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# Discrete-Time Closed-Form Solution of Time-Optimal Seek Control for Hard Disk Drives

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*Abstract*—**This paper derives a discrete-time closed-form solution of time-optimal seek control based on the 3rd-order hard disk drive model including the back-EMF effect and coil inductance. Using the proposed method, online reference signal generation for 2-DOF control structure is discussed for short and long seeking cases. Simulation results show that the reference signals generated are good for smooth settling. Modification of the proposed method for improving acoustics during seeking is demonstrated. A feasible implementation of the proposed method is investigated for efficient usage of digital signal processor and limited memory available. The feed-forward current for seek control is obtained considering the power amplifier saturation so the proposed method has better features of adaptation for varying operating conditions.**

## I. Introduction

The demand for data storage such as semiconductor memory and hard disk drive (HDD) is increasing as there is more need for storing digital data in everyday life. Unlike the semiconductor memory, HDD operation requires mechanical movement of head-slider-arm system.

As track per inch (TPI) is increasing, the HDD actuator must achieve precise positioning of the read/write head on a desired track (track following). The ability for the actuator to seek from one track to another target track (track seeking) is also very important because the data retrieval performance of the drive (IOPs) is directly affected by how quickly and precisely the head seeks from one track to another.

As a HDD servo system, a controller mode-switching method is widely used; a velocity controller is employed in seeking and a position feedback controller such as a PID controller or state-space regulating controller is used in track following.

During seeking, the disk head actuator such as a voice coil motor (VCM) is driven by a bang-bang current profile to achieve time-optimal control, but due to the presence of resonances, the ideal bang-bang profile is smoothed out in actual implementation.

The seek algorithm that has been widely used in industry is called the proximate-time-optimal-servomechanism (PTOS) [1]. There have been approaches on the refinement of PTOS, focusing on the simultaneous minimization of seek time and resonance excitation using command input shaping method [7],[8],[9]. These approaches are all derived using the velocity and position error relationship (phase-plane), employing mode-switching control.

On the other hand, there have been several attempts to develop unifying control algorithms, which work for both track following and seeking. Such control algorithms utilize the two-degree-of-freedom (2-DOF) control structure [6].

Feed-forward control is an important factor of the 2-DOF servomechanism. It is required to design feed-forward input well to shorten the time needed for head positioning, although the way to use the feed-forward signal depends on the control structure. A reference model plays an important role in generating the feed-forward signals. A reference signal generation approach was proposed to effectively use a VCM actuator for fast track seeking [5]. The approach requires two steps. In the first stage, offline simulations are iterated to set up a lookup table between control voltages and seek length-seek time relation. In the second stage, a reference model including back EMF effect generates reference trajectories online using the data extracted in the first stage. The idea of structural vibration minimized acceleration trajectory (SMART) [10] is utilized to adjust the non-zero errors at the end of seek.

The approach lacks the functionality of determining the proper control voltage levels in advance for the desired seek length so it requires iterations to obtain control voltage levels, with inherent limitation of non-zero final states.

This paper proposes a new reference signal generation algorithm based on a discrete-time closed-form solution of time-optimal seek control using "*pseudo-inverse of matrix*" [2]. The control voltage level for the desired seek length can be directly calculated online therefore the adjustment of non-zero final states utilizing SMART is not needed in application. The modification of the proposed solution for improving acoustics and reducing residual vibration is also presented. The feasible implementation of the proposed method is investigated for efficient usage of the limited computational power of digital signal processor and memory available in commercial disk drives.

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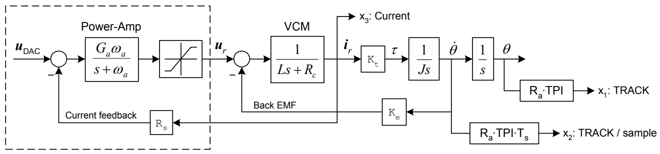



