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Multi-rate Short-seeking Control of Dual-actuator Hard Disk Drives for Computation Saving

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Abstract— **This paper is concerned with multi-rate short-seeking control for a dual-actuator hard disk drive (HDD). Multi-rate control has been proposed to reduce the real time computation in HDD servos, and it has been shown that in track following control of HDDs, computation can be saved greatly without performance degradation by using multi-rate controllers. In this paper, we propose a novel short-seeking control method for dual-actuator HDDs based on multi-rate track following control and initial value adjustment. This method, which utilizes the same multi-rate controllers and same servo structure as track following, adjusts the initial values of the track-following controller for short seeking. Real time computation is greatly saved in two ways: (1) computation is saved by multi-rate scheme; (2) initial value adjustment of the feedback controller makes the use of the feedforward controller and reference trajectory unnecessary, which implies no real time computation for feedforward control is needed. Simulation and experimental results verify the effectiveness of the proposed method**.

Keywords—**Multi-rate control, Short seeking, Dual stage actuation, Hard disk drives**

I. Introduction

The demands for larger storage capacities and higher access speed continue to increase in the hard disk drive (HDD) industry. To meet these requirements, the servo bandwidth of the head positioning system must be increased to reject a wider range of disturbances such as disk flutter vibrations, spindle run-out, windage, and external vibration. Dual-stage actuation has been recognized as a promising candidate to expand the servo bandwidth. The dual-actuator disk file system consists of two actuators: coarse and fine actuators. The coarse actuator is of low bandwidth with a large operating range and it is used for coarse positioning; the fine actuator is of high bandwidth with a small operating range and it is used for fine positioning. The voice coil motor (VCM) and piezoelectric transducer (PZT) are the most popular coarse actuator and fine actuator, respectively. Dual-stage actuators can improve both track-following and track-seeking performance. Many research efforts have been devoted to the design of dual-actuator servo system [1]-[7]. Several designs for dual stage track-following servos have been proposed, such as a decoupled type [1], a parallel type [2], a master-slave type [3], a u-synthesis MIMO type [4], and PQ method [5]. Dual stage short seeking control can be achieved by a two-degree-of-freedom (TDOF) servo structure, in which both the feed-forward controller and reference trajectory need to be designed and implemented [6]-[7].

The HDD industry strives for lowering cost and the control algorithm must be implemented on a low end DSP, which may be performing various other tasks. Thus, the amount of computation for real time control is a major concern. Recently, multi-rate control has been proposed to reduce the real time computation in HDD servo systems [8], [9]. Multi-rate scheme reduces real time computation by updating different components of the controller at different rates. In dual-stage servo system, the controller for the low bandwidth actuator (VCM) can be updated less frequently than the controller for the high bandwidth actuator (PZT) to reduce the computation load. It has been shown that this scheme greatly saves computation without performance degradation in the implementation of the track following controller for dual-actuator HDDs [9].

If a standard two-degree-of-freedom (TDOF) method is applied to dual-stage short seek control, the feedforward control term will increase the amount of real time computation relative to track following control. In this paper, we propose a novel method for dual-stage short seeking control based on multi-rate track following control [9] and initial value adjustment (IVA). This method, which uses the same multi-rate controller and same servo structure as track following, tunes the initial values of the track-following controller for short seeking. The proposed method significantly saves real time computation in two ways: first, multi-rate scheme inherently saves computation by updating the controller for the low bandwidth actuator at a slow rate; second, by tuning the initial values of the feedback controllers, the desired transient characteristics in short-seek can be obtained without the use of feed-forward controller and reference trajectory, which implies no real time computation for feed-forward control is needed.

The remainder of this paper is organized as follows. In Section II, the multi-rate short seeking control with IVA is proposed. In Section III, the proposed method is applied to a dual-actuator HDD, and simulation and experiment results are given. Conclusions are given in Section IV.

