2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

An Actuator Fault Isolation Strategy for Linear and Nonlinear Systems

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***Abstract*— This paper investigates actuator fault isolation problem for linear systems. Then, as an extension, actuator fault isolation problem for nonlinear systems is considered next. For both linear and nonlinear systems, two cases are studied. In the first case, we assume that all states are available. The second case assumes that only outputs are available. To accomplish fault isolation, we use a bank of observers for all possible faulty models. The paper considers constant actuator faults, that is, the outputs of some actuators are stuck at fixed undesirable constant values. The fault isolation strategy is based on combining the conventional observer design techniques with adaptation techniques. Based on the designed observers, a bank of residuals are defined correspondingly. The actuator faults can be isolated if only one residual goes to zero while the others do not. The faulty model with residual approaching zero identifies the faulty actuators. For linear and nonlinear systems with all states available, new sufficient conditions for fault isolation are derived, which require only that the distribution matrices of the actuators are of full column rank. For linear and nonlinear systems with only outputs available, new sufficient conditions for fault isolation are also derived, which require additional conditions besides the full column rank condition of the distribution matrices of the actuators. Some simulation results are given to show the effectiveness of the proposed fault isolation methods.**

I. INTRODUCTION

Model-based fault diagnosis has been studied for over thirty years, and large amount of results have been published in the literature, see for example, related references in survey papers by Willsky [1], Iserman [2], Gertler [3], Frank [4], Patton [5], and Frank [6].

Fault detection is the first step for fault diagnosis. It can inform the presence of faults, and the early detection of faults is very important for the safety of real systems. After the detection of faults, it is often desired to locate the faults, which is the task of fault isolation and is very crucial for fault accommodation. Fault isolation is generally a more difficult task than fault detection and this is the motivation for this work.

Of the early results on fault isolation, two schemes based on a bank of observers are noteworthy. One is called *dedicated observer scheme* and is proposed by Clark [16]. In this scheme, to isolate one fault among N possible faults, N observers are designed to generate N residuals and the ith residual is designed only sensitive to the ith fault but decoupled from all other faults, where $1 \leq i \leq N$. The scheme can only be designed for some special cases and can only be used to detect and isolate one single fault.

The other one is called *generalized observer scheme* and was proposed by Frank [4]. Similarly, N observers are also designed to generate N residuals. The idea different here is to make the ith residual sensitive to all faults except the ith one. Once such residuals can be designed, the decision for fault isolation becomes straightforward. In our paper, we adopt this scheme to do fault isolation.

Another line of work dealing with actuator faults is that of fault-tolerant control strategy. Usually, fault-tolerant control is accomplished by reconfiguring the controller to accommodate the possible actuator faults. Results in this direction can be found in [9], [10], [11],[13]-[15], to name only a few. No fault diagnosis was performed in any the above mentioned results except in [11]. However, the need for faulty actuator isolation is still there.

In [12], a robust isolation scheme was proposed, which can be used for actuator fault isolation. To use the scheme, all states must be available and known compact sets including the unknown parameters are also assumed available. Wang and Daley [8] proposed a method for the faults that can be modeled by $\Theta^T u$ with Θ a constant unknown parameter matrix. To achieve fault isolation, excitation signals may be required to obtain accurate parameter estimation. Also, some actuators faults may not be defined that way. Although they called their observers FDI observers in [11], the observers are only used for fault accommodation. Because adaptive approach is applied, there are no guarantees that the estimated parameters are approaching to their true values. Therefore, whether faults can be isolated is not clear. Moreover, all states are required to be available which in practice is a restrictive requirement.

In this paper, we investigate constant actuator fault isolation problems for a class of linear systems first. Sufficient actuator fault isolation conditions are derived for the case that all states are measured and the case that only outputs are available. The actuator fault isolation schemes for linear systems are then extended to a class of nonlinear systems.

The rest of paper is arranged as follows. In Section 2, we first formulate our fault isolation problem for linear systems. Then a fault isolation scheme is proposed for the case that all states are available and a sufficient condition for fault isolation is derived. Finally, a fault isolation scheme is designed for the more difficult case that only outputs are available and a sufficient condition for fault isolation is also obtained. In Section 3, we extend the schemes and results for linear systems to nonlinear systems in the same way.

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In Section 4, some simulation examples are given to show the effectiveness of our actuator fault isolation schemes. Conclusion and remarks are made in the last section.

