = 1$),
2) $\leftarrow$ indicates that agent $i$ travels one step to the left (corresponding to application of $u_i = -1$), and
3) $|$ indicates that agent $i$ stays at its current position (corresponding to application of $u_i = 0$).

It should be noted that if the agents start in State 2, then the locally optimal control action is for agent 1 to go left ($\leftarrow$) and agent 2 to go right ($\rightarrow$). If $d_1(0) < d_2(0)$ agent 1 will reach its final position (stationary point) before agent 2 reaches its own stationary point and the configuration will switch to State 1. Likewise, if $d_2(0) < d_1(0)$ agent 2 reaches its stationary point first and the configuration changes to State 4.

Similarly, if the agents start in State 3, then the locally optimal control action is for agent 1 to go right ($\rightarrow$) and agent 2 to go left ($\leftarrow$) until one of the agents reaches its stationary point. If agent 1 reaches its stationary point first, then the system transitions to State 4. Otherwise, agent 2 reaches its final position first and the system transitions to State 1.

It should also be noted that State 4 is completely symmetric with respect to State 1. Thus for any control law that is optimal for State 1, we only need to swap the

| Condition | Agent 1 | Agent 2 |
|---|---|---|
| $d_1^k \geq 2d_2^k + 2$ | $\rightarrow$ | $\leftarrow$ |
| $2d_2^k \leq d_1^k \leq 2d_2^k + 1$ | $\rightarrow$ | $\mid$ |
| $\frac{d_2^k}{2} < d_1^k < 2d_2^k$ | $\rightarrow$ | $\rightarrow$ |
| $2d_1^k \leq d_2^k \leq 2d_1^k + 1$ | $\mid$ | $\rightarrow$ |
| $d_2^k \geq 2d_1^k + 2$ | $\leftarrow$ | $\rightarrow$ |

TABLE II
CONTROL LAWS

agent commands with each other to generate the optimal control commands for the system in the configuration of State 4. Therefore in the remainder of the paper the only configuration that is discussed is State 1. The control laws related to the agents starting in other configurations are simple extensions.

By letting $d_1^k = |x_{1,f} - x_1(k)|$ and $d_2^k = |x_{2,f} - x_2(k)|$, one can determine the steepest descent control law for agents starting in State 1, *i.e.*, search through all possible moves and find the one which minimizes the cost function at that time step. Table II shows the criterion on the relationship between $d_1^k$ and $d_2^k$ and the corresponding optimal control laws that minimize the cost at each time step $k$.

*Proposition 1:* The control algorithm described in Table II globally optimizes the cost function defined in equation (2) for any initial condition consistent with State 1.

*Proof:* Clearly if $d_1(0) > d_2(0)$, then $d_1(k) > d_2(k)$ for all $k$. A similar result holds for the case where $d_2(0) > d_1(0)$. Therefore without loss of generality one can assume $d_1(0) > d_2(0)$. It is then true that the globally optimal action for agent 1 is to go toward its destination, while agent 2 has three possible directions to move in, as indicated in Table II.

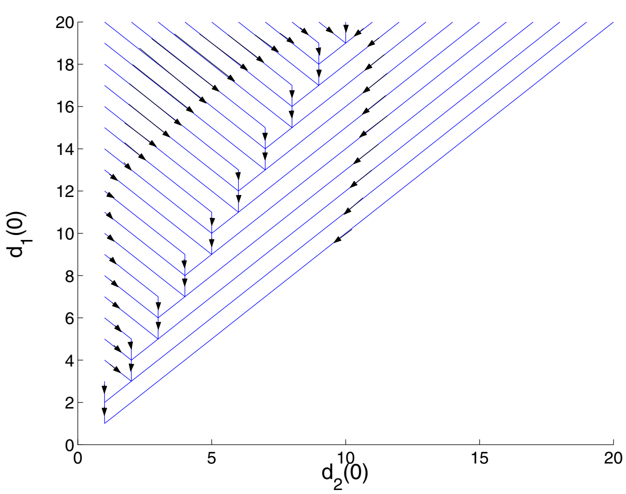


Fig. 2. Optimal Path for $d_2(0) \leq d_1(0) \leq 20$

We use a Dynamic Programming approach (see, e.g., [2]) to generate and subsequently plot the optimal path for some finite pairs $(d_1(0), d_2(0))$. If we let $C(i,j)$ denote the cost associated with the initial condition where $d_1(0) = i,\ d_2(0) = j$, then using dynamic programming, we have:

$$C(i,j) = \min\{C(i-1,j-1), C(i-1,j), C(i-1,j+1)\} + i^2 + j^2 + (i-j)^2 \quad (3)$$

