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Nonuniform Multi-rate Sampled-data $\mathcal{H}_\infty$ Following Control of HDDs

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***Abstract*— This paper is concerned with the use of the sampled-data $\mathcal{H}_\infty$ servo controller with generalized hold for the track-following control of hard disk drives (HDDs). Conventional input multi-rate control normally utilizes a fast zero order hold that is uniformly time-divided piecewise constant function. However, there is no need to assume such an uniformity in hold duration. Elimination of this assumption increases the design degree-of-freedom. In this paper, we propose to invoke the sampled-data $\mathcal{H}_\infty$ servo control theory to have an optimal piecewise continuous hold function. The derived function is used as a basis of our proposed non-uniformly time-divided piecewise constant function, which is introduced to implement it on standard systems with digital-to-analogue conversion devices. We conducted a set of simulations as well as experiments, and confirmed that the proposed method can improve the control performance.**

I. Introduction

Control performance in sampled-data systems is in most part governed by the sampling frequency. Hard Disk Drives (HDDs) are not the exceptions, however, a hold function in HDDs does not have much limitation on its running frequency compared to the sampler, thus the use of the input multi-rate scheme can gain some degree-of-freedom that has a lot of advantages.

Multi-rate control has been developed since around the late 1950's, and an early attempt for application to HDDs can be found in [1]. This idea was attractive because of its simplicity, and was extended by [2] to have a smoother inter-sample estimation even under the existence of exogenous disturbances and model uncertainty. However, the robustness and the optimality were still key concerns.

More recently, multi-rate $\mathcal{H}_\infty$ control design methodology for HDDs [3]-[6] has been discussed to improve their efficiency, and have successfully been used in industry. However, in any of the aforementioned schemes, a hold function is pre-determined. To be specific, they assumes a zero-order hold (ZOH) with the hold duration being constant everywhere between the sampling instances. This assumption is reasonable to synthesize/design the control system because of its simplicity. However, the only limitation imposed on both of the sampler and the hold of HDDs is the sampling time. Thus, such a pre-determination in control design is not appropriate, and should be eliminated to gain an additional design degree-of-freedom for the performance improvement. Motivated by the above discussion, [7] utilizes nonuniform time-divided hold. It derived such a multi-rate hold so as to eliminate the unstable zeros which arises when evaluate the system at the sampling instances. It also proposes a way of obtaining the hold so as to minimize the control input power at mechanical resonant frequency ranges.

On the other hand, the use of the generalized sampled-data hold function (GHF or GSHF) [8][9] draws an authors' attention from the design point of view, since it does not assume any hold in the design procedure. In [10], a way to implementing it on a practical system was proposed simply by approximating the optimal hold with an uniformly time-divided fast zero order hold, which is nothing but the conventional input multi-rate scheme. However, this scheme is not necessarily appropriate, since the obtained optimal hold function has complicated distribution with respect to time, and hence it is not implementable.

In this paper, we propose the use of a non-uniformly time-divided fast zero order hold as an approximation of an optimal hold for HDDs. By its use, the approximation is necessarily improved compared to the uniformly time-divided fast zero order hold.

II. Sampled-data $\mathcal{H}_\infty$ servo controller with optimal hold

There are several methods to design an optimal or suboptimal hold function. One way is to utilize the $\mathcal{H}_\infty$ control scheme, since the robustness against a class of uncertainty can be considered systematically, by which we can have the best performance. In [9], the GHF scheme in [8] was successfully introduced in the sampled-data $\mathcal{H}_\infty$ control problem. In [11], the loop-shaping procedure proposed by [12] was considered with the use of the GHF. [13] extended the idea of [9] to weaken the assumptions as well as to deal with the servo problem required in the HDDs control design. The method has been implemented in a computer aided control systems design (CACSD) software [14], and we will follow the scheme to design the better sampled-data controller in this paper.