Fig. 1. A hard disk dynamic model including back EMF

## II. Reference Signal Generation

### A. System Modeling

The block diagram of HDD dynamic model used for the derivation of the proposed algorithm is shown in Fig. 1, where $J$ is the moment of inertia, $K_t$ is the torque constant, $K_e$ is the back-EMF constant, $L$ is the coil inductance, $R_c$ is the coil resistance, $R_s$ is the current sense resistance, and $R_a$ is the arm length. The state variables of the dynamic model are selected as,

$$x_1 = R_a\theta \cdot TPI = \text{TRACK}$$

$$x_2 = R_a\dot{\theta} \cdot TPI \cdot T_S = \text{TRACK / sample}$$

$$x_3 = i_r = \text{Current}$$

Then, the dynamic description of each state variable is,

$$\dot{x}_1 = R_a\dot{\theta} \cdot TPI = x_2 / T_S$$

$$\dot{x}_2 = R_a\ddot{\theta} \cdot TPI \cdot T_S = R_a \frac{\tau}{J} TPI \cdot T_S = \frac{K_t R_a \cdot TPI \cdot T_S}{J} x_3$$

$$\dot{x}_3 = (u_r - K_e\dot{\theta} - R_c \cdot i_r)/L = \frac{u_r}{L} - \frac{K_e}{(R_a \cdot TPI \cdot T_S)L} x_2 - \frac{R_c}{L} x_3$$

$$\dot{X} = AX + Bu_r \tag{1}$$

$$A = \begin{bmatrix} 0 & 1/T_S & 0 \\ 0 & 0 & \frac{K_t(R_a \cdot TPI \cdot T_S)}{J} \\ 0 & -\frac{K_e}{L(R_a \cdot TPI \cdot T_S)} & -\frac{R_c}{L} \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ 1/L \end{bmatrix}$$

To develop the proposed algorithm, the continuous-time model (1) is converted to a discrete-time model using sampling time $T_s$ as:

$$X(k+1) = \Phi X(k) + \Gamma\, u_r(k) \tag{2}$$

where $k$ denotes the discrete-time instance, which is called hereafter the sample number.

### B. Time-Optimal Seek Control

In HDD seeking control, the process is normally divided into two stages such as short seeking and mid-long-seeking due to the velocity safety limit and the saturation of power amplifier.

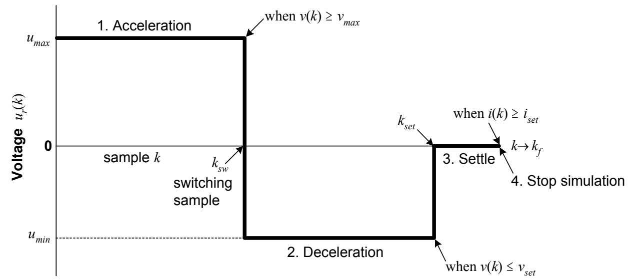



Fig. 2. Control voltage input for short-seek simulation

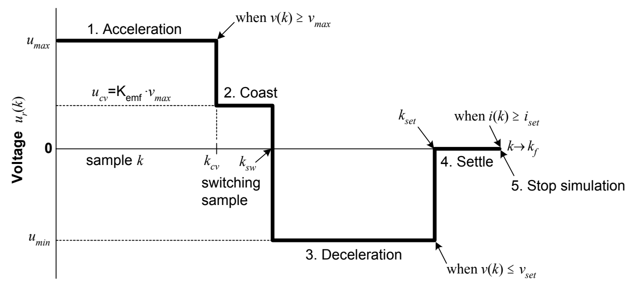



Fig. 3. Control voltage input for mid- and long-seek simulation

The control input in feedback control structure is the demand current equivalent, $u_{DAC}$. The torque generated by VCM actuator is actually due to power amplifier output, $u_r$, not the demand current. The basic idea of the reference signal generation [5] is summarized as: 1) look for proper control voltage for desired seek length; 2) apply the control voltage level to the discrete-time system model which generates online reference signals such as position (TRACK), velocity (TRACK/sample), and current; 3) the current signal is fed as a demand current equivalent, $u_{DAC}$ and reference position and velocity signals are fed to the feedback controller in the 2-DOF control structure.

The approach lacks the functionality of determining the proper control voltage in advance for the desired seek length so the offline iterations are required to set up a lookup table for seek time-seek length relationship.

Here, we develop a discrete-time closed-form solution of time-optimal seek control using "*pseudo-inverse of matrix*" [2], which does not require iterations and lookup table, with negligible non-zero final states.