II. ACTUATOR FAULT ISOLATION FOR LINEAR SYSTEMS

In this section, we give the actuator fault isolation schemes for a class of linear systems for the case that all states are available and the case that only outputs are available. Sufficient conditions for actuator fault isolation are given for the two cases.

A. Actuator Fault Isolation for Linear Systems

1) Case I: State vector is available: Consider the following linear system

$$\dot{x} = Ax + Bu \tag{1}$$

where $x \in R^n$ is the state vector and $u \in R^m$ is the input vector (the output of actuators). The matrix B (function one $B(x)$ in later section) before the control u is called the distribution matrices of the actuators in this paper.

Throughout this paper, we assume that only constant actuator faults can occur, that is, $u_j^f(t) \equiv \theta_j$ for $t \geq t_f$ and $limit_{t\to\infty}|u_j(t) - \theta_j| \neq 0$, where θ_j is a constant and u_j^f is the actual output of the jth actuator when it is faulty, $j \in 1, 2, \cdots, m$ while $u_j(t)$ is the expected output when it is healthy.

Remark 1: This type of faults can often occur in practice. For example, in aircraft control systems, one may encounter these faults [13], [11]. Another reason for considering this type of faults is that adaptation approaches can be incorporated into the design of observers.

Throughout this paper, we consider the following fault isolation(FI) problem.

It is desired to design an observer based scheme to isolate the faulty actuators when faults have occurred.

For simplicity, we consider the case that only one single actuator is faulty at one time because extensions to multi-faults situation are straightforward.

Since all states are available, the detection of faults is very easy. Actually, if any state is not acting as desired, it can be concluded that faults have occurred. The problem is to find out which actuator is faulty.

If any actuator is faulty, the resulting system from (1) is called faulty model in this paper. Since we have m actuators in total, we have m possible faulty models. If the lth actuator is faulty, the corresponding faulty model is given below.

$$\begin{aligned} \dot{x} &= Ax + \sum_{j\neq l} b_j u_j + b_l u_l^f(t) \\ &= Ax + \sum_{j\neq l} b_j u_j + b_l \theta_l \end{aligned} \tag{2}$$

where $B = (b_1 \cdots b_m)$. The measured states are the states of this model.

By using adaptation technique, we design a bank of observer for all possible faulty models below.

$$\begin{aligned} \dot{\hat{x}}_i &= H(\hat{x}_i - x) + Ax + \sum_{j\neq i} b_j u_j + b_i \hat{\theta}_i \\ \dot{\hat{\theta}}_i &= -2\gamma \tilde{x}_i^T P b_i, \ 1 \leq i \leq m \end{aligned} \tag{3}$$

where $\tilde{x}_i = \hat{x}_i - x$, H is Hurwitz matrix which can be chosen freely and γ is a design constant, and P is a positive definite matrix which is a solution of the following matrix equation.

$$H^T P + PH = -Q \tag{4}$$

where Q is a chosen positive definite matrix.

We have the following result.

Theorem 1: If the lth actuator is faulty, then for $i = l$, we have

$$lim_{t\to\infty}\tilde{x}_i = lim_{t\to\infty}\tilde{x}_l = 0 \tag{5}$$

and for $i \neq l$, we have

$$\dot{\tilde{x}}_i = H\tilde{x}_i + b_l(u_l - \theta_l) - b_i(u_i - \hat{\theta}_i) \tag{6}$$

Proof:

For $i = l$, it follows from (2) and (3) that

$$\dot{\tilde{x}}_l = H\tilde{x}_l + b_l\tilde{\theta}_l \tag{7}$$

Let's choose a Lyapunov function as

$$V = (\tilde{x}_l)^T P \tilde{x}_l + \frac{1}{2\gamma}\tilde{\theta}^2 \tag{8}$$

where $\tilde{\theta} = \hat{\theta}_l - \theta_l$.

Differentiate the above Lyapunov function with respect to the time t, by using (7) and the second equation in (3), it is easy to derive that

$$\begin{aligned} \dot{V} &= -\tilde{x}_l^T Q \tilde{x}_l \\ &+ \frac{1}{\gamma}\tilde{\theta}[\dot{\hat{\theta}}_l + 2\gamma(\tilde{x}_l)^T P b_l] \\ &= -\tilde{x}_l^T Q \tilde{x}_l \leq 0 \end{aligned} \tag{9}$$

Using the above result, it is now a standard technique to show that $lim_{t\to\infty}\tilde{x}_l = 0$.