II. MULTI-RATE SHORT SEEKING CONTROL WITH INITIAL VALUE ADJUSTMENT

Figure 1 shows a schematic diagram of short seeking control with IVA for dual-actuator HDDs. P_{VCM} and P_{PZT} represent the dynamics of VCM (low bandwidth actuator) and PZT (high bandwidth actuator), respectively. In this study, the parallel track-following controllers are used to achieve high-accuracy track following, and C_1 and C_2 are, respectively, track following controllers for the VCM and PZT actuators. D is the decimator. The proposed system has the following features:

(1) The overall structure during seeking is a one-degree-of-freedom structure.
(2) Reference input is set to be a step signal for short seek, i.e., $r(k)=r$ for $k \geq 0$, where r is the seeking distance. The output y is sampled with period T_f.
(3) The feedback controllers are the same as the multi-rate track following controllers. The PZT controller C_2 is updated at the measurement sampling rate ($1/T_f$), while the VCM controller C_1 is updated m times slower than the measurement sampling rate with period $T_s=mT_f$. The multi-rate ratio m is an integer greater than 1.
(4) Nonzero initial values are set to both controllers for short seeking.

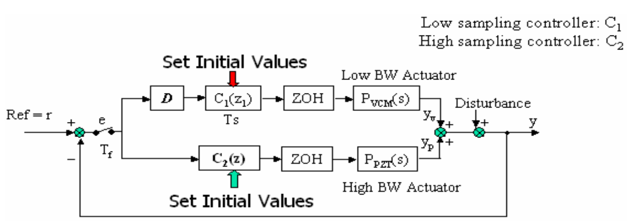

Fig.1 Multi-rate short-seek control with IVA for dual-actuator HDDs

Notice that the adjustment of the initial values of the controllers improves the transient response without changing closed-loop characteristics such as stability and sensitivity. Since we already have the design methodology for multi-rate track following controllers [8], [9], finding the initial values of the multi-rate controllers becomes the main issue. A methodology on how to find the optimal initial values of the dual-stage multi-rate controllers is presented in this paper. In the following sub-section *B*, the multi-rate control system depicted in Fig. 1 is converted to an equivalent MIMO single-rate system. Then in sub-section *C*, IVA is formulated as an optimization problem by finding the optimal initial states of the controller for the equivalent MIMO system to minimize the performance index based on the tracking error. Design of performance index for short-seeking is discussed in sub-section *D*. Sub-section *E* discusses the real-time computation efforts of the proposed method.

A. Representation of the Multi-rate Control System as a Single-rate system via Lifting

Assume that the dynamic models of VCM plant and PZT plant in continuous time are

$$\dot{x}_v(t) = A_{01}x_v(t) + B_{01}u_v(t), \quad y_v(t) = C_{01}x_v(t) \tag{1}$$

and

$$\dot{x}_p(t) = A_{02}x_p(t) + B_{02}u_p(t), \quad y_p(t) = C_{02}x_p(t) \tag{2}$$

where $x_v \in \Re^{n_v}$, u_v and y_v are the state vector, control input and output of VCM, and $x_p \in \Re^{n_p}$, u_p and y_p are the state vector, control input and output of PZT.

The discrete time equivalents of (1) and (2) with measurement sampling period T_f can be expressed by

$$x_v(k+1) = A_1 x_v(k) + B_1 u_v(k), \quad y_v(k) = C_1 x_v(k) \tag{3}$$

and

$$x_p(k+1) = A_2 x_p(k) + B_2 u_p(k), \quad y_p(k) = C_2 x_p(k) \tag{4}$$

where $x_v(k) := x_v(kT_f), x_p(k) := x_p(kT_f)$,

$$A_1 \in \Re^{n_v^2}, B_1 \in \Re^{n_v}, C_1 \in \Re^{1\times n_v}, A_2 \in \Re^{n_p^2}, B_2 \in \Re^{n_p}, C_2 \in \Re^{1\times n_p}$$