### C. Four-stage seek control for short-seeking

As seen in Fig. 2, there are four stages in short seeking control; 1) acceleration; 2) deceleration; 3) settle; and 4) stop.

From (2), the recursive solution of dynamic state equation is described as,

$$X(k_f) = \Phi^{k_f} X(0) + \sum_{j=0}^{k_f-1} \Phi^{k_f-1-j} \Gamma \cdot u(j) \tag{3}$$

where the states are $X(k)$=[pos($k$), vel($k$), cur($k$)]$^T$.

The proposed algorithm is derived assuming that the desirable seek time vs. seek length is given in advance before seek control, as shown in Fig. 4. For a given seek length, the desired seek time is to be looked up in the Fig. 4 and the corresponding seek time is interpreted as the number of samples for a head to move to the desired track.

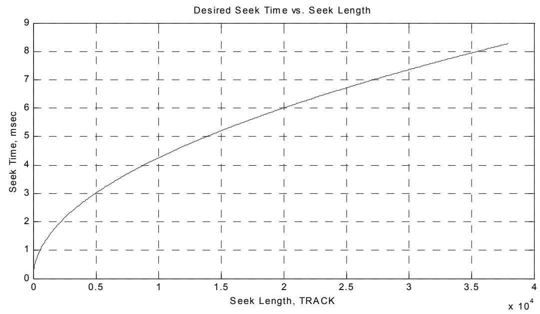



Fig. 4. Desired seek time vs. seek length

Equation (3) is developed with zero initial conditions as,

$$X(k_f) = \left( \sum_{j=0}^{k_{SW}-1} \Phi^{k_f-1-j} \Gamma \right) u_{\max} + \left( \sum_{j=k_{SW}}^{k_{SETTLE}-1} \Phi^{k_f-1-j} \Gamma \right) u_{\min} \quad (4)$$

where $k_f$ is the seek time equivalent sample number, $k_{sw}$ is the switching sample, and $k_{SETTLE}$ is the sample number when the control voltage is set to zero. Equation (4) is simplified after matrix calculations are convolved as,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \cdot \begin{Bmatrix} u_{\max} \\ u_{\min} \end{Bmatrix} = \begin{Bmatrix} Y_f \\ 0 \\ 0 \end{Bmatrix}, \ AU = Y \quad (5)$$

where $Y_f$ is the target track. The *primary advantage* of this algorithm is that the final states of velocity and current can be set to zero, whereas the approach in [5] requires *non-zero* values of velocity and current which should be kept in lookup table.

The dimension of matrix $A$ in (5) is (3×2) and that of vector $U$ is (2×1) so the above simultaneous linear equation is *over-determined* [2]. Therefore, the solution for (5) is solved in a "*weighted least-squares approximation*" sense using "*pseudo inverse of matrix A*". The symmetric non-singular matrix $R$ with a dimension of (3×3) is selected properly for final states tolerance. The solution for control voltage levels is thus obtained as,

$$U = \left( A^T R^{-1} A \right)^{-1} A^T R^{-1} \cdot Y \quad (6)$$

In the 2-DOF control structure, the objective of seeking control is to move the actuator to the target track following the reference trajectories with minimum position error and overshoot so the reference trajectories are also important as well as the feed-forward current.

For example, if the final states of position, velocity, and current are weighted in order, then the matrix $R$ can be chosen as,

$$R = \begin{bmatrix} 0.25 & 0 & 0 \\ 0 & 0.5 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

### D. Five-stage seek control for mid- and long-seeking

In mid- or long-seeking, the seek control is performed in five stages due to the velocity safety limit and power amplifier saturation, as seen in Fig. 3: 1) acceleration; 2) coast; 3) deceleration; 4) settle; and 5) stop.