For $i \neq l$, on one hand, the faulty model is

$$\dot{x} = Ax + \sum_{j\neq i,l} b_j u_j + b_i u_i + b_l \theta_l \tag{10}$$

On the other hand, the ith observer is

$$\begin{aligned} \dot{\hat{x}}_i &= H(\hat{x}_i - x) + Ax + \sum_{j\neq i,l} b_j u_j + b_l u_l + b_i \hat{\theta}_i \\ \dot{\hat{\theta}}_i &= -2\gamma(\tilde{x}_i)^T P b_i \end{aligned} \tag{11}$$

Hence, we have

$$\dot{\tilde{x}}_i = H\tilde{x}_i + b_l(u_l - \theta_l) - b_i(u_i - \hat{\theta}_i) \tag{12}$$

This completes the proof. ■

Let's define residuals as $r_i(t) = \|\tilde{x}_i(t)\|^2$ for $1 \leq i \leq m$. If there exists $l(1 \leq l \leq m)$ such that $limit_{t\to\infty}r_l(t) = 0$ while $limit_{t\to\infty}r_i(t) \neq 0$ for all $i \neq l$, then the lth actuator is faulty. The question now is under what conditions we can achieve this? To answer this question, a sufficient condition for actuator fault isolation based on the above theorem is given as follows.

Theorem 2: If the distribution matrix of the actuators B is of full column rank, that is, $b_1, \cdots, b_m$ are independent, then the fault actuator isolation can be achieved by evaluating the residuals resulting from the observers (3).

Proof: After faults occur, we know from Theorem 1 that at least the residual corresponding to the faulty model, that is, $r_l(t)$ will go to zero. If we can show $r_i(t)$ for all $i \neq l$ do not tend to zero, then we are done.

Now for any $i \neq l$, we have b_l and b_i are independent because B is of column rank. This implies that $b_l(u_l - \theta_l) - b_i(u_i - \hat{\theta}_i)$ is nonzero for all t if $u_l - \theta_l \neq 0$. Note that we have $limit_{t\to\infty}|u_l - \theta_l| \neq 0$ by the definition of faults, we conclude that

$$limit_{t\to\infty} b_l(u_l - \theta_l) - b_i(u_i - \hat{\theta}_i) \neq 0$$

for all $i \neq l$. This implies that $r_i(t)$ does not tend to zero for any $i \neq l$.

This completes the proof.

■

Remark 2: Note that Theorem 2 only presents a sufficient condition for fault isolation. If B is not of full column rank, it does not mean the fault can not be isolated. It only means our scheme might fail and other techniques should be used.

Remark 3: The design of observer bank is similar to that in Boskovic and Mehra [11], where the authors focus on fault accommodation. However, the focus here is on fault isolation, and the adaptive laws for unknown parameters we use are different from theirs.

2) *Case 2: Output vector is available:* Consider the following linear system

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{13}$$

where $y \in R^q$ is the output.

For this system, only the output y is available. This problem is more difficult than the one we considered in the first case because $\tilde{x}$ is not available for the adaptive laws. To design a bank of fault isolation observers, we need the following assumption, which will be used later.

Observer Existence Assumption(OEA): There exist two positive definite matrices P and Q and two matrices L and D with suitable dimensions such that

$$\begin{aligned} (A - LC)^T P + P(A - LC) &= -Q \\ PB &= (DC)^T \end{aligned} \tag{14}$$

Remark 4: The above equation has exactly the same form as that proposed for sliding mode observers in [17]. A systematic algorithm for finding P,Q,L and D can be found in [18] and [19].

As in last subsection, we have m possible faulty models. If the lth actuator is faulty, the corresponding faulty model is given below.

$$\begin{aligned} \dot{x} &= Ax + \sum_{j\neq l} b_j u_j + b_l \theta_l \\ y &= Cx \end{aligned} \tag{15}$$

The measured output is the output of the above faulty model.