The optimal path is from point $(i,j)$ to one of the three points $(i-1,j-1), (i-1,j),$ or $(i-1,j+1)$. Dynamic programming principles dictate selection of the points with the smallest cost. Figure 2 graphically illustrates the structure of the optimal solution generated using dynamic programming principles. A comparison shows that the globally optimal control law is in fact the same as the control laws in Table II.

Using mathematical induction we can show that these control laws remain valid for all initial conditions $(d_1(0), d_2(0))$ provided that $d_1(0) \geq d_2(0)$. To prove this, it is enough to show that

$$C(i,0) \geq \cdots \geq C(i,j-1) \geq C(i,j),$$

and

$$C(i,i) \geq \cdots \geq C(i,j+1) \geq C(i,j), \quad (4)$$

where $j = (i+1)/2$ if $i$ is odd and $j = i/2$ if $i$ is even. The following induction steps complete the proof.

1) When $i = 4$, inequality (4) holds.
2) Assume that when $i = 2k$ inequality (4) holds, where $k \in \mathbf{Z}$ and $k \geq 2$. We can then compute $C(i,j)$ as in equation (3). Some analysis shows that if inequality (4) holds for $i = 2k$, it also holds for $i = 2k+1$. Furthermore, if the relationship holds for $i = 2k+1$, it also holds for $i = 2k+2$.
3) From the results of steps 1 and 2, the inequality (4) holds for any $i \in \mathbf{Z}$ and $i \geq 0$.

### *B. Partial Communication*

In the following discussion we continue to assume that the information is transmitted over an error free channel; however we now limit the number of bits transmitted per agent per time step. When we limit the communication in this manner the performance of the system may be degraded and the cost function associated with the optimal path may increase as a function of the number of bits that are used. In order to quantify the system degradation a ratio of the system performance with perfect communication to that of the performance with limited communication is examined. The ratio used for the purpose of this study is defined such that the results presented are valid only in cases where the agents initial positions, $x_i(0)$, are initially far from their desired positions, $x_{i,f}$.

Suppose the two agents are given $j$ bits to communicate each round, then we are interested in following quantity:

$$r_j = \inf_{\sigma} \limsup_{d_1(0),d_2(0)} \frac{J_j(\sigma, d_1(0), d_2(0))}{J_\infty(d_1(0), d_2(0))}$$

where:

$\sigma \equiv$ any control law.
$J_\infty(d_1(0), d_2(0)) \equiv$ the total cost with perfect information; and
$J_j(\sigma, d_1(0), d_2(0)) \equiv$ the total cost with $j$ bits of communication per agent per time step $k$.

The cost $J_\infty(d_1(0), d_2(0))$ is obtained using the optimal control law described in section III-A. Note that $r_j$ is in some sense the worst case performance hit since we take the supremum over all possible initial conditions.

The following lemmas illustrate the salient features of the ratio $r_j$ for different values of $j$.

