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Faults Isolation For Nonlinear Hessenberg Systems

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Abstract— This paper deals with nonlinear upper and lower Hessenberg systems and the structure of this kind of systems is used to study the detectability and isolability of simultaneous faults. In particular structural analysis tools are used to evaluate the estimation redundancy degree of the Hessenberg form with two faults. From this characteristic, isolability conditions are deduced and specific parity equations are proposed. The key of the isolation issue is the determination of monitorable subsets of the bipartite graph which can be jointed, despite the faults. An illustrative example of a transport phenomena process shows how this result can be used in practice and the simplicity of the structural analysis to get generic properties for the isolability of faults in a dynamic system.

I. INTRODUCTION

In the last decade, the interest to determine reduced order models decoupled of unknown input for nonlinear systems has been increase. One of the reason is that the fault isolation task for dynamic nonlinear systems requires observable subsystems sensitive to some faults and robust to the rest. The solution of this issue is based on redundant information and the consistence between normal and actual process behavior obtained by on-line measurement data [1].

A general theory for the nonlinear fault detection and isolation, FDI, issues is still missing. There are attempts to overcome the difficulties using diverse tools and particular class of nonlinearities, as example [2], [3], [4], [5]. The conditions given with these formulations are difficult to test and satisfy in complex systems and do not help to study, which additional assumptions are required to get a residual generator.

Taking into account that the faulty modes generate usually a family of models with uncertain parameters and disturbances, to analyze the solvability of a FDI task for complex systems, the use of generic structural models is more appropriate than the analytical models. In particular, the Staroswiecki's school suggests the structural analysis theory [6] to study the monitorability of systems. This framework has two advantages: allows to cope with complex system, and requires few parameters information. To know if there is a solution, only the structure of the system plays an important role with this approach.

On the other hand, there are some physical models associated to biological and transport processes with structural coupled properties in the states which can be exploited to simplify the FDI task [7] and study its solvability. This fact motivated this contribution in which is shown that in the case of upper and lower nonlinear Hessenberg systems with a particular fault structure, exists a solution for the diagnosis task of two simultaneous faults. Moreover, the parity equations can be straightforward derived.

The paper is organized as follows. In section II, background of the structural analysis are presented. Section III is dedicated to the specific solvability conditions for the fault detection and isolation issues for the upper and lower Hessenberg form. An illustrative example is presented in section IV and some concluding remarks are given in section V.

II. STRUCTURAL ANALYSIS

Structural Analysis, SA, is a tool to explore properties of a system using theory of graphs [8]. Specifically, a structure graph describes which bonds exist between variables parameters and constraints in a system without describing their elements in details.

Consider the general state-space model

$$\dot{\theta} = 0 \tag{1}$$

$$\dot{x}(t) = g(x(t), u(t), \theta, f, d) \tag{2}$$

$$y(t) = h(x(t), u(t), \theta) \tag{3}$$

$$0 = m(x(t), u(t)) \tag{4}$$

with state $x \in \Re^n$, input $u \in \Re^m$, fault $f \in \Re^f$, disturbance $d \in \Re^d$, output $y \in \Re^p$, parameters $\theta \in \Re^q$, which can be considered as variables with known and unknown constant values, and the functions m static constraints. Then, the set of variables and constraints of system (1-4) is given respectively by

$$\mathcal{Z} = \mathcal{X} \cup \mathcal{U} \cup \mathcal{Y} \cup \Theta \cup \mathcal{F} \cup \mathcal{D} \quad \mathcal{C} = g \cup h \cup m$$

To analysis the structure of the general system given in (1-4) the following definitions taken from the theory of graphs will be used.

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Definition 1: The structure graph of a system with constraints set $\mathcal{C}$, variables set $\mathcal{Z}$ and parameter set $\Theta \subset \mathcal{Z}$ is a bipartite graph

$$G = \{\mathcal{C}, \mathcal{Z}, \mathcal{E}\} \tag{5}$$

with two sets of vertices whose set of edges $\mathcal{E} \subset \mathcal{C} \times \mathcal{Z}$ is defined by

$$(ci, zj) \in \mathcal{E} \Leftrightarrow zj \text{ appears in } ci \tag{6}$$

A representation of a structure graph is given by its incidence matrix where the rows of the matrix are the set of constraints, the columns the set of variables and the edge (ci, zj) is associated with a 1 in the intersection of row i and column j.