We design a bank of observer for all possible faulty models below.

$$\begin{aligned} \dot{\hat{x}}_i &= A\hat{x}_i - L(\hat{y}_i - y) + \sum_{j\neq i} b_j u_j + b_i \hat{\theta}_i \\ \dot{\hat{\theta}}_i &= -2\gamma(e_{y_i})^T d_i, \ 1 \leq i \leq m \end{aligned} \tag{16}$$

where $\hat{y}_i = C\hat{x}_i$,$e_{y_i} = \hat{y}_i - y = C\tilde{x}_i$, γ is a design constant, and P, Q, L and $D^T = (d_1 \cdots d_m)$ satisfy (14).

Similarly as Theorem 1, we can prove the following result.

Theorem 3: If the lth actuator is faulty, then for $i = l$, we have

$$lim_{t\to\infty}\tilde{x}_i = lim_{t\to\infty}\tilde{x}_l = 0 \tag{17}$$

and for $i \neq l$, we have

$$\dot{\tilde{x}}_i = (A - LC)\tilde{x}_i + b_l(u_l - \theta_l) - b_i(u_i - \hat{\theta}_i) \tag{18}$$

Unlike case I, we define residuals as $r_i(t) = \|e_{y_i}(t)\|^2$ for $1 \leq i \leq m$. If only one residual goes to zero while the others do not, then the faulty actuator is identified as the one corresponding to the residual approaching zero. Now, by using Theorem 3, we can give a sufficient condition for actuator the FI problem as follows.

Theorem 4: If CB is of full column rank, then, under OEA, the actuator FI problem can be solved by evaluating the residuals resulting from the observers (16).

Proof: If the lth actuator is faulty, we know from Theorem 3 that $\tilde{x}_l$ will tend to zero. Hence, by definition, $r_l(t)$ will go to zero. If we can show $r_i(t)$ for all $i \neq l$ do not tend to zero, then we are done.

Let $G_i(s) = C(sI - A_L)^{-1}b_i, 1 \leq i \leq L$, where $A_L = A - LC$, then for any $j \neq i$, $G_i(s)$ and $G_j(s)$ are independent because Cb_i and Cb_j are independent. Therefore $r_i(t) = G_l(s)(u_l - \theta_l) - G_i(s)(u_i - \hat{\theta}_i)$ will generally not tend to zero because $u_l - \theta_l$ does not tend to zero. This completes the proof. ■

Remark 5: The condition that CB is of full column rank is a natural extension of the condition that B is of full column rank. Similar remarks can be made here as in Remark 2

Remark 6: The design of the bank of observers here does not require all states to be available as in Case I, and in [11]. Also, the adaptive laws for unknown parameters we use are different the ones in Case I and the ones in [11].

III. ACTUATOR FAULT ISOLATION FOR NONLINEAR SYSTEMS

In this section, the fault isolation schemes for linear systems will be extended to nonlinear systems.

A. Actuator Fault Isolation for Nonlinear Systems

1) Case I: State vector is available: In this case, we will study how to extend the isolation scheme for linear systems to a class of affine nonlinear systems when all states are available. Consider the following nonlinear system

$$\dot{x} = A(x) + B(x)u \tag{19}$$

where $A(x)$ is a nonlinear vector function from R^n to R^n and $B(x) \in R^{n\times m}$ is a matrix function whose elements are nonlinear functions.

If the lth actuator is faulty, the corresponding faulty model is given below.

$$\dot{x} = A(x) + \sum_{j\neq l} b_j(x)u_j + b_l(x)\theta_l \tag{20}$$

where $B(x) = (b_1(x)\cdots b_m(x))$. The measured states are the states of this model.

Similar to the linear system case, by using adaptation technique, we design a bank of observers for all possible faulty models below.

$$\begin{aligned} \dot{\hat{x}}_i &= H(\hat{x}_i - x) + A(x) + \sum_{j\neq i} b_j(x)u_j + b_i(x)\hat{\theta}_i \\ \dot{\hat{\theta}}_i &= -2\gamma\tilde{x}_i^T P b_i(x),\ 1 \leq i \leq m \end{aligned} \tag{21}$$

where $\tilde{x}_i = \hat{x}_i - x$, H is Hurwitz matrix which can be chosen freely and γ is a design constant, and P and Q satisfy (4)

We have the following result.

