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Adaptive Output Feedback Design for Actuator Failure Compensation Using Dynamic Bounding: Output Regulation *

Xidong Tang and *Gang Tao*
Department of Electrical and Computer Engineering
University of Virginia
Charlottesville, VA 22903

Suresh M. Joshi
Mail Stop 132
NASA Langley Research Center
Hampton, VA 23681

Abstract

A new adaptive actuator failure compensation scheme using output feedback is presented in this paper for a enlarged class of nonlinear systems with state-dependent nonlinearities, which are bounded by both linear parameterized static functions and dynamic signals. By introducing the dynamic bounding technique to construct a backstepping control law, the control scheme ensures asymptotic output regulation and closed-loop stability via a Lyapunov based design in the presence of unknown actuator failures as well as system parameter and dynamic uncertainties. Simulation results are included to illustrate the control effectiveness.

1 Introduction

Actuator failures that are often unknown in terms of failure patterns, failure times and failure values may cause system dysfunction and accidents. Hence actuator failure compensation is crucial in control design with many open issues. Recently there have been many results in the literature on control of systems with actuator failures under several different directions such as multiple models, switching and tuning designs [3], [4], fault detection and diagnosis based designs [9], [10], and adaptive designs [1], [2], [11].

Compared with other control design methods, direct adaptive design has a simple structure by using one adaptive controller to accommodate both actuator failure uncertainties and system uncertainties without any explicit fault detection technique. For nonlinear systems, adaptive compensation for actuator failures has been addressed for some subclasses based on different conditions and control objectives. An adaptive actuator failure compensation scheme using output feedback for systems in the output-feedback form is designed in [6]. In [7], adaptive compensation using output feedback design is studied for a more general class of nonlinear systems, which contains unknown state-dependent nonlinearities bounded by a nonlinear function of the output. A robust adaptive controller is developed to handle actuator failures as well as parameter and dynamic uncertainties based on a shifted Lyapunov function. This paper revisits the actuator failure compensation problem using output feedback for the class of nonlinear systems with unknown state-dependent nonlinearities. The state-dependent nonlinearities are bounded not only by some static output-dependent functions but also by some signals dynamically dependent on the output. By introducing dynamic bounding signals, a new adaptive scheme of actuator failure compensation is presented for such an enlarged class of systems. Based on different backstepping design and Lyapunov analysis from those used in [7], the adaptive control scheme ensures asymptotic output regulation and closed-loop stability in the presence of unknown actuator failures. In another submitted paper, under a relaxed condition, a robust adaptive scheme is proposed to achieve desired output tracking with an application to aircraft control.

The paper is organized as follows. In Section 2, the control problem is formulated. In Section 3, an adaptive compensation scheme is developed for output regulation via the backstepping technique [5]. A generic example is presented with simulation to illustrate the effectiveness of the design.

2 Problem Statement

Consider the nonlinear system in the form of

$$\begin{aligned}
\dot{x}_i &= x_{i+1} + \phi_i(x,t) + \varphi_i(y),\ i = 1,2,\ldots,\rho-1 \\
\dot{x}_\rho &= x_{\rho+1} + \phi_\rho(x,t) + \varphi_\rho(y) + \sum_{j=1}^{m} b_{n^*,j}\beta_j(y)v_j \\
&\cdots \\
\dot{x}_{n-1} &= x_n + \phi_{n-1}(x,t) + \varphi_{n-1}(y) + \sum_{j=1}^{m} b_{1,j}\beta_j(y)v_j \\
\dot{x}_n &= \phi_n(x,t) + \varphi_n(y) + \sum_{j=1}^{m} b_{0,j}\beta_j(y)v_j \\
y &= x_1,
\end{aligned} \tag{2.1}$$

where u_j, $j = 1,2,\ldots,m$, are the control inputs whose actuators may fail during system operation, $x = [x_1, x_2, \ldots, x_n]^T$ is the vector of unmeasured states, y is the measured output, $b_{r,j}$, $r = 0,1,\ldots,n^* = n-\rho$, $j = 1,2,\ldots,m$, are unknown constants, $\varphi_i(y)$, $i = 1,2,\ldots,n$, and $\beta_j(y)$, $j = 1,2,\ldots,m$, are smooth nonlinear functions that are known and $\beta_j(y) \neq 0$ for $\forall y \in R$, and $\phi_i(x,t)$, $i = 1,2,\ldots,n$, are unknown func-

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tions that are not necessarily smooth. For $\phi(x,t)=[\phi_1(x,t), \phi_2(x,t),\ldots,\phi_n(x,t)]^T$, we have the following condition:

(A1) There exists a vector of functions $\psi(y)=[\psi_1(y),\psi_2(y),\ldots,\psi_{l_1}(y)]^T\in R^{l_1}$, a diagonal matrix of functions $\Psi(y)=\mathrm{diag}\{\Psi_1(y),\Psi_2(y),\ldots,\Psi_{l_2}(y)\}\in R^{l_2\times l_2}$, and a vector of signals $\rho(t)=[\rho_1(t),\rho_2(t),\ldots,\rho_{l_2}(t)]^T\in R^{l_2}$ with $\rho_i(t)=F_i(s)[\eta_i(y)]$, $\eta_i(y)\geq 0$, $F_i(s)$ is a stable and strictly proper, and $F_i(s-\delta_i)$ with $\delta_i>0$ is stable, $i=1,2,\ldots,l_2$, such that

$$\|\phi(x,t)\|\leq\theta_1+\theta_{20}^T\psi(y)+\theta_{30}^T\Psi(y)\rho(t), \qquad (2.2)$$

where $\psi(y)$, $\Psi(y)$, and $\eta(y)=[\eta_1(y),\eta_2(y),\ldots,\eta_{l_2}(y)]^T\in R^{l_2}$ are smooth and known, a positive constant δ_0 such that $\delta_0\leq\min\{\delta_i\}$ is known, and $\theta_1\in R$, $\theta_{20}=[\theta_{201},\theta_{202},\ldots,\theta_{20l_1},]^T\in R^{l_1}$, and $\theta_{30}=[\theta_{301},\theta_{302},\ldots,\theta_{30l_2},]^T\in R^{l_2}$ are unknown constants. For asymptotic output regulation, we assume that

$$\psi(y)=y\psi_0(y),\ \Psi(y)=y\Psi_0(y), \qquad (2.3)$$

where $\psi_0(y)=[\psi_{01}(y),\psi_{02}(y),\ldots,\psi_{0l_1}(y)]^T\in R^{l_1}$ with $\psi_{0i}^2(y)$ to be smooth, $i=1,2,\ldots,l_1$, and $\Psi_0(y)=\mathrm{diag}\{\Psi_{01}(y),\Psi_{02}(y),\ldots,\Psi_{0l_2}(y)\}\in R^{l_2\times l_2}$ with $\Psi_{0i}^2(y)$ to be smooth, $i=1,2,\ldots,l_2$, are both known.