The basic tool for the SA is the concept of causal matching, which associates variables with constraints from which they can be evaluated.

Definition 2: A Matching $\mathcal{M}$ is a subset of edges $\mathcal{E}$ such that the projections on $\mathcal{C}$,

$$pc : \mathcal{E} \to \mathcal{C} \quad e = (\alpha, \beta) \mapsto pc(e) = \alpha \tag{7}$$

and on $\mathcal{Z}$,

$$pz : \mathcal{E} \to \mathcal{Z} \quad e = (\alpha, \beta) \mapsto pz(e) = \beta \tag{8}$$

are injective, i.e.

$$\forall \quad e_1, e_2 \in \mathcal{M} : \quad e_1 \neq e_2$$

$$\Rightarrow pc(e_1) \neq pc(e_2) \quad \text{and} \quad pz(e_1) \neq pz(e_2).$$

The matching process is in essence a way to specify which constraints in a system are needed to find a solution for its variables. Algorithms to match a system are reported in [9]. Causality means, in the graph context, that a matched variable with a unique value can be calculated from the constraint by whom is matched.

Definition 3: An alternated path of constraints and variables between nodes z_i, and z_j in a causal graph is denoted $P(z_i, z_j)$ and its length or ranking t is the number of constraints that crossed along the path.

Theorem 1: [8] Any bipartite graph of finite external dimension can be uniquely decomposed in three parts

1) Over-constrained: $\mathcal{S}^+ = (\mathcal{C}^+, \mathcal{Z}^+)$ such that $Q(\mathcal{C}^+) = Z^+$, and a complete matching exists on $\mathcal{Z}^+$ but not on $\mathcal{C}$ implying the existence of redundant information.
2) Just-constrained: $\mathcal{S}^0 = \left(\mathcal{C}^0, \mathcal{Z}^0\right)$ such that $Q(\mathcal{C}^0) = Z^0 \cup Z^+$, and a complete matching exists on both $\mathcal{Z}^0$ and on $\mathcal{C}^0$.
3) Under-constrained: $\mathcal{S}^- = (\mathcal{C}^-, \mathcal{Z}^-)$ such that $Q(\mathcal{C}^-) = Z^- \cup Z^0 \cup Z^+$ and a complete matching exists on $\mathcal{C}^-$ but not on $\mathcal{Z}^-$.

Definition 4: A variable z_j is reachable from z_i if $\exists\ P(z_i, z_j)$. A subset $\mathcal{Z}_i$ is reachable from $\mathcal{Z}_j$ if $\forall\ z \in \mathcal{Z}_i$ is reachable from an element of $\mathcal{Z}_j$.

Theorem 2: [6] Necessary and sufficient conditions for the system (1-4) to be structurally observable under derivative causality. i.e. (constraints $c_i : \dot{x}_j = \frac{dx_j}{dt}$) is that,

1) All the elements of the unknown set $\mathcal{X}$ are reachable from the known set $\mathcal{K}$.
2) The just-constrained and over-constrained subgraphs $\mathcal{S}^0$ and $\mathcal{S}^+$ are causal.
3) The under-constrained subgraph is empty.

Since, the key of any FDI procedure is the redundancy to evaluate the unknown variables in the model and this is captured in $\mathcal{S}^+$, the SA for our study will be based on this subsystem for each particular set of faults.

A. Monitorability of a Graph

Starting from the structure of (5) with $\mathcal{Z} = \mathcal{K} \cup \mathcal{X}$, one can study the detection and isolation ability w.r.t a set of faults $\mathcal{F}_1$ and $\mathcal{F}_2$ on complex systems.

Definition 5: A Fault $\mathcal{F}$ in G is defined as a change in a set of constraints $\mathcal{C}_f \subset \mathcal{C}$ which can modify the members of $\mathcal{K}$, or $\mathcal{X}$.

Considering the subset of observable variables of the structure $\mathcal{X}_{obs}$, one can define a measure of the redundancy in a graph as follows.

Definition 6: The estimation redundancy degree of an observable set $\mathcal{X}_{obs} \subset \mathcal{X}$ w.r.t a set $\mathcal{C}_i$, denoted $\delta(\mathcal{X}_{obs}, \mathcal{C}_i)$, is defined by the number of estimation subgraphs whose target is $\mathcal{X}_{obs}$ and which are not affected by the $\mathcal{C}_i$. This degree characterizes the number of system configurations or paths which still allow to estimate $\mathcal{X}_{obs}$.