Theorem 5: If the lth actuator is faulty and all elements of $B(x)$ are bounded, then for $i = l$, we have

$$lim_{t\to\infty}\tilde{x}_i = lim_{t\to\infty}\tilde{x}_l = 0 \tag{22}$$

and for $i \neq l$, we have

$$\dot{\tilde{x}}_i = H\tilde{x}_i + b_l(x)(u_l - \theta_l) - b_i(x)(u_i - \hat{\theta}_i) \tag{23}$$

Proof: With some slightly modifications, this theorem can be proved in the same way as Theorem 1 ∎

Remark 7: The boundedness of all elements of $B(x)$ does not mean that x has to be bounded. When $B(x)$ is a constant matrix, the boundedness requirements of $B(x)$ is automatically satisfied.

If we define $r_i(t) = \|\tilde{x}_i(t)\|^2$ for $1 \leq i \leq m$, then we can give a sufficient condition for actuator fault isolation based on the above theorem as follows.

Theorem 6: If $B(x)$ is of full column rank for all $x \in R^n$, then the actuator FI problem can be solved by evaluating the residuals $r_i(t)$ for $1 \leq i \leq m$.

Proof: This theorem can be proved in the same way as that we did in the proof of Theorem 2. ∎

Remark 8: We can also make similar remarks as those made in Remark 2.

2) Case 2: Output vector is available: In this case, we will study how to extend the isolation scheme for linear systems to a class of affine nonlinear systems when only outputs are available.

Consider the following nonlinear system

$$\begin{aligned} \dot{x} &= Ax + f(y) + g(x) + Bu \\ y &= Cx \end{aligned} \tag{24}$$

where $y \in R^q$ is the output, $f(y)$ is any nonlinear vector function of the outputs, and $g(x)$ is a vector function satisfies $\|g(x) - g(z)\| \leq \delta\|x - z\|$. For this system, only the outputs y are available. The states(that is, x) need to be estimated.

Because x is not available, we can not deal with the general affine system studied in previous case. Instead, we consider a special class of nonlinear system given by (24). For this system, the fault detection observer design is the most difficult one among all cases we have considered. To perform fault detection, we design the following observer

$$\dot{\hat{x}} = A\hat{x} - L(C\hat{x} - y) + f(y) + g(\hat{x}) + Bu \tag{25}$$

For the above detection observer, we have the following result.

Theorem 7: If (A, C) is detectable and if there exist two positive definite P and Q such that

$$\begin{aligned} (A - LC)^T P + P(A - LC) &= -Q \\ 2\|P\|\delta &< \lambda_{min}(Q) \end{aligned} \tag{26}$$

where $\lambda_{min}(Q)$ the smallest eigenvalue of Q, δ is the Lipschitz constant defined earlier.

Then we have

$$lim_{t\to\infty}\tilde{x} = 0 \tag{27}$$

where $\tilde{x} = \hat{x} - x$.

Proof: If the system has no fault, it follows from (24) and (25) that

$$\dot{\tilde{x}} = (A - LC)\tilde{x} + g(\hat{x}) - g(x) \tag{28}$$

Choose $V = (\tilde{x})^T P\tilde{x}$, it follows from (28) that

$$\begin{aligned} \dot{V} &= -(\tilde{x})^T Q\tilde{x} + 2(\tilde{x})^T P[g(\hat{x}) - g(x)] \\ &\leq -\lambda_{min}(Q)\|\tilde{x}\|^2 + 2\|\tilde{x}\|\|P\|\delta \\ &= -(\lambda_{min}(Q) - 2\|P\|\delta)\|\tilde{x}\|^2 \end{aligned} \tag{29}$$

Because $2\|P\|\delta < \lambda_{min}(Q)$, we know that $\tilde{x}$ tends to zero exponentially. This completes the proof. ∎

Define $r(t) = \|C\hat{x} - y\|$, if it does not tend to zero, faults are detected.

To design a bank of fault isolation observers, we need the following assumption, which will be used later.

Isolation Observer Existence Assumption(IOEA): There exist two positive definite matrices P and Q and two matrices L and D with suitable dimensions such that

$$\begin{aligned} (A - LC)^T P + P(A - LC) &= -Q \\ PB &= (DC)^T \\ 2\|P\|\delta &< \lambda_{min}(Q) \end{aligned} \tag{30}$$

Remark 9: The above assumption is stronger than that needed for fault detection. The first two equations in (30) have exactly the same form as that used for fault isolation problem of linear system with only output available. Unlike the linear system problem, for our nonlinear systems, more constraint on P and Q(that is, $2\|P\|\delta < \lambda_{min}(Q)$) is needed.
If the lth actuator is faulty, the corresponding faulty model is given below.

$$\begin{aligned} \dot{x} &= Ax + f(y) + g(x) + \sum_{j \neq i} b_j u_j + b_i \theta_i \\ y &= Cx \end{aligned} \tag{31}$$

The measured output is the output of the above faulty model.