As in [7], the actuator failures are modeled as

$$u_j(t)=\bar{u}_j,\ t\geq t_j,\ j\in\{1,2,\ldots,m\}, \qquad (2.4)$$

where the failure value $\bar{u}_j$, the failure time instant t_j, and the failure pattern, that is, the index j, are unknown. More general failures modeled as $u_j(t)=\bar{u}_j+\sum_{i=1}^{n_j}\bar{d}_{ji}f_{ji}(t)$, $j=1,2,\ldots,m$, with $\bar{u}_j$ and $\bar{d}_{ji}$, unknown constants, and $f_{ji}(t)$, known signals, can also be handled similarly.

Suppose that there are p_k actuators failed at a time instant t_k, $k=1,2,\ldots,q$, and $t_0<t_1<t_2<\ldots<t_q<\infty$. Due to up to $m-1$ failures, it follows that $\sum_{k=1}^q p_k\leq m-1$. In other words, at time $t\in(t_k,t_{k+1})$, $k=0,1,\ldots,q$, with $t_{q+1}=\infty$, there are $p=\sum_{i=1}^k p_i$ failed actuators, that is, $u_j(t)=\bar{u}_j$, $j=j_1,\ldots,j_p$, $0\leq p\leq m-1$, and $u_j(t)=v_j(t)$, $j\neq j_1,\ldots,j_p$, where $v_j(t)$, $j=1,2,\ldots,m$, are applied control inputs to be designed.

The control objective is to design an output feedback scheme for the nonlinear plant (2.1) with p, the number of failed actuators, changing at time instants t_k, $k=1,2,\ldots,q$, such that the plant output $y(t)$ converges to zero asymptotically and that all closed-loop signals are bounded in the presence of unknown actuator failures, plant parameters, and state-dependent nonlinearities.

3 Adaptive Compensation Control

Since the failure pattern, $j=j_1,\ldots,j_p$, $0\leq p\leq m-1$, is unknown in this problem, a desirable adaptive design is expected to achieve the control objective for any possible failure pattern. For a fixed failure pattern, there are a set of failed actuators and another set of active actuators. For this set of active actuators, there is a resulting pattern of zero dynamics for the system (2.1). For the closed-loop stability, the zero dynamics of the system (2.1) need to be stable. Since zero dynamics depend on the pattern of active actuators, the number of which may range from 1 to m, there is a set of zero dynamics corresponding to a chosen control design. We specify the following condition with which a stable adaptive scheme can be developed to achieve the control objectives.

(A2) The polynomials $\sum_{j\neq j_1,\ldots,j_p}\mathrm{sign}[b_{n^*,j}]B_j(s)$ are stable, $\forall\{j_1,\ldots,j_p\}\subset\{1,2,\ldots,m\}$, $\forall p\in\{0,1,2,\ldots,m-1\}$, where

$$B_j(s)=b_{n^*,j}s^{n^*}+b_{n^*-1,j}s^{n^*-1}+\cdots+b_{1,j}s+b_{0,j}. \qquad (3.1)$$

For an adaptive design, another assumption is required.

(A3) The sign of $b_{n^*,j}$ is known, for $j=1,2,\ldots,m$.

For adaptive actuator failure compensation, we use the proportional actuation scheme

$$v_j=\mathrm{sign}[b_{n^*,j}]\frac{1}{\beta_j(y)}v_0,\ j=1,2,\ldots,m, \qquad (3.2)$$

where v_0 is a control signal generated by a backstepping design procedure consisting of ρ steps, which will be given next.

With the actuation scheme (3.2) and following definition

$$\begin{aligned}\varphi(y) &= [\varphi_1(y),\varphi_2(y),\ldots,\varphi_n(y)]^T, &(3.3)\\ k_{1,r} &= \sum_{j\neq j_1,\ldots,j_p}\mathrm{sign}[b_{n^*,j}]b_{r,j},\ r=0,1,\ldots,n^*, &\\ k_{2,rj} &= b_{r,j}\bar{u}_j,\ r=0,1,\ldots,n^*,\ j=j_1,\ldots,j_p, &\\ k_{2,rj} &= 0,\ r=0,1,\ldots,n^*,\ j\neq j_1,\ldots,j_p, &(3.4)\end{aligned}$$

and rewrite the system (2.1) with p actuator failures as

$$\begin{aligned}\dot{x} &= Ax+\phi(x,t)+\varphi(y)+\sum_{r=0}^{n^*}e_{n-r}\sum_{j=1}^{m}k_{2,rj}\beta_j(y)+\sum_{r=0}^{n^*}e_{n-r}k_{1,r}v_0\\ y &= c^Tx, \qquad (3.5)\end{aligned}$$

where $A\in R^{n\times n}$ is a canonical form describing a chain of n integrators, $c=[1,0,\ldots,0]^T$, and e_i is the ith coordinate vector in R^n. It follows from Assumption (A2) that the polynomial $k_{1,n^*}s^{n^*}+k_{1,n^*-1}s^{n^*-1}+\cdots+k_{1,1}s+k_{1,0}$ is stable, and, in addition, from (3.3), that $k_{1,n^*}>0$.

3.1 State Observation

Since the states of the system (2.1) are not available for feedback, an observer is needed to provide auxiliary signals for handling the unmeasured states in control design.