Suppose track following controllers have already been designed to meet performance specifications for the dual-actuator HDD, so that the closed loop system is stable and steady state error is zero. And $C_1(z)$ and C_2 (z) can be expressed in time domain as follows:

$$x_{vc}(k+1) = A_{c1}x_{vc}(k) + B_{c1}e(k), \quad u_v(k) = C_{c1}x_{vc}(k) + D_{c1}e(k) \tag{5}$$

and

$$x_{pc}(k+1) = A_{c2}x_{pc}(k) + B_{c2}e(k), \quad u_p(k) = C_{c2}x_{pc}(k) + D_{c2}e(k) \tag{6}$$

where $x_{vc}(k) := x_{vc}(kT_f), x_{pc}(k) := x_{pc}(kT_f)$,

$$x_{vc} \in \Re^{n_{vc}}, A_{c1} \in \Re^{n_{vc}^2}, B_{c1} \in \Re^{n_{vc}}, C_{c1} \in \Re^{1\times n_{vc}}, D_{c1} \in \Re$$

$$x_{pc} \in \Re^{n_{pc}}, A_{c2} \in \Re^{n_{pc}^2}, B_{c2} \in \Re^{n_{pc}}, C_{c2} \in \Re^{1\times n_{pc}}, D_{c2} \in \Re .$$

Referring to Fig. 1, C_1 is updated m times slower than C_2. The control system in Fig. 1 cannot be described in the standard state-space form because it is a time-periodic system. Thus, we introduce a discrete-time lifting technique to describe the multi-rate system by an equivalent single-rate system, which may be accomplished in the following four steps.

STEP 1) *The VCM controller (slow-rate controller) is transformed to an equivalent MIMO controller in the time domain with slow period T_s.*

Lemma 1: Under the proposed multi-rate implementation scheme, the controller described by (5) can be represented by a MIMO system in time domain as follows:

$$\begin{aligned} x_{vc}[k+1] &= A_{vc}x_{vc}[k] + B_{vc}\,\bar{e}[k] \\ \overline{U}_v[k] &= C_{vc}x_{vc}[k] + D_{vc}\,\bar{e}[k] \end{aligned} \tag{7}$$

where $x_{vc}[k] := x_{vc}(kT_s)$ (8)

$\bar{e}[k], \overline{U}_v[k]$ are the lifted position error signal and lifted control signal of the VCM controller:

$$\bar{e}[k] = [e[k,0], e[k,1], ..., e[k,m-1]]^T$$

$$\overline{U}_v[k] = [u_v[k,0],\ u_v[k,1], ..., u_v[k,m-1]]^T \tag{9}$$

where $e[k,i] := e(kT_s + iT_f)$, $u_v[k,i] := u_v(kT_s + iT_f)$

and $A_{vc} \in \Re^{n_{vc}\times n_{vc}}, B_{vc} \in \Re^{n_{vc}\times m}, C_{vc} \in \Re^{m\times n_{vc}}, D_{vc} \in \Re^{m\times m}$ are:

$$A_{vc} = A_{c1}^m,\ B_{vc} = [\sum_{j=0}^{m-1} A_{c1}^j B_{c1},\ \mathrm{O}]\ C_{vc} = \begin{bmatrix} C_{c1} \\ \vdots \\ C_{c1} \end{bmatrix}, D_{vc} = \begin{bmatrix} D_{c1} & O \\ .... & \\ D_{c1} & O \end{bmatrix} \tag{10}$$

Proof: Under the proposed multi-rate scheme, controller $C_1(z)$ is updated at slow rate ($1/T_s$), so the input to the controller is held constant during one slow period T_s, i.e.

$$e(kT_s) = e(kT_s + iT_f) \quad \text{for } i = 0,1,...,m\text{-}1 \tag{11}$$

So,
$$\begin{aligned} x_{vc}[(k+1)T_s] &= x_{vc}(kT_s + mT_f) \\ &= A_{c1}x_{vc}(kT_s + (m-1)T_f) + B_{c1}e(kTs + (m-1)T_f) \\ &= A_{c1}[A_{c1}x_{vc}(kT_s + (m-2)T_f) + B_{c1}e(kT_s + (m-2)T_f)] \\ &\quad + B_{c1}e(kT_s + (m-1)T_f) \\ &\ldots\ldots \\ &= A_{c1}^m x_{vc}(kT_s) + \sum_{i=0}^{m-1} A_{c1}^{m-1-i} B_{c1} e(kT_s + iT_f) \\ &= A_{c1}^m x_{vc}(kT_s) + \sum_{j=0}^{m-1} A_{c1}^j B_{c1} e(kT_s) \end{aligned} \tag{12}$$