By using adaptation technique, we design a bank of observers for all possible faulty models below.

$$\begin{aligned} \dot{\hat{x}}_i &= A\hat{x}_i - L(\hat{y}_i - y) + f(y) + g(\hat{x}) + \sum_{j \neq i} b_j u_j + b_i \hat{\theta}_i \\ \dot{\hat{\theta}}_i &= -2\gamma (e_{y_i})^T d_i, \ 1 \leq i \leq m \end{aligned} \tag{32}$$

where $\hat{y}_i = C\hat{x}_i$,$e_{y_i} = \hat{y}_i - y = C\tilde{x}_i$, γ is a design constant, and P, Q, L and $D^T = (d_1 \cdots d_m)$ satisfy (30).

We have the following result.

Theorem 8: If the lth actuator is faulty, then for $i = l$, we have

$$lim_{t \to \infty} \tilde{x}_i = lim_{t \to \infty} \tilde{x}_l = 0 \tag{33}$$

and for $i \neq l$, we have

$$\dot{\tilde{x}}_i = (A - LC)\tilde{x}_i + g(\hat{x}) - g(x) + b_l(u_l - \theta_l) - b_i(u_i - \hat{\theta}_i) \tag{34}$$

Proof: By combing the techniques used in proving Theorem 3 and Theorem 26, this theorem can be proved easily. ■

Because x is not available and only y is available, to solve the isolation problem, we define residuals as $r_i(t) = \|e_{y_i}(t)\|^2$ for $1 \leq i \leq m$. If only one residual goes to zero and the others do not, then the fault actuator isolation is achieved. Now, by using Theorem 8, we can give a sufficient condition for actuator fault isolation as follows.

Theorem 9: For any $i \neq l$, if there does not exist a time $T > T_f$ such that $Ce^{A_L t}[(g(x) - g(y)) + b_l(u_l - \theta_l) - b_i(u_i - \hat{\theta}_i)] \equiv 0$ for any $x, y \in R^n$ and any $t > T$, then the actuator FI problem can be solved under IOEA.

Proof: Because $Ce^{A_L t}[(g(x) - g(y)) + b_l(u_l - \theta_l) - b_i(u_i - \hat{\theta}_i)]$ is continuous with respect to t. Using the property of continuity, this theorem can be proved easily. ■

Remark 10: The fault isolation condition in this theorem may be very difficult to check. In practice, we can just monitor all the residuals directly without checking them. If only one residual goes to zero, the fault isolation can be performed. However, we need to explore further when more than two residuals go to zero and at least one does not. In such a case, other techniques are needed for complete isolation.

IV. EXAMPLES AND SIMULATION RESULTS

In this section, we will first give two examples corresponding to the linear and nonlinear cases with only output is available to show the effectiveness of the proposed fault isolation schemes. In all simulations, we assume that the first actuator has a constant fault at time step 400 but this is not known to the detection and isolation system. The fault isolation observers run simultaneously with the systems.

A. Example For Linear System With Known Outputs

The system is given as

$$\begin{aligned} \dot{x} &= Ax + Bu \\ y &= Cx \end{aligned} \tag{35}$$

where

$$A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}, \ B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}, C = B^T$$

It is easy to show that the system is passive. It follows that $u = -y$ makes the closed-loop system stable and $limit_{t \to \infty} x(t) = 0$ if the actuators are healthy.

If we choose $L = C^T$ and $Q = 4I + C^T C$, then with $P = I$ and $D = I$, it is easy to check that (14) is satisfied. The fault isolation observers can be designed using (16).

Since $A_L = A - LC = A - C^T C$ is symmetric, we know that $e^{A_L t}$ is positive definite for any t. Note that $B = C^T$ is of full column rank, we know that $Ce^{A_L t} C^T$ is of full column rank for any t. From the sufficient condition we derived for this case, we conclude that the constant actuator fault isolation can be accomplished.