Choosing a vector $l\in R^n$ such that $A_o=A-lc^T$ is stable, with knowledge of $k_{1,r}$ and $k_{2,rj}$, $j=1,2,\ldots,m$, $r=0,1,\ldots,n^*$, we have a nominal state observer for x as

$$x^*=\xi+\xi^*+\sum_{r=0}^{n^*}\sum_{j=1}^{m}k_{2,rj}\zeta_{rj}+\sum_{r=0}^{n^*}k_{1,r}\mu_{rj}, \qquad (3.6)$$

with the filters defined as

$$\begin{aligned}\dot{\xi} &= A_o\xi + ly + \varphi(y),\\ \dot{\xi}^* &= A_o\xi^* + \phi(x,t),\\ \dot{\zeta}_{rj} &= A_o\zeta_{rj} + e_{n-r}\beta_j(y),\ 0\le r\le n^*,\ 1\le j\le m,\\ \dot{\mu}_r &= A_o\mu_r + e_{n-r}v_0,\ 0\le r\le n^*. \end{aligned}\qquad (3.7)$$

With $\varepsilon = x - x^*$, it follows from (3.5), (3.6) and (3.7) that $\dot{\varepsilon} = A_o\varepsilon$ and $\lim_{t\to\infty}\varepsilon(t) = 0$ exponentially. The signal x^* in (3.6) is indeed the nominal observation of x in the sense that the error $x(t) - x^*(t)$ converges to zero exponentially. However, there are two issues related to using this x^* as an estimate of the state vector x. The first issue is that the parameters $k_{1,r}$ and $k_{2,rj}$, $j = 1,2,\ldots,m$, $r = 0,1,\ldots,n^*$, are unknown, which can be solved by using their adaptive estimates. The second issue, which is more challenging, is that the nonlinear function $\phi(x,t)$ is not available, as the system state x is not available. Hence, in addition to using adaptive estimates of $k_{1,r}$ and $k_{2,rj}$, we use bounding functions to replace the unavailable $\phi(x,t)$ based on Assumption (A1). In this case, by using bounding functions, an explicit expression of x^* and its adaptive version may not be available.

To solve this problem, in [7], we introduce an auxiliary signal generated from an observer with $\phi(x,t) = 0$ to approximate the nominal state estimate x^* and such that the closed-loop stability and desired tracking performance are established based on a shifted Lyapunov function. In this paper, we propose a new design for handling the uncertain nonlinear function $\phi(x,t)$ that appears in (3.7) through ξ^* based on the ideal nominal observer (3.6) for the system (3.5) with the unavailable signal ξ^*. However, this seemingly natural way will face a new problem: the second component of the vector ξ^* , i.e., ξ_2^* (that appears in the backstepping design procedure, e.g., added to $\phi_1(x,t)$ in (3.16)) is dynamically dependent on the unavailable $\phi(x,t)$. To deal with the unavailable component of ξ^*, we introduce dynamic bounding signals $\chi_0(t) = [\chi_{01}(t),\chi_{02}(t),\ldots,\chi_{0l_2}(t)]^T \in R^{l_2}$ and $\bar{\chi}(t) = [\bar{\chi}_1,\bar{\chi}_2,\ldots,\bar{\chi}_{\bar{L}}]^T \in R^{\bar{L}}$, $\bar{L} = l_1 + l_2$, which are generated from

$$\dot{\chi}_0(t) = -\delta_0\chi_0(t) + \eta(y),\ \dot{\bar{\chi}}(t) = -\bar{\delta}\bar{\chi}(t) + \bar{\Phi}(y), \qquad (3.8)$$

where $\bar{\Phi}=[|\psi_1(y)|,\ldots,|\psi_{l_1}(y)|,|\Psi_1(y)\chi_{01}(t)|,|\Psi_2(y)\chi_{02}(t)|,\ldots,|\Psi_{l_2}(y)\chi_{0l_2}(t)|]^T=[\bar{\Phi}_1,\bar{\Phi}_2,\ldots,\bar{\Phi}_{\bar{L}}]^T\in R^{\bar{L}}$, $\bar{\delta}>0$ is chosen such that $A_o+\bar{\delta}I$ is stable, $\delta_0>0$ defined in Assumption (A1), $\psi(y)$, $\Psi(y)$, and $\eta(y)$ satisfies Assumption (A1).

First recall Assumption (A1) and notice that $\delta_0 \le \min\{\delta_i\}$ such that $F_i(s-\delta_0)$, $i = 1,2,\ldots,l_2$, are stable. It can be concluded from Lemma 2.6 in [8] that

$$|\rho_i(t)| \le \delta_{fi}\frac{1}{s+\delta_0}\eta_i(y) = \delta_{fi}\chi_{0i}(t),\ i = 1,2,\ldots,l_2, \quad (3.9)$$

where $\delta_{fi} = \|f_{i\delta_0}(\cdot)\|_1$ with $f_{i\delta_0}(t)$ denoting the impulse response function of $F_i(s-\delta_0)s$, and $\chi_{0i}(t)$ is the ith component of $\chi_0(t)$. From (3.9), it in turn follows that

$$\theta_{20}^T\psi(y) + \theta_{30}^T\Psi(y)\rho(t) \le \bar{\theta}^T\bar{\Phi}, \qquad (3.10)$$

where $\bar{\theta} = [\theta_2^T,\theta_3^T]^T = [\bar{\theta}_{(1)},\bar{\theta}_{(2)},\ldots,\bar{\theta}_{(\bar{L})}]^T \in R^{\bar{L}}$, and $\theta_2 \in R^{l_1}$ and $\theta_3 \in R^{l_2}$ are unknown vectors whose components are $\theta_{2i}=|\theta_{20i}|\ge 0$, $i = 1,2,\ldots,l_1$, and $\theta_{3i}=|\theta_{30i}|\delta_{fi}\ge 0$, $i = 1,2,\ldots,l_2$.