Definition 7: A subsystem is monitorable w.r.t $\mathcal{C}_m$ if its set of unknown variables $\mathcal{X}$ can be determined by paths $P(\mathcal{K})$, whether $\mathcal{C}_m$ satisfied or not.

One can say that the monitorability is related w.r.t a subset of constraints which in the case of diagnosis is $\mathcal{F}$ and the evaluation of $\mathcal{X}$ implies the existence of redundancy in the system.

Theorem 3: [6] Two equivalent necessary conditions for a fault $\mathcal{F}$ to be monitorable in a subsystem are:

1) $\mathcal{X}_f = \mathcal{Q}(f) \cap \mathcal{X}$ is structurally observable in the subsystem $(\mathcal{C} \backslash \mathcal{C}_f, \mathcal{Z})$.
2) $\mathcal{F}$ belongs to the structurally observable over-determined part of the subsystem $(\mathcal{C}, \mathcal{Z})$.

Since, in the traditional FDI framework, one speaks about the fault detectability instead of monitorability, here one defines the detectability in the context of SA as follows.

Definition 8: Consider the structure (5) and the subset of unknown variables $\mathcal{X}_f = \mathcal{Q}(f) \cap \mathcal{X}$ appearing in $\mathcal{C}_f$, the fault f is detectable, if at least one member of $\mathcal{X}_f$ is reachable from the known set $\mathcal{K}$ without using the set of constraints $\mathcal{C}_f$.

This definition can be considered as a weak case of the structural monitorability. Two conditions to study the detectability problem of $\mathcal{F}$ can be derived on the base of this.

Condition 1: Consider the structure G and the subset of unknown variables $\mathcal{X}_f = \mathcal{Q}(f) \cap \mathcal{X}$ appearing in the subset $\mathcal{C}_f$, a sufficient condition for the detectability of fault $\mathcal{F}$, is

$$\mathcal{X}_f \subseteq \overline{\mathcal{X}}_{obs}$$

with $\overline{\mathcal{X}}_{obs}$ the observable subset of the structure without $\mathcal{C}_f$. Moreover, any member of subset $\mathcal{C}_f$ can be used to generate a residual.

Demonstration: If the unknown subset $\mathcal{X}_f$ appearing in the constraints $\mathcal{C}_f$ can be estimated without them, Theorem 3 satisfies and the structure is monitoreable and then is detectable with respect to $\mathcal{F}$. Moreover, putting the subset $\mathcal{X}_f$ into any member of subset $\mathcal{C}_f$, one can generate a residual testing whether the result is ZERO or not.

Condition 2: Consider the structure G and the unknown variables subset $\mathcal{X}_f = \mathcal{Q}(f) \cap \mathcal{X}$ appearing in the subset $\mathcal{C}_f$, necessary conditions for the detectability of fault $\mathcal{F}$ are

1) the existence of a no empty subset $\hat{\mathcal{X}}_f \subseteq \mathcal{X}_f$ which can be evaluated without passing by $\mathcal{C}_f$;
2) the existence of a no empty subset $\hat{\mathcal{C}}_f \subseteq \mathcal{C}_f$ such that at least one member of $\hat{\mathcal{X}}_f$ can be evaluated using this subset and generating an output ZERO in normal conditions.

Moreover, the inconsistency in fault condition, between the calculation of any member of $\hat{\mathcal{X}}_f$ passing by $\mathcal{C}_f$ and avoiding it, generates a residual.

Demonstration: If 1) satisfies and at least one member of $\hat{\mathcal{X}}_f$ is reachable passing by subset $\mathcal{C}_f$ means that there is a redundancy, since two paths exist to calculate at least one unknown variable in normal condition. Moreover, exists an inconsistency in the calculation under fault condition. Then, testing whether the difference between the calculation is ZERO or not generate a residual.

Because, faults isolation demand a particular handling in the context of FDI, the robust monitorability of a subsystem with respect to the non-interest faults must be considered as disturbance $\mathcal{D}$. In the framework of SA, two suggestions have been reported; one way consists to define the augmented unknown variables $\mathcal{X} \cup \mathcal{D}$ and the other way is to start the analysis considering the subsystem $\mathcal{C} \backslash \mathcal{C}_d$ (see [10]).