The fault isolation effect based on the observers is shown in Figure 1. In the simulation, $x(0) = (0.3, 0.3, 0.3)^T$ while other initial values are all set to zero, and $\gamma = 1$. From the figure, we see that $r_1(t)$ goes to zero while $r_2(t)$ does not. Therefore, we isolate the faulty actuators correctly. This agrees with the result from our sufficient condition.

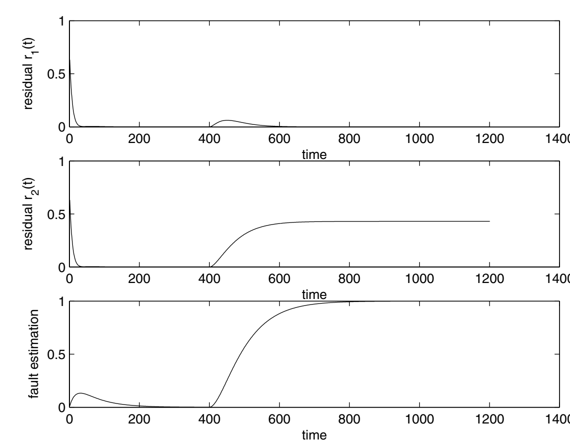

Fig. 1. Fault isolation of linear system with known outputs.

B. Example For Nonlinear System With Known outputs

The system is given as

$$\begin{aligned} \dot{x} &= Ax + f(y) + g(x) + Bu \\ y &= Cx \end{aligned} \quad (36)$$

where $A = \begin{pmatrix} -2 & 0 & 0 \\ 0 & -2 & 0 \\ 0 & 0 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 1 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$, $C = B^T$, $f(y) = \begin{pmatrix} y_1^2 + y_2^2 \\ y_1^2 + y_2^2 \\ 0 \end{pmatrix}$ $g(x) = \begin{pmatrix} 0.2ln(x_1^2+1) \\ 0.1sin(x_2) \\ 0 \end{pmatrix}$.

If we choose $u_1 = y_1$ and $u_2 = -y_2 - (y_1^2 + y_2^2)$, it is easy to show that the closed-loop system is stable and $limit_{t\to\infty} x(t) = 0$ if the actuators are healthy.

If we choose $L = C^T$ and $Q = 4I + C^T C$, then with $P = I$ and $D = I$, it is easy to check that (30) is satisfied. The fault isolation observers can be designed using (32).

Here, we do not check whether the system satisfies the sufficient condition derived for this case, we just monitor the two residuals directly. The fault isolation effect based on the observers is shown in Figure 2. In the simulation, $x(0) = (0.3, 0.3, 0.3)^T$ while other initial values are all set to zero, and $\gamma = 1$. From the figure, we see that $r_1(t)$ goes to zero while $r_2(t)$ does not. Therefore, we isolate the faulty actuators correctly. ¿From Figure 1 to Figure 2, if we look at the estimation of the fault, the estimations all go to actual values of the constant faults.

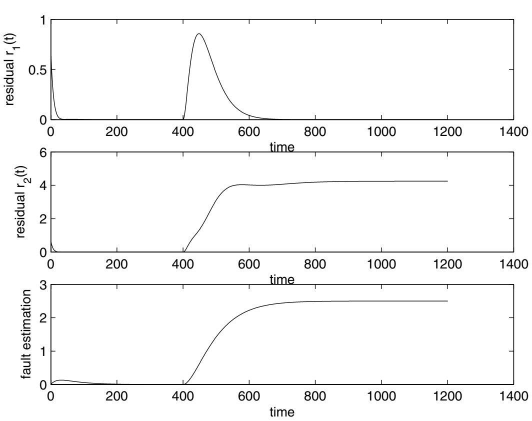

Fig. 2. Fault isolation of nonlinear system with known outputs.

V. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper, we have proposed actuator fault isolation schemes based on a bank of observers for linear systems with known states, linear systems with known outputs, nonlinear systems with known states, and nonlinear systems with known outputs respectively. Sufficient conditions for actuator fault isolation are derived. Some examples and simulation results show that the proposed schemes are effective.

B. Future Work

One future consideration is to extend our research to systems with unknown system parameters by using an adaptive technique.

VI. ACKNOWLEDGMENTS

This research was supported by Natural Sciences and Engineering Research Council (NSERC) of Canada through its *Discovery Grant Program.*

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