Let us recall the sampled-data $\mathcal{H}_\infty$ servo configuration in [13], where the generalized plant is given by

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$$G_d : \begin{bmatrix} \dot{x}(t) \\ z(t) \\ y(t) \end{bmatrix} = \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t) \\ w(t) \\ u(t) \end{bmatrix}. \quad (1)$$

Following all the assumptions in [13], we have the resultant controller of the form

$$\begin{aligned} x[k] &= A_c x[k-1] + B_c y_d[k] \\ u(t) &= H(t)x[k], \end{aligned} \quad (2)$$

where A_c and B_c are resultant constant matrices, and $H(t)$ is a resultant hold function. Notice that the control input $u(t)$ is a continuous-time signal while the measurement $y_d[k]$ is a discrete-time signal.

III. Practical design of optimal hold for HDDs

The target drive considered is a 3.5" with 10kRPM and 10kHz sampling frequency, whose frequency response is shown in **Fig. 1**. Double integrator model of F_s^2/s^2 with some time delay is used for a nominal plant, where F_s is the sampling frequency. The resonant modes are regarded as model error and they will be taken care of in the weighting function setting.

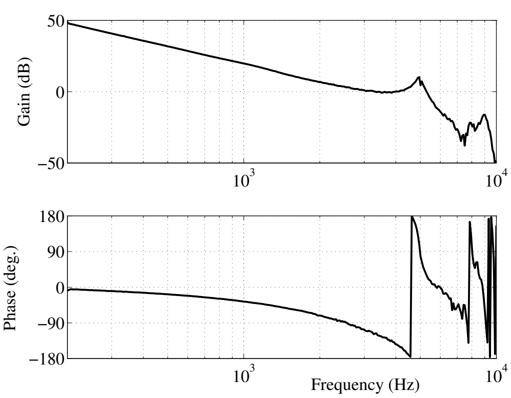

Fig. 1. Plant frequency response.

A. Generalized plant

The generalized plant proposed here for the track-following control of HDDs is depicted in **Fig. 2**. This configuration is intended to evaluate the unstructured model uncertainty as multiplicative error by the signal from w_1 to z_1 and the disturbance rejection performance by the signals from w_2, w_3 to z_2. $\mathcal{P}$ is a controlled plant, and η, g, and d are all positive scalar design parameters that leave us the space to adjust the performance of the controller, which plays the most important role in the practical design. W_s and W_t are frequency dependent weights for disturbance rejection and robust stability, respectively. Q_e is a positive scalar design parameter to specify the controller's integrator gain.

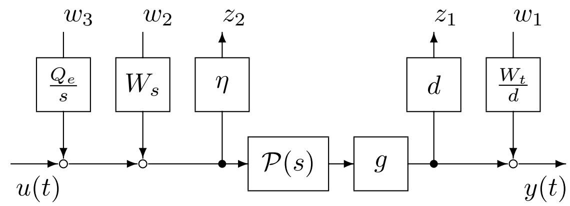

Fig. 2. Generalized plant $G_d(s)$ for sampled-data $\mathcal{H}_\infty$ servo problem with GHF

B. Disturbance rejection specification

The disturbance rejection performance or the sensitivity function of the system is specified by Q_e/s in conjunction with the frequency weighting function W_s. W_s must be selected as a strictly proper function by assumption in (1) where D_{11} is assumed to be zero. We have selected $W_s = \beta/(s+\alpha)$ for simplicity. Thus, overall specification to the disturbance rejection becomes as follows.

$$\frac{Q_e}{s} + W_s = \frac{Q_e}{s} + \frac{\beta}{s+\alpha} = \frac{(Q_e+\beta)s + Q_e\alpha}{s(s+\alpha)} \; (=: W) \quad (3)$$

These free parameters, α, β, and Q_e are adjusted to represent the estimated exogenous disturbance spectrum shown in **Fig. 3**.