III. FDI PROPERTIES OF A HESSENBERG FORM

Consider the differential system defined on a domain $\Omega \in \Re^n$

$$\Sigma \begin{cases} \dot{x} = f(x,u) + F_1(x)p_1 + F_2(x)p_2 \\ x(0) = x_0 \quad u \in U \\ y = h(x) \in \Re^2 \end{cases} \tag{9}$$

where the following properties satisfy:

1) The system is both strictly linked lower and upper Hessenberg. This means for any indexes (i,j) such that

$$j > i+1, \quad \frac{\partial f_i(x,u)}{\partial x_{j+1}} = 0, \quad \frac{\partial f_i(x,u)}{\partial x_{i+1}} \neq 0$$

and if

$$j < i+1, \quad \frac{\partial f_i(x,u)}{\partial x_{j-1}} = 0, \quad \frac{\partial f_i(x,u)}{\partial x_{i-1}} \neq 0$$

2) The system has only 2 outputs and for any $x \in \Omega$, $y_1 = h(x_1)$ with $\frac{dh(x_1)}{dx_1} \neq 0$ and $y_2 = h(x_n)$ with $\frac{dh(x_n)}{dx_n} \neq 0$. These output properties are called upper and lower measured respectively.
3) Independent on the set of admissible $u(t)$, the existence of a fault p_i produces a deviation of the output such that the $\|y(t) - y_0(t)\| \neq 0$, where $y_0(t)$ is the output of the system without fault p_i. This property is called detectability of the fault [11].
4) The fault distribution vectors $F_1(x)$ and $F_2(x)$ have $n-1$ zero elements, with only the component i and j different from zero respectively.

Then, the objective is now to study the monitorability of system (9) and to determine measurements point to guaranty the detectability and isolation ability for at most two faults in the system.

The start point of the SA are the constraints

$$\mathcal{C} = \{c_1, ..., c_n, d_1, ..., d_n\},$$

obtained from each state equation of system (9)

$$\dot{x}_i = f_i(x,u) + F_{1,i}(x)p_1 + F_{2,i}(x)p_2 \tag{ci}$$

and the members

$$\dot{x}_i := \frac{dx_i}{dt} \tag{di}$$

the known variables

$$\mathcal{K} = \{x_1, x_n, u_1, u_2\},$$

the unknown variables

$$\mathcal{X} = \{x_2, x_3, ..., x_n, \dot{x}_1, \dot{x}_2, ...\dot{x}_n\},$$

and the unknown variables (faults)

$$\Theta_f = \{p_1, p_2\}$$

To illustrate the incidence matrix form, Table 1 shows its for the special case of $n = 7$.

In absence of fault $|\mathcal{C}| > |\mathcal{X}|$, and exist diverse paths to evaluate $\mathcal{X}$.

Considering the properties (1) and (2) of the system, the alternated paths obtained from each output are

$$P_1 : x_1 - d_1 - \dot{x}_1 - c_1 - x_2 - d_2 - \dot{x}_2 - c_2 - x_3$$
$$... - x_n - d_n - \dot{x}_n \tag{10}$$

↘	$x1$	x_2	x_3	x_4	x_5	x_6	x_7	$\dot{x}_1$	$\dot{x}_2$	$\dot{x}_3$	$\dot{x}_4$	$\dot{x}_5$	$\dot{x}_6$	$\dot{x}_7$	f_1	f_2	f_3
c_1	1	1						1									
c_2	1	1	1						1						1		
c_3		1	1	1						1							
c_4			1	1	1						1					1	
c_5				1	1	1						1					
c_6					1	1	1						1				1
c_7						1	1							1			
d_1	1							1									
d_2		1							1								
d_3			1							1							
d_4				1							1						
d_5					1							1					
d_6						1							1				
d_7							1							1			

TABLE I

Incidence Matrix for the example of section 4

$$P_2 : x_n - d_n - \dot{x}_n - c_n - x_{n-1} - d_{n-1} - \dot{x}_{n-1} \\ ... - c_2 - x_1 - d_1 - \dot{x}_1 \quad (11)$$

and allow to calculate $\mathcal{X}$ with redundancy. Since the whole $\mathcal{X}$ can be calculated with one path, the system is structurally observable with any of the two outputs.