From the dynamic equations of ξ^* in (3.7), it follows that

$$\xi_2^*(t) = H_1^*(s)[\phi_1(x,t)] + \cdots + H_n^*(s)[\phi_n(x,t)], \qquad (3.11)$$

where $H_i^*(s) = \frac{N_i(s)}{D(s)}$ is the transfer function from $\phi_i(x,t)$ to ξ_2^*, $i = 1,2,\ldots,n$, with $D(s) = \det(sI - A_o)$. Since $A_o + \bar{\delta}I$ is stable, we have that $D(s-\bar{\delta})$ is stable. It is also noted that $H_i^*(s)$ is strictly proper. With the definition that $H_i(s) = (s+\bar{\delta})H_i^*(s) = \frac{N_i(s)(s+\bar{\delta})}{D(s)}$ for $i = 1,2,\ldots,n$, the signal ξ_2^* in the input-to-output form (3.11) is rewritten as

$$\xi_2^*(t)=H_1(s)\frac{1}{s+\bar{\delta}}[\phi_1(x,t)]+\cdots+H_n(s)\frac{1}{s+\bar{\delta}}[\phi_n(x,t)], \quad (3.12)$$

where $H_i(s)$ is a proper transfer function for $i = 1,2,\ldots,n$.

Based on Lemma 2.6 in [8], it follows from (3.12) that

$$\begin{aligned}|\xi_2^*(t)| &\le \delta\frac{1}{s+\bar{\delta}}\left(\theta_2^T\psi(y) + \theta_{30}^T\Psi(y)\rho(t)\right)\\ &\le \delta\frac{1}{s+\bar{\delta}}\bar{\theta}^T\bar{\Phi} = \delta\bar{\theta}^T\bar{\chi}(t), \end{aligned}\qquad (3.13)$$

where $\delta = \|h_{1\bar{\delta}}(\cdot)\|_1 + \|h_{2\bar{\delta}}(\cdot)\|_1 + \cdots + \|h_{n\bar{\delta}}(\cdot)\|_1$, and $h_{i\bar{\delta}}(t)$ is the impulse response function of $H_i(s-\bar{\delta})$, $i = 1,2,\ldots,n$.

3.2 Adaptive Design for Output Regulation

The backstepping technique [5] is now applied with a design procedure consisting of ρ steps to derive a stable adaptive control scheme for output regulation, in the presence of unknown actuator failures and system dynamics and parameters.

Define unknown constant $k=[k_{1,n^*},k_1^T,k_2^T]^T$ and k_{30} with

$$\begin{aligned}k_1 &= [k_{1,0},k_{1,1},\ldots,k_{1,n^*-1}]^T,\\ k_2 &= [k_{2,01},k_{2,02},\ldots,k_{2,0m},k_{2,11},\ldots,k_{2,1m},\ldots,k_{2,n^*m}]^T,\\ k_{30} &= [\theta_{21}^2,\theta_{22}^2,\ldots,\theta_{2l_1}^2,\theta_{31}^2,\theta_{32}^2,\ldots,\theta_{3l_2}^2]^T, \end{aligned}\qquad (3.14)$$

and $\kappa=\frac{1}{k_{1,n^*}}$, and vectors of signals $\upsilon_i=[\mu_{n^*,i},\omega_i^T,\varepsilon_i^T]^T$, ς_0 as

$$\begin{aligned}\omega_i &= [\mu_{0,i},\mu_{1,i},\ldots,\mu_{n^*-1,i}]^T,\ i = 1,2,\ldots,n,\\ \varepsilon_i &= [\zeta_{01,i},\zeta_{02,i},\ldots,\zeta_{0m,i},\zeta_{11,i},\ldots,\zeta_{1m,i},\ldots,\zeta_{n^*m,i}]^T,\\ \varsigma_0 &= [\psi_{01}^2(y),\psi_{02}^2(y),\ldots,\psi_{0l_1}^2(y),\Psi_{01}^2(y)\chi_{01}^2(t),\\ &\quad \Psi_{02}^2(y)\chi_{02}^2(t),\ldots,\Psi_{0l_2}^2(y)\chi_{0l_2}^2(t)]^T, \end{aligned}\qquad (3.15)$$

where $\mu_{r,i}$ and $\zeta_{rj,i}$, $i = 1,2,\ldots,n$, are the ith component of μ_r and ζ_{rj}, $r = 0,1,\ldots,n^*-1$, $j = 1,2,\ldots,m$. Let $\hat{\kappa}$, $\hat{k}_{1,n^*}$, $\hat{k}_1$, $\hat{k}_2$, $\hat{k}_{30}$, and $\hat{k}$ denote the estimates of κ, k_{1,n^*}, k_1, k_2, k_{30}, and k.

Step 1: Defining the output tracking error $z_1 = y$, we have

$$\begin{aligned}\dot{z}_1 &= \varepsilon_2 + x_2^* + \phi_1(x,t) + \varphi_1(y)\\ &= \varepsilon_2 + \upsilon_2^T k + \xi_2 + \xi_2^* + \phi_1(x,t) + \varphi_1(y). \end{aligned}\qquad (3.16)$$

Choose the auxiliary signal

$$\begin{aligned} z_2 &= \mu_{n^*,2} - \alpha_1,\ \alpha_1 = \hat{\kappa}\bar{\alpha}_1, \\ \bar{\alpha}_1 &= -c_1 z_1 - d_1 z_1 - \lambda_0 z_1 \varsigma_0^T \hat{k}_{30} \\ &\quad -\omega_2^T \hat{k}_1 - \varepsilon_2^T \hat{k}_2 - \xi_2 - \varphi_1(y). \end{aligned} \tag{3.17}$$

Substituting (3.17) into (3.16) results in

$$\begin{aligned} \dot{z}_1 =& -c_1 z_1 - d_1 z_1 - \lambda_0 z_1 \varsigma_0^T k_{30} + \varepsilon_2 + \xi_2^* + \phi_1(x,t) \\ &+ \hat{k}_{1,n^*} z_2 - k_{1,n^*}\tilde{\kappa}\bar{\alpha}_1 - \tilde{k}_{1,n^*}\alpha_1 + \upsilon_2^T \tilde{k} + \lambda_0 z_1 \varsigma_0^T \tilde{k}_{30}, \end{aligned} \tag{3.18}$$

where $\tilde{k}_{1,n^*}=k_{1,n^*}-\hat{k}_{1,n^*}$, $\tilde{k}=k-\hat{k}$, $\tilde{k}_{30}=k_{30}-\hat{k}_{30}$, and $\tilde{\kappa}=\kappa-\hat{\kappa}$, and c_1, d_1, and λ_0 are some positive constants to be chosen.