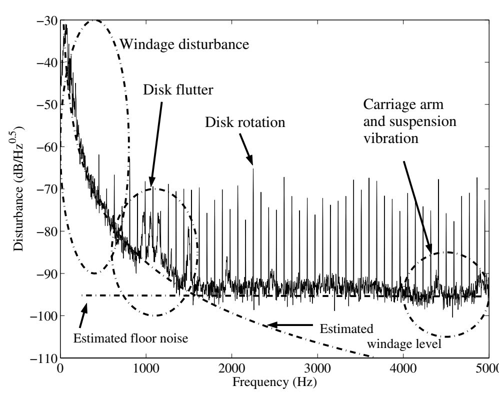

Fig. 3. Disturbances and noises in HDDs.

C. Robust stability specification

Robust stability is specified by the frequency weighting function W_t as the multiplicative error of the plant. W_t is assumed to be a strictly proper transfer function since D_{21} is assumed to be zero. It should be pointed out that this may violate the needs to make the controller's gain at higher frequency range smaller because of the plant model uncertainty. However, as [4] pointed out, in this type of configuration, W_t is not only the weight for robust stability but for the evaluation of the measurement noise. Too much weight at higher frequency range tends to limit the achievable performance due to the unexpected aliasing consideration, which may not exist in a real system. Taking these facts into account, we selected it as follows.

$$W_t = \frac{s^2 + 2\zeta_1\omega_1 s + \omega_1^2}{s^2 + 2\zeta_2\omega_2 s + \omega_2^2} \cdot \frac{k_3}{s + \omega_3} \tag{4}$$

The first factor is not only to notch out the dominant resonant modes but also to consider the uncertainty in higher frequency range, which is usually from 5 to 10kHz. They can be shaped by ω_2,ζ_2 and ω_1,ζ_1. The second factor is to satisfy the condition on D_{21} as well as to avoid the conservatism.

D. Design result

We obtained a controller after several trial-and-error procedures. Corresponding design parameters are $Q_e = 638$, $\eta = 1$, $\alpha = 2\pi 10^4$, $\beta = 10^4$, $d = 80$, $g = 0.316$, $\zeta_1 = 0.4$, $\omega_1 = 2\pi 4000$, $\zeta_2 = 0.05$, $\omega_2 = 2\pi 4800$, $k_3 = 2.52 \times 10^4$, $\omega_3 = 2.5 \times 10^4$, and $|W_s|$ and $|W_t|$ are plotted in **Fig. 4**. The gain of resultant sensitivity function and complementary sensitivity function are depicted in **Fig. 5**, which are calculated by using the technique described in [10].

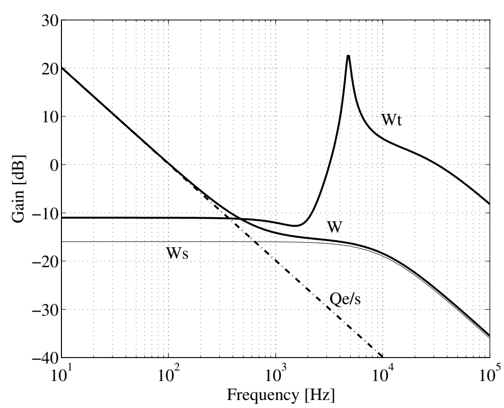

Fig. 4. Specifications on disturbance rejection W_s and robust stability W_t.

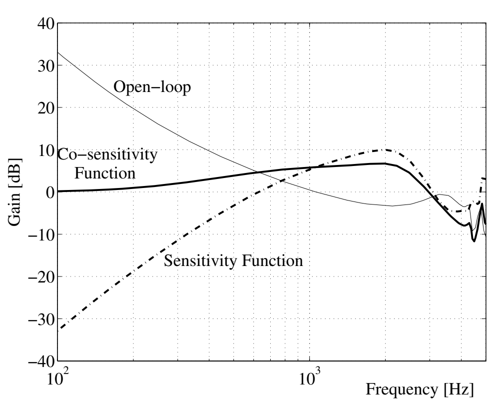

Fig. 5. Sensitivity function and complementary sensitivity function.