Equation (3) is decomposed into three parts considering back-EMF and velocity safety limit.

$$X(k_f) = \left( \sum_{j=0}^{k_{CV}-1} \Phi^{k_f-1-j} \Gamma \right) u_{\max} + \left( \sum_{j=k_{CV}}^{k_{SW}-1} \Phi^{k_f-1-j} \Gamma \right) u_{cv} + \left( \sum_{j=k_{SW}}^{k_{SETTLE}-1} \Phi^{k_f-1-j} \Gamma \right) u_{\min} \quad (7)$$

where $k_{CV}$ is the sample number when velocity coasting starts. Then, (7) is simplified as,

$$\begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \cdot \begin{Bmatrix} u_{\max} \\ u_{\min} \end{Bmatrix} + \begin{Bmatrix} C_1 \\ C_2 \\ C_3 \end{Bmatrix} \cdot u_{cv} = \begin{Bmatrix} Y_f \\ 0 \\ 0 \end{Bmatrix}$$

$$AU + Y_{cv} = Y_f \quad (8)$$

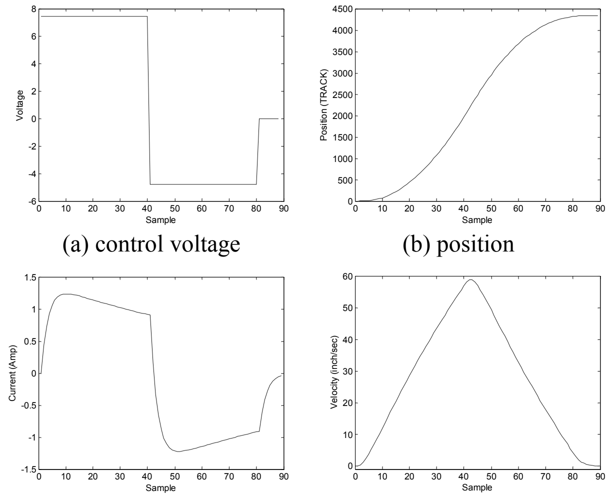
We have an extra term in (8) compared to (5) due to velocity coasting, but the additional term is a constant vector as the final states. Therefore, the solution for control voltage for five stage seek control is obtained as,

$$U = \left( A^T R^{-1} A \right)^{-1} A^T R^{-1} \cdot \left( Y_f - Y_{cv} \right) \quad (9)$$

## III. Simulation Example and Discussions

The proposed closed-form solutions of time-optimal seek control in discrete-time domain were explained in section II. The method is simulated to verify the usefulness in four- and five-stage seek control and the results are shown in Figures 5 and 6. Once the proper control voltage level for target track is calculated using (6) or (9), the reference signals are recursively generated in (3). The back-EMF effect and coil inductance are included in the 3rd-order HDD dynamic model so we can notice that the generated reference signals are naturally decayed at the end of seek, which is good for smooth settling.

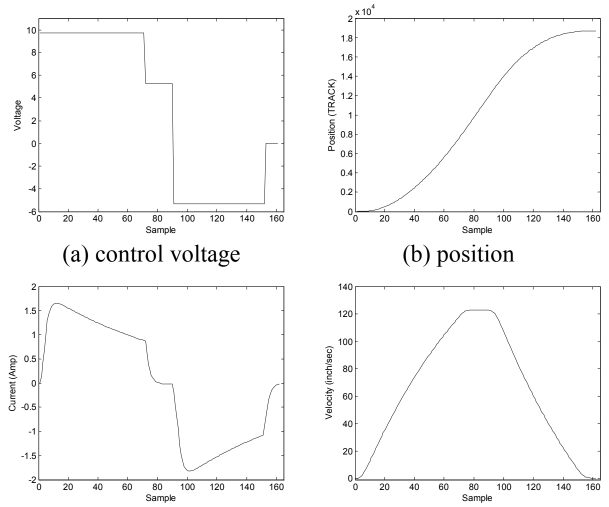
By investigating the current profiles of Figures 5 and 6, it is found that the maximum rate of change of current $|di/dt|_{\max}$ is high especially around the transition from acceleration to deceleration, which results in bad acoustics during seeking. The current profile is pretty much determined by the control voltage setting. Therefore, the proposed approach is to be modified for improving acoustics during seeking. The modification is to be described below.



(a) control voltage (b) position

(c) current (d) velocity

Fig. 5. Four-stage seek control example



(a) control voltage (b) position

(c) current (d) velocity

Fig. 6. Five-stage seek control example

By investigating the current profiles of Figures 5 and 6, it is found that the maximum rate of change of current $|di/dt|_{\max}$ is high especially around the transition from acceleration to deceleration, which results in bad acoustics during seeking. The current profile is pretty much determined by the control voltage setting. Therefore, the proposed approach is to be modified for improving acoustics during seeking. The modification is to be described below.