Furthermore, from state equation (5),

$$u_v(kT_s) = C_{c1}x_{vc}(kT_s) + D_{c1}e(kT_s) \tag{13}$$

and by noting that the control signal is not updated during one slow period T_s, i.e. $u_v(kT_s+iT_f) = u_v(kT_s)$, for $i=1,2,\ldots, m\text{-}1$, we obtain $u_v(kT_s + iT_f) = C_{c1}x_{vc}(kT_s) + D_{c1}e(kT_s)$ (14)

Combining Eqs. (12), (13), (14), and using (8) and (9), we can represent the controller as a time-invariant system with the slow update period T_s as described by (7). □

STEP 2) *The PZT controller (fast rate controller) is transformed to an equivalent MIMO controller in the time domain with slow period T_s.*

By defining the lifted signal of $u_p(k)$ as:

$$\overline{U}_p[k] = [u_p[k,0], u_p[k,1], ..., u_p[k,m-1]]^T \tag{15}$$

where $u_p[k,i] := u_p(kT_s + iT_f)$, $u_p(k) := u_p(kT_f)$

and using (9), the PZT controller (6) can be converted to an equivalent MIMO controller given by

$$\begin{aligned} x_{pc}[k+1] &= A_{pc}x_{pc}[k] + B_{pc}\,\bar{e}[k] \\ \overline{U}_p[k] &= C_{pc}x_{pc}[k] + D_{pc}\,\bar{e}[k] \end{aligned} \tag{16}$$

where $x_{pc}[k] := x_{pc}(kT_s)$,

$$A_{pc} = A_{c2}^m,\ B_{pc} = [A_{c2}^{m-1}B_{c2}, A_{c2}^{m-2}B_{c2}, ..., B_{c2}],$$

$$C_{pc} = \left[C_{c2}{}^T,\ (C_{c2}A_{c2})^T,\ ...,\ (C_{c2}A_{c2}^{m-1})^T\right]^T$$

STEP 3) *The VCM plant and PZT plant are expressed by equivalent MIMO plants in the time domain with the underlying slow period T_s.*

By lifting the signals, $y_v(k)$ and y_p (k), as

$$\begin{aligned} \overline{Y}_v[k] &= [y_v[k,0],\ y_v[k,1], ..., y_v[k,m-1]]^T, \\ \overline{Y}_p[k] &= [y_p[k,0], y_p[k,1], ..., y_p[k,m-1]]^T \end{aligned} \tag{17}$$

where $y_v[k,i] := y_v(kT_s + iT_f)$, $y_p[k,i] := y_p(kT_s + iT_f)$

and using (9) and (15) , the VCM plant and PZT plant can be expressed by equivalent MIMO plants given by

$$\begin{aligned} x_v[k+1] &= A_v x_v[k] + B_v\,\overline{U}_v[k] \\ \overline{Y}_v[k] &= C_v x_v[k] + D_v\,\overline{U}_v[k] \end{aligned} \tag{18}$$

and
$$\begin{aligned} x_p[k+1] &= A_p x_p[k] + B_p\,\overline{U}_p[k] \\ \overline{Y}_p[k] &= C_p x_p[k] + D_p\,\overline{U}_p[k] \end{aligned} \tag{19}$$

where $x_v[k] := x_v(kT_s)$, $x_p[k] := x_p(kT_s)$,

$$A_v = A_1^m, B_v = [A_1^{m-1}B_1, A_1^{m-2}B_1, ..., B_1],\ C_v = [C_1^T, (C_1A_1)^T, ..., (C_1A_1^{m-1})^T]^T$$

$$A_p = A_2^m,\ B_p = [A_2^{m-1}B_2, A_2^{m-2}B_2, ..., B_2], C_p = [C_2^T, (C_2A_2)^T, ..., (C_2A_2^{m-1})^T]^T$$