Consider the partial Lyapunov function $V_1 = \frac{1}{2}z_1^2 + \frac{k_{1,n^*}}{2\gamma_1}\tilde{\kappa}^2 + \frac{1}{2}\tilde{k}_{30}^T\Gamma_{30}^{-1}\tilde{k}_{30}$, where $k_{1,n^*} > 0$ due to Assumption (A2), and $\gamma_1 > 0$ and $\Gamma_{30} = \Gamma_{30}^T > 0$ are adaptive gains.

Choose the adaptive laws for $\hat{\kappa}$ and $\hat{k}_{30}$ as

$$\dot{\hat{\kappa}} = -\gamma_1 z_1 \bar{\alpha}_1,\ \dot{\hat{k}}_{30} = \lambda_0 z_1^2 \Gamma_{30} \varsigma_0 \tag{3.19}$$

and the tuning functions for $\hat{\kappa}$ and $\hat{k}$ as

$$\tau_1 = z_1[(\mu_{n^*,2} - \alpha_1), \omega_2^T, \varepsilon_2^T]^T. \tag{3.20}$$

Then the time-derivative of V is

$$\begin{aligned} \dot{V}_1 &= -c_1 z_1^2 - d_1 z_1^2 - \lambda_0 z_1^2 \varsigma_0^T k_{30} + z_1 \varepsilon_2 \\ &\quad + z_1 \xi_2^* + z_1 \phi_1(x,t) + \hat{k}_{1,n^*} z_1 z_2 + \tau_1^T \tilde{k}, \end{aligned} \tag{3.21}$$

where $\hat{k}_{1,n^*} z_1 z_2$ is cancelled at the next step and $z_1\varepsilon_2$, $z_1\xi_2^*$, $z_1\phi_1(x,t)$ are to be handled later on.

Step $i = 2,3,4,\ldots,\rho$: Define z_i, $i = 2,3,\ldots,\rho$, as

$$z_i = \mu_{n^*,i} - \alpha_{i-1}. \tag{3.22}$$

Differentiating (3.22), we obtain

$$\begin{aligned} \dot{z}_i =& \mu_{n^*,i+1} - l_i \mu_{n^*,1} - \frac{\partial \alpha_{i-1}}{\partial \hat{\kappa}}\dot{\hat{\kappa}} - \frac{\partial \alpha_{i-1}}{\partial \hat{k}_{30}}\dot{\hat{k}}_{30} \\ &- \frac{\partial \alpha_{i-1}}{\partial \chi_0}(-\bar{\delta}\chi_0 + \eta(y)) - \sum_{q=1}^{i} \frac{\partial \alpha_{i-1}}{\partial \upsilon_q}(\upsilon_{q+1} - l_q \upsilon_1) \\ &- \frac{\partial \alpha_{i-1}}{\partial y}(\varepsilon_2 + \upsilon_2^T k + \xi_2 + \xi_2^* + \phi_1(x,t) + \varphi_1(y)) \\ &- \sum_{q=1}^{i} \frac{\partial \alpha_{i-1}}{\partial \xi_q}(\xi_{q+1} - l_q \xi_1 + l_q y + \varphi_q(y)) - \frac{\partial \alpha_{i-1}}{\partial \hat{k}}\dot{\hat{k}}. \end{aligned} \tag{3.23}$$

Choose the stabilizing function α_i as

$$\begin{aligned} \alpha_i =& -a_i z_{i-1} - c_i z_i - d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i + l_i \mu_{n^*,1} \\ &+ \frac{\partial \alpha_{i-1}}{\partial \hat{\kappa}}\dot{\hat{\kappa}} + \frac{\partial \alpha_{i-1}}{\partial \hat{k}_{30}}\dot{\hat{k}}_{30} + \frac{\partial \alpha_{i-1}}{\partial \chi_0}(-\bar{\delta}\chi_0 + \eta(y)) \\ &+ \frac{\partial \alpha_{i-1}}{\partial y}(\upsilon_2^T \hat{k} + \xi_2 + \varphi_1(y)) + \sum_{q=1}^{i} \frac{\partial \alpha_{i-1}}{\partial \xi_q}(\xi_{q+1} - l_q \xi_1 + l_q y \\ &+ \varphi_q(y)) + \sum_{q=1}^{i} \frac{\partial \alpha_{i-1}}{\partial \upsilon_q}(\upsilon_{q+1} - l_q \upsilon_1) \\ &+ \frac{\partial \alpha_{i-1}}{\partial \hat{k}}\Gamma\tau_i - \sum_{q=2}^{i-1} \frac{\partial \alpha_{q-1}}{\partial \hat{k}}\Gamma\frac{\partial \alpha_{i-1}}{\partial y}\upsilon_2 z_q, \end{aligned} \tag{3.24}$$

where $a_2 = \hat{k}_{1,n^*}$ and $a_i = 1$, $i = 3,4,\ldots,\rho$, $c_i > 0$ and $d_i > 0$, $i = 2,3,\ldots,\rho$, are design constants, $\Gamma = \Gamma^T > 0$ is the adaptive gain, and τ_i, $i = 2,3,\ldots,\rho$, are the tuning functions given by

$$\tau_i = \tau_{i-1} - \frac{\partial \alpha_{i-1}}{\partial y}\upsilon_2 z_i. \tag{3.25}$$

Defining $z_{i+1} = \mu_{n^*,i+1} - \hat{\kappa} y_r^{(i)} - \alpha_i$, we rewrite (3.23) as

$$\begin{aligned} \dot{z}_i =& -z_{i-1} - c_i z_i - d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i - \frac{\partial \alpha_{i-1}}{\partial y}\varepsilon_2 \\ &- \frac{\partial \alpha_{i-1}}{\partial y}\xi_2^* - \frac{\partial \alpha_{i-1}}{\partial y}\phi_1(x,t) + z_{i+1} - \frac{\partial \alpha_{i-1}}{\partial y}\upsilon_2^T \tilde{k} \\ &+ \frac{\partial \alpha_{i-1}}{\partial \hat{k}}\Gamma\tau_i - \sum_{q=2}^{i-1} \frac{\partial \alpha_{q-1}}{\partial \hat{k}}\Gamma\frac{\partial \alpha_{i-1}}{\partial y}\upsilon_2 z_q - \frac{\partial \alpha_{i-1}}{\partial \hat{k}}\dot{\hat{k}}. \end{aligned} \tag{3.26}$$