IV. Consideration on the implementation

Currently, there is no widely-used hold device but the digital-to-analog conversion devices (DAC), and HDDs unexceptionally utilize it. So the consideration on the implementation with the DAC is inevitable. In [10], we have exemplified the use of an uniformly time-divided piecewise constant hold to approximate the GHF and to implement it on a DSP, which is exactly the same as the conventional multi-rate control used in HDD industry. However, this method is not necessarily the best representation of the obtained hold function since it tends to have complex distribution with respect to time, especially when the stabilizing solution has dominant complex numbers in closed-loop system.

A. Basic idea of non-uniformly time-divided hold

First, let us see the obtained hold function $H(t)$. The order of the resultant controller is 6, meaning that $H(t)$ is, by definition, a piecewise continuous function with respect to time, with dimension of $H(t) \in \mathbb{R}^{1\times 6}$ as illustrated in **Fig. 6**. When we try to implement this as a 4-times multirate controller, for example, as is depicted in **Fig. 7**, it is clear that the uniformly time-divided piecewise holds will have almost the same values in the first and second pieces, since the function does not vary much up to $t = 0.7T_s$. This poses our motivation to optimize the way to divide it with respect to time.

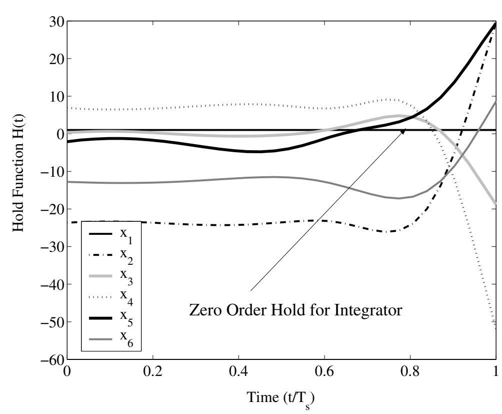

Fig. 6. Hold function.

One way to optimize the timing to switch as well as the magnitude to hold is to formulate the problem as a cost function minimization problem as in [16][7]. First, let us introduce the non-uniformly time-divided hold $\tilde{H}_i \in \mathbb{R}$ as a suboptimal representation of the given optimal hold $H(t)$, where i represents the i-th switching point, and the corresponding cost function J.

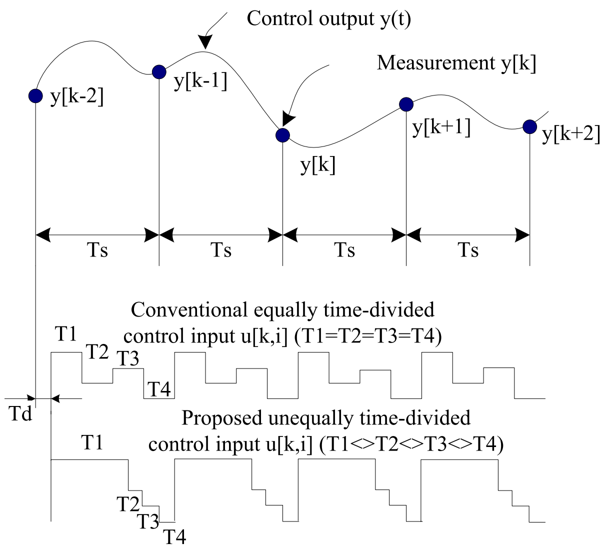

Fig. 7. Basic idea of non-uniformly time-divided hold function.

$$\tilde{H}(\tau) := \begin{cases} \tilde{H}_1, 0 \le \tau < t_1 \\ \tilde{H}_2, t_1 \le \tau < t_2 \\ \vdots \\ \tilde{H}_r, t_{r-1} \le \tau < t_r = T_s \end{cases}, \tau \in [\ 0 \quad T_s\ ) \tag{5}$$

$$J := \sum_{i=1}^{r} \int_{t_{i-1}}^{t_i} \left(\left\| H(\tau) - \tilde{H}(\tau) \right\| \right) d\tau \tag{6}$$

This can be viewed as a nonlinear minimization of J over a set of parameters H_i and T_i with constraints $T_i > 0 (i = 1, \cdots, r)$ and $\sum_{i=1}^{r} T_i = T_s$, which can be dealt with by the sequential quadratic programming (SQP) method [17].