STEP 4) *Combining the controllers and plants equations from STEPS 1, 2 and 3, we obtain an MIMO single-rate representation of the closed loop system in lifted form.*

From the controllers equations, (7) and (16), and plants equations, (18) and (19), and referring to Fig. 2, we obtain the closed loop system:

$$\begin{aligned} x[k+1] &= Ax[k] + B\,\bar{r} \\ \overline{Y}[k] &= Cx[k] + D\,\bar{r} \end{aligned} \tag{20}$$

where $x[k] := [x_v[k]^T, x_p[k]^T, x_{vc}[k]^T, x_{pc}[k]^T]^T \in \Re^{n_v + n_p + n_{vc} + n_{pc}}$

$\bar{r} = [r, r, ..., r]^T \in \Re^m$, $\overline{Y} = [y[k,0], y[k,1], ..., y[k,m-1]]^T$

Matrices *A, B, C* and *D* can be computed from the system matrices in (7), (16), (18) and (19).

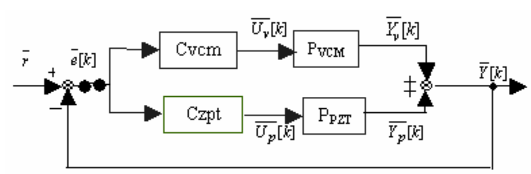

Fig. 2 Lifted system equivalent to the multi-rate short-seeking control system

Now the multi-rate control system in Fig. 1 is converted to an equivalent MIMO single-rate system descried by (20). The lifted system equivalent to the multi-rate short-seeking control system is shown in Fig. 2. Notice that the state of the closed loop system $x[k]$ can be separated into two parts: the states of two plants ($x_k[k]$) and the states of two controllers ($x_c[k]$) :

$$x_k[k] := \begin{bmatrix} x_v[k] \\ x_p[k] \end{bmatrix}, \; x_c[k] := \begin{bmatrix} x_{vc}[k] \\ x_{pc}[k] \end{bmatrix}$$

The resulting closed-loop system (20) is asymptotically stable because the multi-rate track following controller has been designed to stabilize the system. The steady state of (20) is obtained by: $x[\infty] = (I - A)^{-1} B\bar{r}$, and the error dynamics of the closed loop system is:

$$e_x[k+1] = Ae_x[k] \tag{21}$$

where $e_x[k] = x[k] - x[\infty], \quad x[\infty] = (I - A)^{-1} B\bar{r}$ (22)

B. Initial Value Adjustment of the Multi-rate Controllers

After the multi-rate control system is converted to an equivalent MIMO single-rate system, the initial value adjustment of the multi-rate controllers can be formulated as the following optimization problem.

Problem: Consider the multi-rate control system depicted in Fig. 1 or the equivalent MIMO single-rate system descried by equation (20). Find the initial value of the controller $x_c[0]$ that minimizes performance index *J:*

$$J = \sum_{k=0}^{\infty} e_x[k]^T Q\, e_x[k], \;\; for\; Q > 0 \tag{23}$$

where $e_x[k]$ is as defined in (22).