For the partial Lyapunov candidate function $V_i = V_{i-1} + \frac{1}{2}z_i^2$, it follows from (3.26) that

$$\begin{aligned} \dot{V}_i =& -\sum_{q=1}^{i} c_q z_q^2 - d_1 z_1^2 - \lambda_0 z_1^2 \varsigma_0^T k_{30} + z_1\varepsilon_2 + z_1\xi_2^* \\ &+ z_1\phi_1(x,t) - \sum_{q=2}^{i} d_q\left(\frac{\partial \alpha_{q-1}}{\partial y}\right)^2 z_q^2 - \sum_{q=2}^{i} z_q \frac{\partial \alpha_{q-1}}{\partial y}\varepsilon_2 \\ &- \sum_{q=2}^{i} z_q \frac{\partial \alpha_{q-1}}{\partial y}\xi_2^* - \sum_{q=2}^{i} z_q \frac{\partial \alpha_{q-1}}{\partial y}\phi_1(x,t) + z_i z_{i+1} \\ &+ \tau_i^T \tilde{k} + \sum_{q=2}^{i} z_q \frac{\partial \alpha_{q-1}}{\partial \hat{k}}(\Gamma\tau_i - \dot{\hat{k}}). \end{aligned} \tag{3.27}$$

Design the control signal $v_0(t)$ in the control law (3.2) as

$$v_0 = \alpha_\rho + \sum_{j=1}^{m} \frac{\partial \alpha_{\rho-1}}{\partial \zeta_{n^* j,\rho}}\beta_j(y) - \mu_{n^*,\rho+1}, \tag{3.28}$$

where α_ρ is the stabilizing function from the ρth step.

Consider a Lyapunov function

$$V = V_\rho + \frac{1}{2}\tilde{k}^T\Gamma^{-1}\tilde{k} + \sum_{i=1}^{\rho} \frac{\bar{L}\delta^2}{2d_i\bar{\delta}} \sum_{q=1}^{\bar{L}} \bar{\theta}_{(q)}^2 \bar{\chi}_q^2 + \sum_{i=1}^{\rho} \frac{1}{d_i}\varepsilon^T P \varepsilon, \tag{3.29}$$

where $P=P^T>0$ satisfying the Lyapunov equation $PA_o + A_o^T P = -I$. Based on Assumptions (A1) and (3.13), we show that

$$\begin{aligned} \dot{V} \le& -\sum_{i=1}^{\rho} c_i z_i^2 - d_1 z_1^2 - \lambda_0 z_1^2 \varsigma_0 k_{30} + z_1\varepsilon_2 + |z_1||\xi_2^*| \\ &+ |z_1||\phi_1(x,t)| - \sum_{i=2}^{\rho} d_i\left(\frac{\partial \alpha_{i-1}}{\partial y}\right)^2 z_i^2 - \sum_{i=2}^{\rho} z_i \frac{\partial \alpha_{i-1}}{\partial y}\varepsilon_2 \\ &+ \sum_{i=2}^{\rho} |z_i \frac{\partial \alpha_{i-1}}{\partial y}||\xi_2^*| + \sum_{i=2}^{\rho} |z_q \frac{\partial \alpha_{i-1}}{\partial y}||\phi_1(x,t)| \\ &- \sum_{i=1}^{\rho} \frac{\bar{L}\delta^2}{d_i} \sum_{q=1}^{\bar{L}} \bar{\theta}_{(q)}^2 \bar{\chi}_q^2 + \sum_{i=1}^{\rho} \frac{\bar{L}\delta^2}{d_i\bar{\delta}} \sum_{q=1}^{\bar{L}} \bar{\theta}_{(q)}^2 \bar{\chi}_q \bar{\Phi}_q(y) - \sum_{i=1}^{\rho} \frac{1}{d_i}\varepsilon^T\varepsilon \\ \le& -\sum_{i=1}^{\rho} c_i z_i^2 - \sum_{i=1}^{\rho} \frac{1}{2d_i}\|\varepsilon\|^2, \end{aligned} \tag{3.30}$$

when λ_0 is chosen such that $\lambda_0 \geq \frac{\bar{L}\delta^2}{2\underline{\delta}^2}\sum_{i=1}^{\rho}\frac{1}{d_i}$.

In summary, with the actuation scheme (3.2) and the filters of the signals ξ, ζ_{rj} and μ_{rj}, $0 \leq r \leq n^*$, $1 \leq j \leq m$ defined in (3.7), the adaptive scheme consists of the control law:

$$\begin{aligned} v_j &= \text{sign}[b_{n^*,j}]\frac{1}{\beta_j(y)}v_0,\ j=1,2,\ldots,m, \\ v_0 &= \alpha_\rho + \sum_{j=1}^{m}\frac{\partial\alpha_{\rho-1}}{\partial\zeta_{n^*j,\rho}}\beta_j(y) - \mu_{n^*,\rho+1}, \end{aligned} \quad (3.31)$$

and the adaptive laws for parameters $\hat{\kappa}$, $\hat{k}$, and $\hat{k}_{30}$:

$$\dot{\hat{\kappa}} = -\gamma_1 z_1\bar{\alpha}_1,\ \dot{\hat{k}} = \Gamma\tau_\rho,\ \dot{\hat{k}}_{30} = \lambda_0 z_1^2\Gamma_{30}\varsigma_0, \quad (3.32)$$

where α_ρ, υ_ρ, τ_ρ, and ν_ρ are derived from the recursive backstepping procedure with ρ steps.