*Lemma 2:* Assume neither agent knows when the system equilibrium has been reached. Let $\sigma_0$ be control law associated with both agents moving one step closer to their desired positions at every time step regardless of the position of the other agent (we call this set of control actions 'racing to the origin'). Then

$$\limsup_{d_1(0), d_2(0)} \frac{J_0(\sigma_0, d_1(0), d_2(0))}{J_\infty(d_1(0), d_2(0))} = 1.2$$

*Proof:* It is clear that

$$\limsup_{d_1(0), d_2(0)} \frac{J_0(\sigma_0, d_1(0), d_2(0))}{J_\infty(d_1(0), d_2(0))} = \lim_{n\to\infty} \frac{J_0(\sigma_0, n, 0)}{J_\infty(n, 0)}.$$

When $d_1(0) = n$, the optimal control law given in Table II shows that:

$$J_\infty(n, 0) = \sum_{i=1}^{n} i^2 + 5\sum_{i=1}^{k} i^2 + \sum_{i=0}^{k} (n-2i)^2,$$

where $k = \frac{n}{3} - 1$ (assuming $\frac{n}{3}$ is an integer, otherwise, take the largest integer near it). After some simplification, we get

$$J_\infty(n, 0) = 15k^3 + \mathcal{O}(k^2)$$

On the other hand, we have

$$J_0(n, 0) = 2\sum_{i=1}^{n} i^2 = 18k^3 + \mathcal{O}(k^2),$$

hence

$$\lim_{n\to\infty} \frac{J_0(\sigma_0, n, 0)}{J_\infty(n, 0)} = \frac{18}{15} = 1.2.$$

*Lemma 3:* The control law $\sigma_0$ defined in Lemma 2 (*i.e.*, 'racing to the origin') achieves $r_0$; *i.e.*, for any other given control law $\sigma$ which achieves a finite cost,

$$\limsup_{d_1(0), d_2(0)} \frac{J_0(\sigma, d_1(0), d_2(0))}{J_\infty(d_1(0), d_2(0))} \geq 1.2.$$

*Proof:* For any given control law that achieves a finite cost $\sigma$, at least one of the agents will reach its stationary point in a finite number of time steps. Otherwise, the cost may continue to increase for an infinite number of time steps and, which would result in $J_0 \to \infty$ as $k \to \infty$.

Selecting a sequence of pairs $s_k = (k, 0)$ as the initial conditions for the two agents and evaluating the quantity

$$\limsup_{s_k} \frac{J_j(\sigma, s_k)}{J_\infty(s_k)}.$$

After a finite amount of time, $d_2$ will become zero for all future time. If we then push $k$ to $\infty$, we can simply ignore the cost that agent 2 has accumulated. Then the only cost contribution of interest is that of agent 1. Now, the best thing agent 1 can do is to go toward the origin, since any other $\sigma$ will incur a cost that is larger than $\sigma_0$, *i.e.*,

$$\lim_{k\to\infty} \frac{J_0(\sigma, s_k)}{J_\infty(s_k)} \geq \lim_{k\to\infty} \frac{J_0(\sigma_0, s_k)}{J_\infty(s_k)}.$$

If we take the supremum of the limit of both sides, we get

$$\limsup_{k\to\infty} \frac{J_0(\sigma, s_k)}{J_\infty(s_k)} \geq \limsup_{k\to\infty} \frac{J_0(\sigma_0, s_k)}{J_\infty(s_k)}.$$

Applying Lemma 2, we get

$$\limsup_{k\to\infty} \frac{J_0(\sigma, s_k)}{J_\infty(s_k)} \geq \lim_{k\to\infty} \frac{J_0(\sigma_0, s_k)}{J_\infty(s_k)} = 1.2.$$

*Lemma 4:* $r_j$ is 1 for all $j > 0$.

*Proof:* Suppose $j = 2$. If this ratio is indeed correct it would represent the minimum achievable value over all communication protocols. Thus in order to prove the validity of Lemma 4 we need to show that there is some protocol over which it is valid. Thus we propose the following protocol: before $l = \max\{\log_2(d_1(0)), \log_2(d_2(0))\}$ rounds of communication, both agents go toward their destination (*i.e.*, they 'race to the origin').

For a coding scheme where '00' represents '0', '01' represents '1' and '10' represents the end of the sequence it will take $l + 1$ rounds, for each agent i to fully transmit $d_i(k)$ to the other agent. At this point the problem reduces to one with perfect information and both agents can follow the optimal control law.

Clearly when $d_1(0)$ or $d_2(0)$ becomes large, the cost accumulated during the first $l$ rounds is negligible compared to the total cost. Hence $r_j = 1$. Similarly, for any $j > 2$, $r_j = 1$.