Moreover, considering P_1 and P_2 simultaneously there exist a estimation redundancy degree $\delta(\mathcal{X}, \phi) = 4$ obtained with unmatched constraints

$$\{c_i, d_{i+1}\} \text{ or } \{c_{i+2}, d_{i+1}\} \quad (12)$$

Since, the system graph $\mathcal{S}^+ \cup \mathcal{S}^0$ is causal for any intersection variable i, and $S^- = \phi$, by Theorems (2) and (3), the structure is observable and the sets (12) guaranty its monitorability with four parity equations. If more than two sensors exist, the redundancy of the system is increased and the extra measures can be used to robustify the diagnosis.

Now if one fault p_1 exists, only constraint (ci) must be eliminated from the paths. The estimation redundancy degree $\delta(\mathcal{X}, c_i)$ is reduced to two and the intersection of the paths P_1 and P_2 can be made eliminating the subpath in which c_i appears. Then, the whole system remains monitorable and the unmatched constraints can be used as parity equations. These results have been as well obtained for the model of a pipeline with leaks by geometric approach. However, the analysis with this approach demands specific parameters for the model, more technical background and the assumption of the existence of an observer [12].

A. Two faults isolation case

More interesting to study is the case of two simultaneous faults, since $|\mathcal{C} \backslash \{c_i, c_j\}| = |\mathcal{X}|$ and then there is not redundancy and the whole system is not monitorable; there is only a FDI solution with particular configurations of faults.

Then, the objective is to determine a subset, which allows to generate a residual sensitive only to one fault. In other words, one searches for detectable subgraphs $\mathcal{X}_m \subset \mathcal{X}$ in which Condition 2 satisfies for each particular combination of faults.

1) Faults pair with $\mathcal{C}_f = \{c_i, c_{i+k}\}$ and $k \leq 2$: Considering property (4) and that the faults p_1 and p_2 affect directly at most the state variables pair (x_i, x_{i+k}), the constraints $\{c_{i-1}, ..., c_{i+3}\}$ associated to these variables are at most given by

$$\dot{x}_{i-1} = f_{i-1}(x_{i-2}, x_{i-1}, x_i, u) \quad \text{(ci-1)}$$

$$\dot{x}_i = f_i(x_{i-1}, x_i, x_{i+1}, u) + F_1(x)p_1 \quad \text{(ci)}$$

$$\dot{x}_{i+1} = f_{i+1}(x_i, x_{i+1}, x_{i+2}, u) \quad \text{(ci+1)}$$

$$\dot{x}_{i+2} = f_{i+2}(x_{i+1}, x_{i+2}, x_{i+3}, u) + F_2(x)p_2 \quad \text{(ci+2)}$$

$$\dot{x}_{i+3} = f_{i+3}(x_{i+2}, x_{i+3}, x_{i+4}, u) \quad \text{(ci+3)}$$

From this set of constraints and paths (10) and (11), one can calculate the unknown variables until point $\dot{x}_i$ starting from x_1 in $P1$, and until $\dot{x}_{i+2}$ starting from x_n in $P2$. These evaluations disregard the pair (c_i, c_{i+k}). Since, the value of x_{i+1} at the time that the faults occur (t_f^+) can be estimated, one can get the evolution

$$\frac{d\hat{x}_{i+1}}{dt} = f_{i+1}(x_i, \hat{x}_{i+1}, x_{i+2}, u) \quad \hat{x}_{i+1}(t_f^+) = x_{tf} \quad (13)$$

increasing the cardinality of $\mathcal{X}$ and getting $|\mathcal{C} \backslash \mathcal{C}_f| = |\mathcal{X}|$. Therefore, Condition (2) satisfies and using constraint (ci) one can define as parity equation for fault p_1

$$r_1(t) = -\hat{x}_{i+1} - x_{i+1}\,|_{P1} \quad (14)$$

considering path $P1$ with c_i for the calculation $x_{i+1}\,|_{P1}$ in the residual r_1, and as parity equation for fault p_2

$$r_2(t) = -\hat{x}_{i+1} - x_{i+1}\,|_{P2} \quad (15)$$

considering now path $P2$ and c_{i+k} for the calculation $x_{i+1}\,|_{P2}$ in the residual r_2.

2) Faults pair with $\mathcal{C}_f = \{c_i, c_{i+k}\}$ and $k > 2$: There is not redundancy in the paths and the subgraph is non monitorable. Moreover, the number of constraints associated to the faults increase, and two break points appear in each path which cannot be related by an intermediary constraint. Then, there is not path to evaluate the unknown variables between the x_i and x_{i+k}. This means, the second part of Condition (2) does not satisfy. New sensors, which joint the paths are required. This means, some variables between constraints c_i and c_{i+k} must be measured to isolate a couple of faults.