3.3 Stability Analysis

The proposed adaptive scheme has the following properties:

Theorem 3.1 *The adaptive output feedback control scheme consisting of the controller (3.31) and the filters (3.7) along with the parameter update laws (3.32) applied to the system (2.1), based on Assumptions (A1), (A2), and (A3), ensures global boundedness of all closed-loop signals and asymptotic output regulation:* $\lim_{t\to\infty} y(t) = 0$.

Proof: For each time interval (t_k, t_{k+1}), $k = 0,1,\ldots,q$, we have a positive definite function V defined as (3.29) whose time-derivative $\dot{V}$ satisfies (3.30). Starting from the first time interval, we conclude that $V(t) \in L^\infty$ for $\forall t \in [t_0,t_1)$, so that z, $\hat{\kappa}$, $\hat{k}_{1,n^*}$, $\hat{k}_1$, $\hat{k}_2$, $\hat{k}_{30}$, and ε are bounded for $t \in [t_0,t_1)$. As $z_1 = y$, y is also bounded for $t \in [t_0,t_1)$.

It follows from (3.7) that ξ, ζ_{rj}, $r = 0,1,\ldots,n^*$, $j = 1,2,\ldots,m$, and ξ^* are bounded because of the boundedness of $\phi(x,t)$ due to Assumption (A1). As in [5], it can be concluded that with $K(s) = \frac{1}{k_{1,n^*}s^{n^*}+\cdots+k_{1,1}s+k_{1,0}}$

$$\begin{aligned} \mu_{r,i} &= e_i^T(sI-A_o)^{-1}e_{n-r}K(s)\left[\frac{d^n y}{dt} - \sum_{i=1}^{n}\frac{d^{n-i}\phi_i(x,t)}{dt}\right. \\ &\quad \left. - \sum_{i=1}^{n}\frac{d^{n-i}\varphi_i(y)}{dt} - \sum_{r=0}^{n^*}\sum_{j=1}^{m}k_{2,rj}\frac{d^r\beta_j(y)}{dt}\right], \end{aligned} \quad (3.33)$$

which results in the boundedness of $\mu_{r,i}$, $r = 0,1,\ldots,n^*$, $i = 1,2,\ldots,n$, because the matrix A_o and the polynomial $k_{1,n^*}s^{n^*} + \cdots + k_{1,1}s + k_{1,0}$ are stable, and $\phi_i(x,t)$, $i = 1,2,\ldots,n$, are bounded based on Assumption (A1), and $\varphi_i(\cdot)$, $i = 1,2,\ldots,n$, and $\beta_j(\cdot)$, $j = 1,2,\ldots,m$, are bounded too because of the smoothness. It follows from (3.6) and the boundedness of ε that x is bounded. According to (3.31), it can also be seen that v_0 is a bounded signal. Since $\beta_j(y) \neq 0$ for $\forall y \in R$, the boundedness of v_j is guaranteed too, $j = 1,2,\ldots,m$. Therefore, all closed-loop signals are bounded for $t \in [t_0,t_1)$.

At time $t = t_1$, p_1 actuator failures occur, which result in the abrupt change of κ, k_{1,n^*}, k_1, and k_2 denoted as $\Delta\kappa$, $\Delta k_{1,n^*}$, Δk_1, and Δk_2 respectively. Since the change of values of these parameters are finite and z, ε, $\hat{\kappa}$, $\hat{k}_{1,n^*}$, $\hat{k}_1$, $\hat{k}_2$, and $\hat{k}_{30}$ are continuous, we have that $V(t_1^+) = V(t_1^-) + \bar{V}_1$ with a finite $\bar{V}_1$. Therefore it can be concluded from (3.30) that $V(t) \in L^\infty$ for $t \in (t_1,t_2)$. By repeating the argument above, the boundedness of all the signals is proved for the time interval (t_1,t_2). Continuing in the same way, finally we have that $V(t) \in L^\infty$ for $t \in (t_q,\infty)$ with $V(t_q^+) = V(t_q^-) + \bar{V}_q$ for a finite $\bar{V}_q$ with a similar form as $\bar{V}_1$. Due to the finite times of actuator failures, it can be concluded that $V(t)$ is bounded for $\forall t > t_0$, and so are all the closed-loop signals.

At the last time interval (t_q,∞) with a positive finite initial $V(t_q^+)$, it follows from (3.30) that $z \in L^2$. In particular, $z_1 \in L^2$. Together with $\dot{z}_1 \in L^\infty$, we conclude that $\lim_{t\to\infty} z_1 = 0$, i.e., asymptotic output regulation is ensured. ∇

3.4 Simulation Study

A generic example is used to illustrate the effectiveness of the proposed adaptive scheme. Consider the following system

$$\begin{aligned} \dot{x}_1 &= x_2 + \vartheta_1 x_1\tan^{-1}(x_2) + \vartheta_2 x_1^2 x_3 \\ \dot{x}_2 &= \vartheta_3 x_1^3 x_3^2 + b_1u_1 + b_2u_2 \\ \dot{x}_3 &= \vartheta_4(e^{x_1}-1) + \vartheta_5 x_1 + \vartheta_6 x_3 \\ y &= x_1, \end{aligned} \quad (3.34)$$

where ϑ_i, $i = 1,2,\ldots,6$, and b_1 and b_2 are unknown constants.

The differential equation for x_3 constructs the zero dynamics of the system (3.34). With $\vartheta_6<0$, the zero dynamics are input-to-state stable such that the minimum-phase condition is satisfied as stated in Assumption (A2). It in turn implies that $|x_3|\leq\rho_1(t)+\rho_2(t)$, where $\rho_1(t)$ is governed by $\rho_1(t)=F_1(s)\eta_1(y)$ with $F_1(s)=\frac{\vartheta_5/2}{s-\vartheta_6}$ and $\eta_1(y)=y^2+1$, and $\rho_2(t)$ is governed by $\rho_2(t) = F_2(s)\eta_2(y)$ with $F_2(s) = \frac{\vartheta_4}{s-\vartheta_6}$ and $\eta_2(y) = e^y - 1$. Define that $\phi(x,t) = [\vartheta_1 x_1\tan^{-1}(x_2) + \vartheta_2 x_1^2 x_3, \vartheta_3 x_1^3 e^{x_3}]^T$, and $k=[(|b_1|+|b_2|),0]^T$, when u_1 and u_2 are both active, and $k=[|b_i|, b_j\bar{u}_j]^T$ if u_i is active and u_j is failed, $i\neq j\in\{1,2\}$, where $\bar{u}_1$ and $\bar{u}_2$ are the unknown values.