B. Practical design and the results

In practice, the constraints discussed in the previous subsection are sometimes not enough, since it tends to yield too short duration in T_i. Too short duration makes the implementation not only difficult but also meaningless from the practical point of view. Hence, we introduced an additional constraint of $T_i \ge 0.05T_s$, and we obtained the solutions as $T_1 = 0.74T_s$, $T_2 = 0.16T_s$, $T_3 = T_4 = 0.05T_s$, whose corresponding non-uniformly time-divided piecewise constant hold function are shown in **Fig. 8** with the original optimal hold function, where only the one for the state variable x_2 is depicted in this figure.

Notice that, ignoring the inter-sample behavior, we may evaluate how close the obtained system is to the optimal one in terms of linear time-invariant system evaluated only at the sampling instances as follows.

First, as in [10], we can get the equivalent open-loop transfer function for the optimal system as follows:

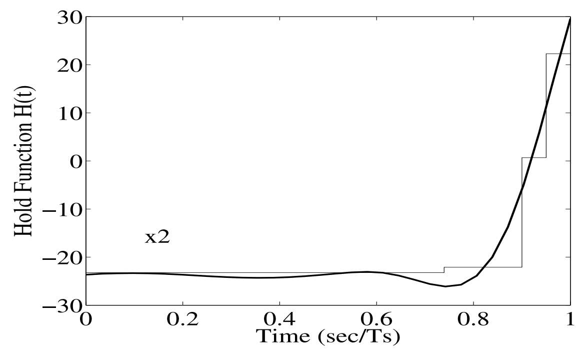

Fig. 8. SQP optimization for representation of the optimal hold function

$$\left[\begin{array}{cc|c} e^{AT_s} & MA_c & MB_c \\ 0 & A_c & B_c \\ \hline C_2 & 0 & 0 \end{array} \right], \tag{7}$$

where

$$M := [\ I \quad 0\ ] \exp\left(\begin{bmatrix} A & B_2C_c \\ 0 & D_c \end{bmatrix} T_s \right) \begin{bmatrix} 0 \\ I \end{bmatrix}.$$

The open-loop equivalent for the proposed system can be obtained by breaking up the signal at the control output with the use of the following lifted model for the controller,

$$\begin{aligned} x[k] &= A_c x[k-1] + B_c y_d[k], \\ u[k] &= H_c x[k], \end{aligned} \tag{8}$$

where

$$H_c := [\ \tilde{H}_1 \quad \tilde{H}_2 \quad \cdots \quad \tilde{H}_r\ ]^T,$$

and for the plant

$$\tilde{\mathcal{P}}[z] := C_{s2} \left(A_s - zI \right)^{-1} \tilde{B}, \tag{9}$$

with

$$\begin{aligned} \tilde{B} &:= [\ \tilde{B}_1 \quad \tilde{B}_2 \quad \cdots \quad \tilde{B}_r\ ], \\ \tilde{B}_i &:= \int_{t_{i-1}}^{t_i} e^{A(t_i - \tau)} B_2 d\tau. \end{aligned}$$

Combining the controller and the plant together yields the open-loop transfer function as follows.

$$\left[\begin{array}{cc|c} A_s & \tilde{B}H_cA_c & \tilde{B}H_cB_c \\ 0 & A_c & B_c \\ \hline C_{s2} & 0 & 0 \end{array} \right] \tag{10}$$

The difference between (7) and (10) lies only in the term M and $\tilde{B}H_c$. Thus the evaluation can be performed by checking the maximum singular value $\bar{\sigma}(M - \tilde{B}H_c)$, which was 0.080 for our proposed nonuniform multi-rate approximation while that of the standard uniform one with $T_1 = T_2 = T_3 = T_4$ was 0.367. This difference is one way to understand the effectiveness of our proposed method. Further notice that this evaluation may be used as a cost function for the optimization as in (6) although the optimization does not take the inter-sample behavior into account.