One can conclude, that if the variables associated to the faults are related and can be compacted in a block of the path, as case 1, one can find parity equations for the isolation task. On the contrary, if the faults cannot be compacted in a subsystem, multi-breaks appear in the paths, and faults cannot be isolated without additional sensors.

IV. EXAMPLE

Consider a nonlinear model of a transport process associated with a fluid in a pipeline divided in four sections which boundaries are given by the faults positions in the line λ_1, λ_2 and λ_3 and faults values $f_1(\lambda_1, x_2)$, $f_2(\lambda_2, x_4)$ and $f_3(\lambda_3, x_6)$ respectively taken from [13]

$$\dot{x}_1 = a_1(u_1 - x_2) - \mu x_1^2 \quad \text{(c1)}$$
$$\dot{x}_2 = a_2(x_1 - x_3 - f_1(\lambda_1, x_2)) \quad \text{(c2)}$$
$$\dot{x}_3 = a_1(x_2 - x_4) - \mu x_3^2 \quad \text{(c3)}$$
$$\dot{x}_4 = a_2(x_3 - x_5 - f_2(\lambda_2, x_4)) \quad \text{(c4)}$$
$$\dot{x}_5 = a_1(x_4 - x_6) - \mu x_5^2 \quad \text{(c5)}$$
$$\dot{x}_6 = a_2(x_5 - x_7 - f_3(\lambda_3, x_6)) \quad \text{(c6)}$$
$$\dot{x}_7 = a_1(x_6 - u_2) - \mu Q_4^2 \quad \text{(c7)}$$
$$\dot{x}_1 = \frac{dx_1}{dt} \quad \text{(d1)}$$
$$\dot{x}_2 = \frac{dx_2}{dt} \quad \text{(d2)}$$
$$\dot{x}_3 = \frac{dx_3}{dt} \quad \text{(d3)}$$
$$\dot{x}_4 = \frac{dx_4}{dt} \quad \text{(d4)}$$
$$\dot{x}_5 = \frac{dx_5}{dt} \quad \text{(d5)}$$
$$\dot{x}_6 = \frac{dx_6}{dt} \quad \text{(d6)}$$
$$\dot{x}_7 = \frac{dx_7}{dt} \quad \text{(d7)}$$

where the flows in the sections are associated to x_i for $i = 1, 3, 5, 7$ and the pressures between each section are the variables x_i for $i = 2, 4, 6$; the known variables are the pressure input $[u_1\ u_2]$ and the flow output $[x_1\ x_7]$. The resulting incidence matrix is shown in Table 1. The SA is used to study the possibility to isolate two leaks in the line. From the incidence matrix one can see that the model satisfies the Upper and Lower Hessenberg form, and the above derivate condition to test the detectability can be straightforward used.

Considering the two leaks case, located in the first and second section of the line with only sensors at the ends of the pipeline, the detectability conditions satisfy and one can selects the parity equations

$$r_1(t) = -\hat{x}_3 - x_3\,|_{P1} \qquad (16)$$

for fault $f_1(.,.)$ and

$$r_2(t) = -\hat{x}_3 - x_3\,|_{P2} \qquad (17)$$

for the second one $f_2(.,.)$. Fig. 1 shows the residuals evolution without leaks. One can see that both residuals remain around zero.

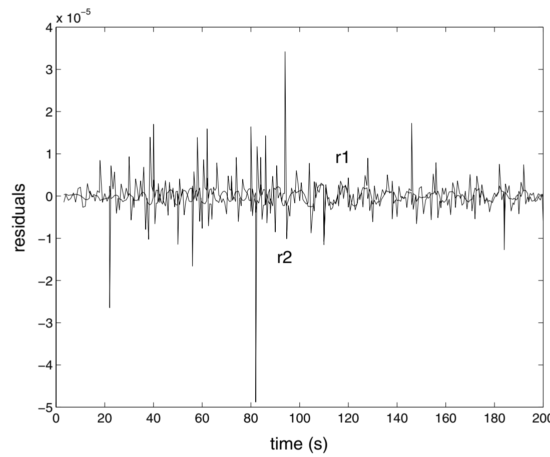

Fig. 1. Residuals in normal conditions

Fig. 2 shows the results when only a leak appears at 50s in x_2, one can see that the residual associated to the second leak in the state equation of x_4, remains around zero.