Instead of changing the control voltage from zero to certain voltage, putting *intermediate* values between the two control voltages makes the rate of change less abrupt. The proposed method can accommodate any intermediate values and still the closed-form solution can be obtained by proper development of the recursive state equation (3).

In order to show the modification procedure, the five-stage seek control is considered here. The four-stage case is just the simplification of the five-stage procedure.

Instead of considering general case, the half value of the two control voltages is chosen as the intermediate value to make the modification procedure simple to follow. The modified form of control voltage for five-stage control is illustrated in Fig. 7.

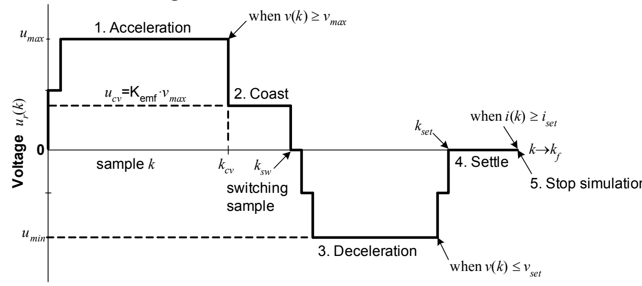



Fig. 7. Modified five-stage seek control

Then, (7) is modified to include the intermediate value as,

$$X(k_f) = \Phi^{k_f-1}\Gamma\left(\frac{u_{\max}}{2}\right) + \left(\sum_{j=1}^{k_{CV}-1}\Phi^{k_f-1-j}\Gamma\right)u_{\max} + \left(\sum_{j=k_{CV}}^{k_{SW}-1}\Phi^{k_f-1-j}\Gamma\right)u_{cv} \quad (10)$$

$$+\Phi^{k_f-1-k_{SW}-1}\Gamma\left(\frac{u_{\min}}{2}\right) + \left(\sum_{j=k_{SW}+2}^{k_{SETTLE}-2}\Phi^{k_f-1-j}\Gamma\right)u_{\min} + \Phi^{k_f-1-k_{SETTLE}+1}\Gamma\left(\frac{u_{\min}}{2}\right)$$

Equation (10) is developed and simplified as follows.

$$\begin{bmatrix} a_{11} \\ a_{21} \\ a_{31} \end{bmatrix}\frac{u_{\max}}{2} + \begin{bmatrix} a_{12} \\ a_{22} \\ a_{32} \end{bmatrix}u_{\max} + \begin{bmatrix} c_1 \\ c_2 \\ c_3 \end{bmatrix}u_{cv} + \begin{bmatrix} b_{11} \\ b_{21} \\ b_{31} \end{bmatrix}\frac{u_{\min}}{2} + \begin{bmatrix} b_{12} \\ b_{22} \\ b_{32} \end{bmatrix}u_{\min} + \begin{bmatrix} b_{13} \\ b_{23} \\ b_{33} \end{bmatrix}\frac{u_{\min}}{2}$$

$$= \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \\ A_{31} & A_{32} \end{bmatrix} \cdot \begin{Bmatrix} u_{\max} \\ u_{\min} \end{Bmatrix} + \begin{bmatrix} C_1 \\ C_2 \\ C_3 \end{bmatrix} \cdot u_{cv} = \begin{Bmatrix} Y_f \\ 0 \\ 0 \end{Bmatrix}$$

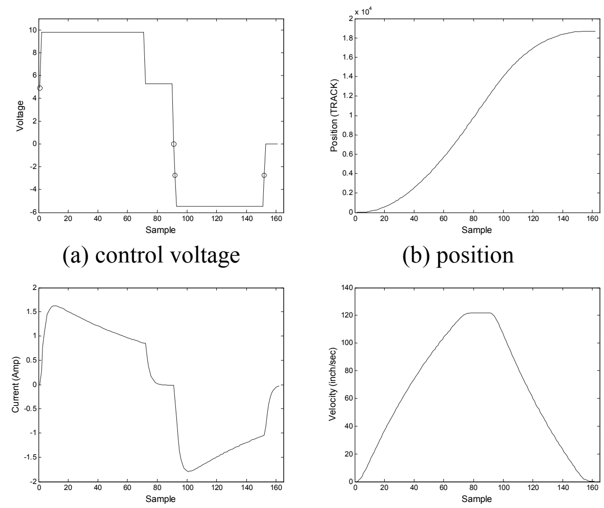
$$AU + Y_{cv} = Y_f \quad (11)$$

It is shown in (11) that the solution procedure is the same when there are intermediate values for improving acoustics during seeking.