Now, let us compare the control performance in terms of power spectral density of position error signal and sensitivity function, which are depicted in **Fig. 9**. The top row illustrates the result for the proposed nonuniform multi-rate controller that is designed in this section by using SQP algorithm. The second one is the case of generalized hold method discussed in the previous section that is driven with the hold functions in **Fig. 6**. The third one is the uniform multi-rate hold method as in [10] and the bottom is the single-rate result that is obtained by simply averaging the generalized hold function over one sampling time. Non-repeatable position error (NRPE) as positioning accuracy of each result is shown on the corresponding figure.

What we can see from two results in the first and second figures is that our proposed method does not deteriorate the positioning accuracy in any frequency range, implying that the method is an efficient and effective way of implementing the optimal hold function. Note that the generalized hold itself is not implementable and that its high accuracy uniform multi-rate approximation such as 32-times multi-rate is very hard to implement due to its computational cost. On the other hand, the computational cost of the proposed method is same as that for 4-times multi-rate case, and hence it can be easily implemented. Moreover, as compared with the results of 4-times uniform multi-rate and single-rate, we have the better positioning accuracy, implying that the design method with the generalized hold function can improve the achievable performance.

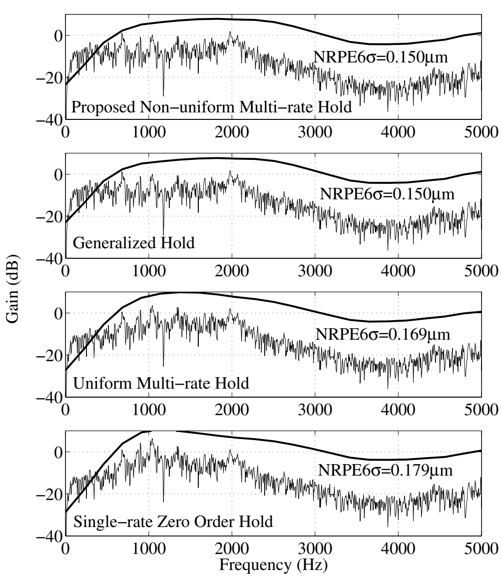

Fig. 9. Comparison of sensitivity function and power spectral density of position error signal.

V. Experiment

We compare the proposed multi-rate controller (MRSDC) with conventional multi-rate observer/regulator scheme (MROBS) as in [2] as well as a continuous-time $\mathcal{H}_\infty$ controller with D-scaling (MRMU) as in [5] that can be obtained through D-K iteration algorithm. MROBS is not optimized much for the positioning accuracy, so it can be regarded as a conservative controller. On the other hand, MRMU is optimized by trial-and-error via frequency weight to have the best positing performance under some design criteria on classical stability margin (5dB/30deg for gain/phase margin).

Fig. 10 shows the open-loop frequency responses. All the controllers have sufficient roll-off at around 5kHz that successfully compensate for the dominant resonant modes. Although the proposed controller is the worst from the view point of classical stability margins, it does not necessarily mean the inferiority from the sensitivity function point of view. **Table I** summarizes the NRPE, bandwidth (Bw), gain margin (Gm), and phase margin (Pm). As we can see,

TABLE I
Summary of the performance of the proposed controller.

K	NRPE	Bw(Hz)	Gm(dB)	Pm(deg)
MROBS[2]	0.1983	670	6.2	31.2
MRMU[5]	0.1773	830	6.8	39.2
MRSDC	0.1665	840	6.6	27.8

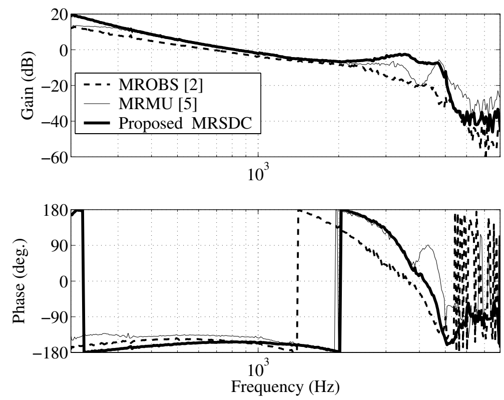

Fig. 10. Comparison of open-loop frequency response.