Fig. 3 shows the results when only a leak appears at 50s in x_4. Here, one can see that the residual associated to the first leak in the state equation x_2 remains around zero.

Fig. 4 shows the residuals when both leaks appear simultaneously in states x_2 and x_4. The magnitude of the both residual deviated from zero at the time the leaks appear.

Since the structure of the system (c1-c7) is symmetric with respect to the output, one gets similar residuals for the combination of faults 2 and 3.

Considering the faults associated to the constraints c_2 and c_6, one has the case $k > 2$ and there is no way to generate a residual for each fault. Therefore the faults are not isolable. Similar conclusion has been obtained using Geometric Approach for the case of leaks in a pipeline [12].

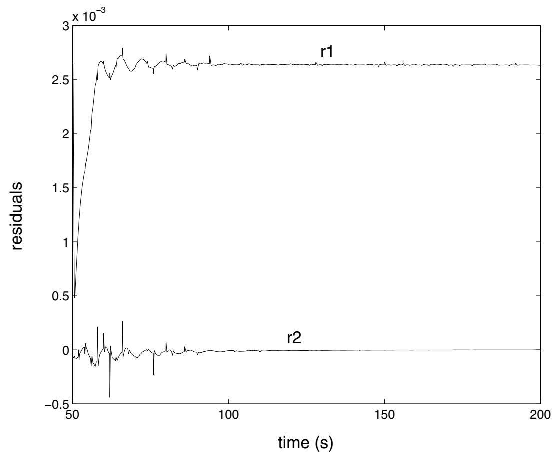

Fig. 2. Residuals with fault combination (1,0)

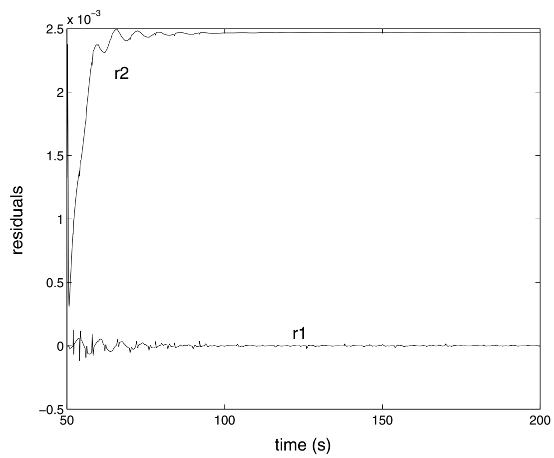

Fig. 3. Residuals for fault combination (0,1)

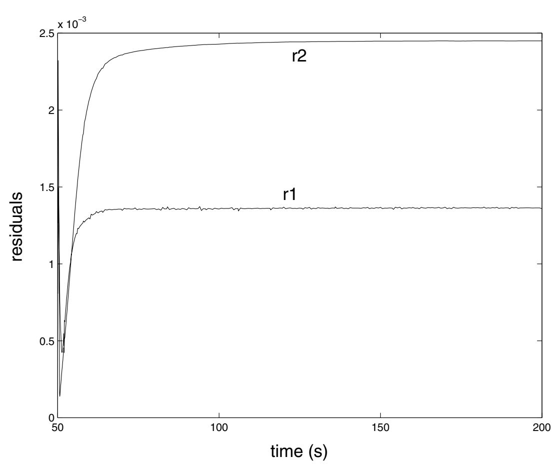

Fig. 4. Residuals for fault combination (1,1)

This example show the simplicity of the Structural Analysis to analysis which possible solutions exists for a FDI problem, before a particular procedure is applied to design the diagnosis system.

V. CONCLUSIONS

It has been introduced, a very general framework to study the properties of the structure of strictly upper and lower Hessenberg nonlinear systems for FDI issues. In particular, the structural analysis helps to identify where sensors must be added to improve the reliability of large systems. In conclusion, the Structural Analysis has so far proved to be a useful aid to design fault detection systems. This analysis allows to determine very general properties of nonlinear systems which are useful for fault tolerant control design. Moreover one can easily determine which kind of fault are impossible to detect or to isolate from the structure of the system. This analysis of the monitorability and redundancy of a model must be done before one selects the appropriated FDI procedure for a particular problem.

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