For any $j = \frac{1}{q} < 2$, the number of rounds required for each of the agents to communicate its $d_i(k)$ to the other agent becomes $2ql$ instead of $l$. However, since $q$ is fixed, when we let $d_1(0)$ approach $\infty$, the relation $r_j = 1$ remains valid.

## IV. Communication with Channel Errors

In this section we attempt to extend the results of the previous section to the case where communication is done over channels that may introduce errors. Suppose we are given a Binary Erasure Channel (BEC) with error probability $p$, $0 < p < 1/2$, *i.e.*, when a '1' is sent, a '1' is received with probability of $1 - p$ and an 'x' is received with probability $p$. Similarly, when a '0' is sent, a '0' is received with probability of $1 - p$ and an 'x' is received with probability $p$. We are again interested in the variation of $r_j$ as a function of $j$? It turns out that a relationship that is very similar to the one described in section III-B holds in a probabilistic form.

*Lemma 5:* For any given $\epsilon$, $0 < \epsilon < 1$, $r_j = 1$ with probability $1 - \epsilon$ for all $j > 0$.

*Proof:* As was justified in the proof of Lemma 4, without loss of generality, we assume that we can send two bits per time step and use the same coding scheme, *i.e.*, '00' represents '0', '01' represents '1' and '10' represents the end of the sequence. It follows then agent $i$ ($i \in \{1, 2\}$) takes $\log d_i(0) + 1$ rounds to send the distance information. For simplicity, let $d_i(0) = n$ for the following discussion.

In the noiseless case, this is all that is required to pass all the necessary information from one agent to the other. However, in the BEC case, any bit during the transmission could be erased by the channel. Hence in order to increase the probability that the receiving party gets the complete information, the sending party needs to send duplicate copies of its message. Therefore, after $\log n + 1$ rounds of communication, agent $i$ keeps sending the same information $T$ times ($T$ is to be determined). It is clear that on the receiving party side, all the odd bits it receives are indication bits, *i.e.*, a leading '0' means the next bit contains distance information and a leading '1' means the end of the sequence.

Now consider only the $T$ copies of the indication bits '1'. During the transmission, '1' could be erased so the receiving agent would get an 'x' instead, where 'x' indicates the error event. As the error probability of the channel is $p < 1/2$, then for any given $\epsilon > 0$, from the weak law of large numbers, there exits $T_1 > 0$, such that if $T > T_1$,

$$\text{Prob}\{\text{number of } 1'\text{s} = T(1-p)\} > 1 - \epsilon.$$

As $T(1-p) > Tp$, there must be at least two consecutive '1's that appear in the odd positions between which there are $2 \log n$ bits conveying the distance information. The receiving part can then break the $2T(\log n + 1)$ bits it has received into $T$ copies of the same information and form an information matrix of size $T \times \log n$. The rows of the matrix are the identical distance information extracted from the even bit positions and the columns represent the received bits for the same bit. For example, if $n = 2601$, then the binary expansion of it is 101000101001. The sending part would send '01' '00' '01' '00' '00' '00' '01' '00' '01' '00' '00' '01' '10' for $T$ times. The receiver part would then form a matrix such as the one depicted in Figure 3.

Notice that this communication scheme would fail if at least one column consists completely 'x's. Let this failing probability be $P_e$, then it is easy to show that

$$P_e \leq (\log n)p^T + C^2_{\log n} p^{2T} + ... + C^{\log n}_{\log n} p^{(\log n)T}, \quad (5)$$

where $C^k_{\log n} = \frac{(\log n)!}{(\log n - k)! k!}$. Therefore,

$$\begin{aligned} C^k_{\log n} p^{kT} &= \frac{(\log n)!}{(\log n - k)! k!} p^{kT}, \\ &\leq ((\log n)p^T)^k. \end{aligned} \quad (6)$$