As an example, the reference signals for modified five-stage seek control are shown in Fig. 8. The recalculated control voltage plot shows intermediate values in the acceleration, transition from acceleration to deceleration, and settle process, as marked by circle in the Fig. 8 (a).

The effect of intermediate values on the rate of change of current (*di/dt*) is studied further and the result is compared in the case of single and double intermediate values, seen in Fig. 9. We can notice that the intermediate values in the control voltage and the generated feed-forward current accordingly reduce jerk (*di/dt*) during seeking. From the results shown, we can infer that the optimal selection of intermediate values and more number of intermediate values improve the acoustics more.

The above procedure can be extended to the general case of control input shaping, minimized jerk, and vibration minimized access control [10]~[14].



(a) control voltage (b) position

(c) current (d) velocity

Fig. 8. Modified five-stage seek control example

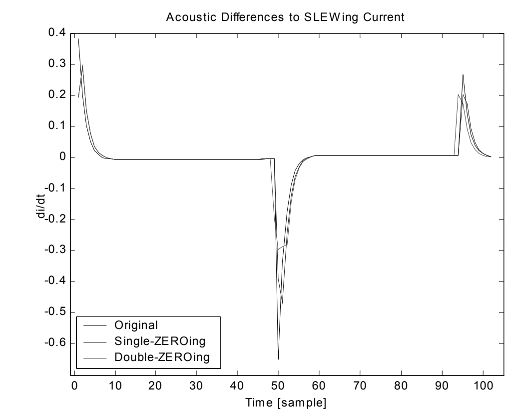



Fig. 9. The rate of change of current profile

## IV. IMPLEMENTATION OF REFERENCE SIGNAL GENERATION

The effectiveness of the proposed method for reference signal generation is demonstrated in the previous sections. With the limited computational power of processor used in commercial hard disk drives, a feasible and efficient way of implementation is discussed here.

The method of solving for control voltage is reviewed first. Equation (6) or (9) that solve for control voltage requires matrix $A$ with dimension of (3×2) for each seek length. The matrix $A$ is the partial summation of convolution for acceleration and deceleration respectively but it requires the long summation of recursive calculation of matrix power product. The inverse of matrix in (6) or (9) can be easily calculated because the dimension of matrix to be inverted is only (2×2). Therefore, if the convolution sums are available in advance, (6) or (9) can be solved fast in online process. Here, a lookup table of *convolutions* for system matrix, $\Phi$ and input vector, $\Gamma$ is proposed. The new table structure only stores the convolutions that are referred to in the Equation (4) or (7).

- New *convolution* lookup table structure

| | array | 1 | 2 | 3 |
|---|---|---|---|---|
| pointer → | $\Gamma$ | | | |
| | $\Phi\,\Gamma$ | | | |
| | $\Phi^2\Gamma$ | | | |
| | $\Phi^3\,\Gamma$ | | | |
| | $\vdots$ | | | |
| | $\Phi^{N-1}\Gamma$ | | | |

The block diagram of the procedure mentioned above is illustrated in Fig. 10.

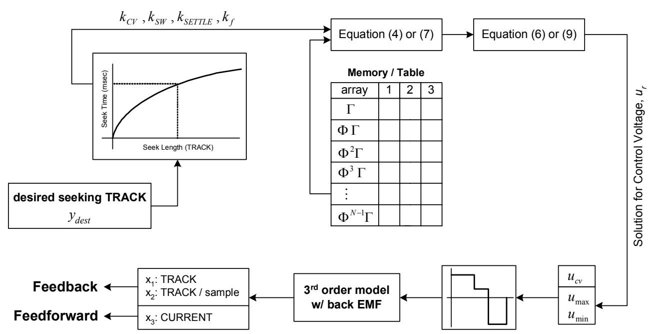



Fig. 10. Implementation of online reference signal generation

The feed-forward current mainly performs an important role in seeking and the feedback control is reducing the error between reference trajectory and moving trajectory due to the system parameter variations. The discussion of feedback controller for seek control is beyond the content of this paper so it is not considered here.