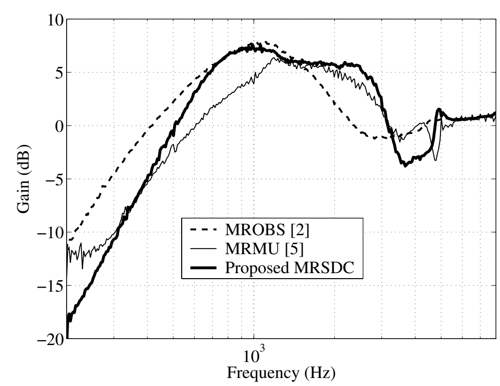

Fig. 11. Comparison of sensitivity function.

MRSDC has the best positioning accuracy, and this can be backed by the sensitivity function (shown in **Fig. 11**) that is optimized to attenuate the disturbances to have the PES spectrum as shown in **Fig. 12**.

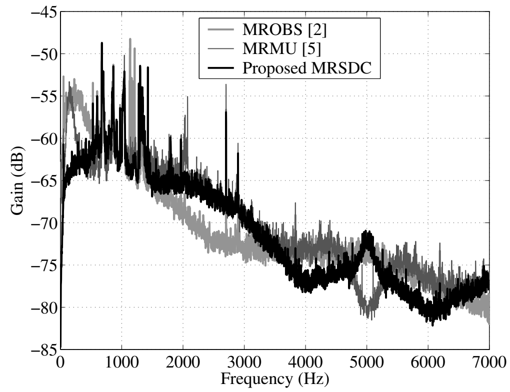

Fig. 12. Comparison of position error signal in power spectral density.

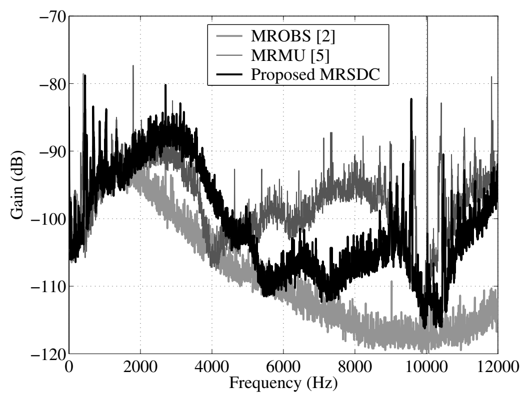

Fig. 13. Comparison of control input in power spectral density.

On the other hand, a large difference can be seen in the power spectrum of control input as depicted in **Fig. 13**. In the continuous design, much control effort is concentrated at the frequency range of 5-9kHz in order to have smaller γ on performance evaluation in $\mathcal{H}_\infty$ optimization, where this frequency range is normally not used because of the aliasing problem and the existence of unmodelled resonant modes. The proposed controller successfully suppress them, meaning the optimization performance can be thought among the best.

VI. SUMMARY

In this paper, we have proposed to invoke the sampled-data $\mathcal{H}_\infty$ servo control theory to have an optimal piecewise continuous hold function. Obtained function is used as a basis of our proposed non-uniformly time-divided piecewise constant function, which is introduced to implement it on standard systems with digital-to-analogue conversion devices.

We have demonstrated its practical use though a set of simulations and experiments. First, we have exemplified the effectiveness of our proposed nonuniform multi-rate hold by clarifying by simulations that the performance deterioration of the proposed one against the original optimal hold is very small. Secondly, we have carried out some experiments to compare its positioning accuracy with that of other controllers already designed by different methods, and have confirmed that it has among the best control performance.

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