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 1 | 0 | 1 | x | x | 0 | 1 | x | 1 | 0 | x | x |
| 1 | 0 | x | 0 | 0 | x | x | 0 | 1 | 0 | 0 | x |
| x | 0 | 1 | x | 0 | 0 | 1 | 0 | x | 0 | 0 | x |
| x | x | x | 0 | 0 | 0 | x | 0 | x | 0 | x | x |
| x | 0 | x | x | 0 | x | 1 | x | 1 | x | 0 | x |
| x | 0 | x | 0 | x | 0 | x | x | 1 | 0 | 0 | x |
| x | 0 | 1 | x | x | 0 | x | x | x | x | 0 | 1 |

Fig. 3. Information pattern at the receiver side

Substitute inequality (5) into (6) we get,

$$\begin{aligned} P_e &\leq \sum_{i=1}^{\log n} \left((\log n)p^T\right)^i \\ &= (\log n)p^T \frac{1 - ((\log n)p^T)^{\log n}}{1 - (\log n)p^T} \end{aligned}$$

Hence, by letting $T = \log n$, we get

$$\lim_{n \to \infty} P_e = 0.$$

Therefore, for large enough $n$ such that $\log n > T_1$, by letting $T = \log n$, the above communication scheme produces error less than any pre-specified number $\epsilon$ in a probabilistic sense. On the other hand, as we have

$$\lim_{n \to \infty} \frac{(\log n)^2}{n} = 0,$$

the cost accumulated during the $(\log n)^2$ rounds of communication is negligible compared to the total cost. Hence, $r_2 = 1$.

We are also interested in how $r_j$ changes when other types of communication channels with behavior more complicated than the BEC are introduced. The simplest example is the Binary Symmetric Channel (BSC). This channel is one such that when a '1' is sent, a '1' is received with probability of $1 - p$ and a '0' is received with probability $p$. Here $0 < p < 1/2$ and the relationship is completely symmetric for the case when a '0' is sent.

*Conjecture 6:* Similar to the BEC case, in BSC, we have that for any given $\epsilon$, $0 < \epsilon < 1$, $r_j = 1$ is of probability $1 - \epsilon$ for all $j > 0$.

While we believe conjecture (6) to be true, the proof is considerably more difficult because it crucially depends on the communication scheme which is used. This added complexity arises from the fact that with a BSC channel the indication bits are harder to decipher and thus cannot help us in the same way. Further, greater system knowledge or added constraints like an upper bound on the $d_i(0)$ are likely to be required. Current work is underway to verify this conjecture.

## V. Conclusions and Future Directions

As an initial step toward developing real-time information theory, we studied the problem of optimal two-agent one dimensional formation control, both with and without communication constraints.

In the first case, we assumed each agent could exchange unlimited information with the other. For this case, we showed that the control laws based on local decisions achieve the globally optimal solution.

In the second case, we limited the amount of information that could be exchanged by each agent during one round (*i.e.*, per time step). When the communication was limited, (a finite number of bits could be transmitted per round), we showed that this communication constraint could degrade system performance. In order to get a quantitative measure of this degradation we introduced the ratio, $r_j$ to describe the lower bound of the performance impairment. Here $j$ is the number of bits that can exchanged each round and $r_j$ is the ratio of cost with j bits of information to the cost with unlimited information. We found $r_0 = 1.2$ and $r_j = 1$ for any $j > 0$ as long as the initial position of each the agents was far enough away from its desired position. Similar results were obtained when we introduced a BEC model into the communication channels. However because the BEC channel behaves in a stochastic fashion only probabilistic results are possible.

In the near future, there are several interesting issues that we can study, such as:

- The development of a systematic method to generate a globally optimal solution, (based on local decisions), for a general multi-agent optimal control system, without communication constraints. Roughly speaking, this problem is very difficult to transfer into a convex problem, but it may be solvable using a dynamic programming approach.
- The addition of agent dynamical behavior to the system model. In this paper, the local rules are pretty simple and no dynamics are involved. The problem may become complicated if the agents had some interesting dynamics or if they were chasing moving targets. Then the dynamics of agents and the targets would affect system performance.
- The extension of the results to a system with a BSC model, especially computation of the performance ratio $r_j$.

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