Choose a vector $l \in R^n$ such that $A_o = A - lc^T$ is stable, where A and c have the the form as those in (3.5). With knowledge of k_1 and k_2, the nominal observer for x is designed as $x^* = \xi + \xi^* + \sum_{j=1}^{2} k_2\zeta + k_1\mu$, with the filters defined as

$$\begin{aligned} \dot{\xi} &= A_o\xi + ly,\ \dot{\xi}^* = A_o\xi^* + \phi(x,t), \\ \dot{\zeta} &= A_o\zeta + e_2,\ \dot{\mu} = A_o\mu + e_2 v_0. \end{aligned} \quad (3.35)$$

Assuming that $|\vartheta_6| > \delta_0 > 0$ and δ_0 is known, we introduce the dynamic bounding signal $\chi_0 = [\chi_{01},\chi_{02}]^T \in R^2$, which is generated from

$$\dot{\chi}_0 = -\delta_0\chi_0 + \eta(y), \quad (3.36)$$

where $\eta(y) = [\eta_1(y),\eta_2(y)]^T \in R^2$. It follows that

$$\|\phi(x,t)\| \ \leq\ \theta^T\bar{\Phi}, \quad (3.37)$$

where $\theta = [\theta_1, \theta_2, \theta_3, \theta_4, \theta_5]^T$, $\theta_1 = \frac{\pi}{2}|\vartheta_1|$, $\theta_2 = |\vartheta_2|\delta_{f_1}$, $\theta_3 = |\vartheta_2|\delta_{f_2}$, $\theta_4 = 2|\vartheta_3|\delta_{f_1}^2$, $\theta_5 = 2|\vartheta_3|\delta_{f_2}^2$, with δ_{f_i}, $i = 1,2$, defined after (3.9), and $\bar{\Phi} = [|x_1|, x_1^2\chi_{01}, x_1^2\chi_{02}, |x_1^3|\chi_{01}^2, |x_1^3|\chi_{02}^2]^T$. Furthermore we define $k_{30} = [\theta_1^2, \theta_2^2, \theta_3^2, \theta_4^2, \theta_5^2]^T$, $\kappa = \frac{1}{k_1}$, and $\varsigma_0 = [1, x_1^2\chi_{01}^2, x_1^2\chi_{02}^2, x_1^4\chi_{01}^4, x_1^4\chi_{02}^4]^T$. With Assumption (A3), the adaptive controller is thus derived as

$$v_j = v_0,\ j = 1,2,\ v_0 = \alpha_2 + \frac{\partial \alpha_1}{\partial \zeta_2}, \qquad (3.38)$$

with the adaptive laws for $\hat{\kappa}$, $\hat{k}$, and $\hat{k}_{30}$:

$$\dot{\hat{\kappa}} \ = \ -\gamma z_1 \bar{\alpha}_1,\ \dot{\hat{k}} = \Gamma \tau_\rho,\ \dot{\hat{k}}_{30} = \lambda_0 z_1^2 \Gamma_{30} \varsigma_0, \qquad (3.39)$$

via the backstepping procedure given in Section 3.2.

In the simulation, the initials of the states are chosen as $x(0) = [-0.2, -0.2, 0.2]^T$, the initials of the estimates $\hat{\kappa}$, $\hat{k}_1$, $\hat{k}_2$, and $\hat{k}_{30}$ are set as $[5, 0.5, 0, 0, 0, 0.1, 0.75, 1]^T$. The gains are $c_1 = c_2 = 0.2$, $d_1 = d_2 = 0.7$, $\lambda_0 = 1$, $\gamma = 1$, $\gamma_1 = 10$, $\gamma_2 = 1$, $\Gamma_{30} = 0.1I$, and $l = [2, 1]$. For simulation, u_1 fails at the 10th second with an unknown value $\bar{u}_1 = 0.1$ and the system response and inputs are shown in Figure 1 and Figure 2 respectively.

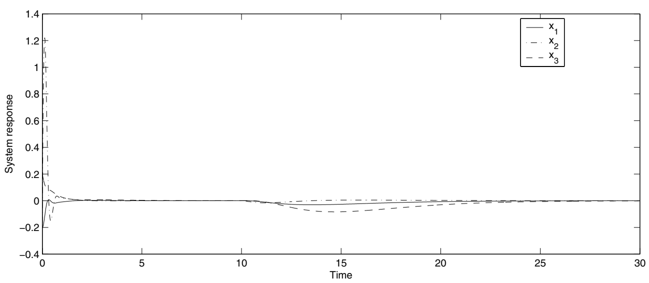

Figure 1: System states.

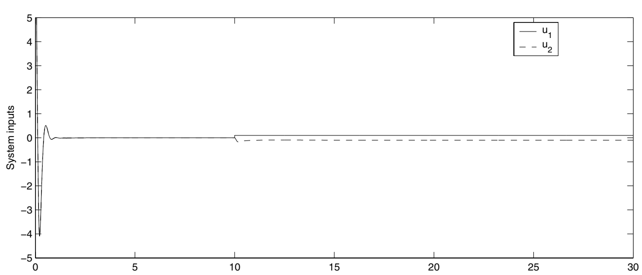

Figure 2: System inputs.

4 Concluding Remarks

Actuator failures have a major influence on the performance and stability of control systems. Compensation of unknown actuator failures is a challenging problem with many open issues, especially for nonlinear systems. In this paper, the actuator failure compensation problem is solved for an enlarged class of nonlinear systems with unknown state-dependent nonlinearities. The state-dependent nonlinearities are bounded not only by some static output-dependent functions, but also by some signals dynamically dependent on the output. An adaptive output feedback scheme is developed to ensure closed-loop stability and asymptotic output regulation in the presence of the unknown actuator failures and uncertain system nonlinearities and parameters. Under a relaxed condition on the bounding functions, a different robust adaptive control scheme is proposed in another paper.

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