The transition samples such as switching ($k_{SW}$) and settle ($k_{SETTLE}$) need to be investigated further for smooth settling but they can be programmed in an approximate polynomial for efficient memory usage. Depending on the operation environment, the power amplifier can be saturated so the demand current is not available under certain conditions. The proposed method described is devised for calculating the equivalent of power amplifier output (control voltage) so the adaptation for varying operating conditions can be facilitated by investigating the real drive operations.

## V. CONCLUSION

A discrete-time closed-form solution of time-optimal seek control was derived based on the 3rd-order hard disk drive system model including back-EMF effect and coil inductance. The feasibility of the method was shown by simulation examples. The proposed method was shown to have advantages compared to the approach [5], such as: 1) direct calculation of control voltage for given seek length; 2) functionality of improving acoustics; 3) efficient use of memory; and 4) adaptation for varying operating conditions. The proposed method can be integrated in any form of 2-DOF control structure.

## REFERENCES


[1] G. F. Franklin, J. D. Powell, and M. Workman, *Digital Control of Dynamic Systems*, 3rd Ed., Addison Wesley, 1998.
[2] W. L. Brogan, *Modern Control Theory*, 3rd Ed., Prentice Hall, 1991.
[3] K. Shishida, T. Yamaguchi, and H. Hirai, "Fast head positioning method of magnetic disk drives", *IEEE Trans. Magn.*, vol. 29, no. 6, pp. 4045-4047, Nov. 1993.
[4] J. Ishikawa, Y. Yanagita, T. Hattori, and M. Hashimoto, "Head positioning control for low sampling rate systems based on two degree-of-freedom control", *IEEE Trans. Magn.*, vol. 32, no. 3, pp. 1787-1792, May 1996.
[5] S. Hara, T. Hara, L. Yi, and M. Tomizuka, "Novel reference signal generation for two-degree-of-freedom controllers for hard disk drives", *IEEE/ASME Trans. Mechatron.*, vol. 5, no. 1, pp. 73-78, Mar. 2000.
[6] L. Yi and M. Tomizuka, "Two-degree-of-freedom control with robust feedback control for had disk servo systems", *IEEE/ASME Trans. Mechatron.*, vol. 4, no. 1, pp. 17-24, Mar. 1999.
[7] Hai T. Ho, "Fast servo bang-bang seek control", *IEEE Trans. Magn*., vol. 33, no. 6, pp. 4522-4527, Nov. 1997.
[8] C. La-orpacharapan and L. Y. Pao, "Shaped time-optimal control for disk drive systems with back EMF, slew rate limits, and different acceleration and deceleration rates", *Proc. Amer. Contr. Conf.*, Denver, CO, pp. 4788-4795, June 2003.
[9] C. La-orpacharapan and L. Y. Pao, "Fast seek control for flexible disk drive systems with back EMF and inductance", *Proc. Amer. Contr. Conf.*, Denver, CO, pp. 3077-3082, June 2003.
[10] Y. Mizoshita, S. Hasegawa, and K. Takaishi, "Vibration minimized access control for disk drives", *IEEE Trans. Magn.*, vol. 32, pp.1793-1798, May 1996.
[11] D. K. Miu and S. P. Bhat, "Minimum power and minimum jerk position control and its applications in computer disk drives", *IEEE Trans. Magn.*, vol. 27, no. 6, pp. 4471-4475, Nov. 1991.
[12] B. Chang and Y. Hori, "Trajectory design considering derivative of jerk for head-positioning of disk drive system with mechanical vibration", *Proc. Amer. Contr. Conf.*, Denver, CO, pp. 4335-4340, June 2003.
[13] M. Muenchhof and T. Singh, "Jerk limited time optimal control of flexible structures", *ASME J. Dyn. Syst. Meas.Contr.*, vol. 125 (1), pp. 139-142, 2003.
[14] M. Muenchhof and T. Singh, "Desensitized jerk limited time optimal control of multi-input systems", *AIAA J. Guid. Contr. Dyn.*, vol. 25 (3), pp. 474-481, 2002.