Theorem: The initial value of the controller $x_c[0]$ that minimizes performance index J is:

$$x_c[0] = K\,\bar{r} \tag{24}$$

where $K = [P_{22}^{-1} P_{12}^T \;\; I\,](I\text{-}A)^{-1} B$ (25)

and P_{22} , P_{12} are obtained by solving the Lyapunov Eq.:

$$A^T PA - P = -Q, \;\; P =: \begin{bmatrix} P_{11} & P_{12} \\ P_{12}^T & P_{22} \end{bmatrix} \tag{26}$$

Proof:

Since A is asymptotically stable, (26) has a unique positive definite solution P>0, and J can be transformed to:

$$J = e_x[0]^T\, P\, e_x[0]$$

since $e_x[0] = x[0] - x[\infty] = \begin{bmatrix} x_k[0] \\ x_c[0] \end{bmatrix} - x[\infty] =: \begin{bmatrix} e_k[0] \\ e_c[0] \end{bmatrix}$ (27)

So, $J = e_k^T[0]\, P_{11} e_k[0] + 2e_k^T[0]\, P_{12} e_c[0] + e_c^T[0]\, P_{22} e_c[0]$

It is easy to show that $\partial J / \partial x_c[0] = 0$ is equivalent to $\partial J / \partial e_c[0] = 0$. Solving $\partial J / \partial e_c[0] = 0$, we get

$$e_c[0] = -P_{22}^{-1} P_{12}^T e_k[0] \tag{28}$$

From (27) and (28), and noting that $x_k[0]$ may be assumed zero for 1-track seek, we conclude that J is minimized for

$x_c[0] = K\,\bar{r}$, where $K=[P_{22}^{-1}P_{12}^{T}, I](I\text{-}A)^{-1}B$

From (24) and (25), we see that the initial value of the controller $x_c[0]$ is determined by $\bar{r}$, A, B and P. We further note that $\bar{r}$ is determined by r and P is determined by Q and A. Thus, given, Q, r, A and B, the initial value of the controller $x_c[0]$ can be computed off-line. In the following section, we will discuss how Q may be designed.

C. Design of Performance Index Function

The performance index J plays an important role in determining the initial value of the controller and thus the short-seeking performance.

Design I- considering tracking error at sampling point

It is straightforward to evaluate the tracking error at sampling points and consider smoothness of the output and control input in the performance index: i.e.

$$J = \sum_{k=0}^{\infty} [e_y[k]^T e_y[k] + q_1 \dot{e}_y[k]^T \dot{e}_y[k]] + q_2 J_u \tag{29}$$

where $e_y[k] = \bar{Y}[k] - \bar{Y}[\infty], \;\; \dot{e}_y[k] := [e_y[k+1] - e_y[k]]/T_s$

$$J_u = \sum_{k=0}^{\infty} [\bar{U}[k] - \bar{U}[\infty]]^T [\bar{U}[k] - \bar{U}[\infty]], \;\; \bar{U}[k] = [\bar{U}_v[k]^T, \; \bar{U}_p[k]^T]^T \tag{30}$$

and q_1, q_2 are weighting factors.

Lemma 2: The performance index J in (29) can be transformed into the standard quadratic form:

$$J = \sum_{k=0}^{\infty} e_x[k]^T\, Q\, e_x[k]$$

where $Q = C^T C + q_1 C_d{}^T C_d + q_2 C_u{}^T C_u$,

$$C_d = C\frac{A-I}{T_s}, \;\; C_u = (I + \begin{bmatrix} D_{vc} \\ D_{pc} \end{bmatrix} [D_v \;\; D_p])^{-1} \begin{bmatrix} -D_{vc}C_v, -D_{vc}C_p, C_{vc}, \,0 \\ -D_{pc}C_v, -D_{pc}C_p, 0, C_{pc} \end{bmatrix} \tag{31}$$

The proof is omitted due to the space limitation.

Design II- considering the inter-sampling behaviors

Design method I only evaluates the tracking error at the sampling instants. However, the plant is a continuous-time system, and it is natural to evaluate the continuous-time signals directly. Especially in dual-stage servos, inter-sampling ripple may take place if the second actuator has mechanical resonance modes at high frequencies beyond sampling frequency, which can be excited by the control input. To incorporate the inter-sampling errors, we modify J in (29) as follows:

$$J = \int_0^{\infty} [e_y(t)^2 + q_1 \dot{e}_y(t)^2] dt + q_2 J_u \tag{32}$$

where $e_y(t) = y(t) - y(\infty)$, J_u is as defined in (30).

To evaluate (32), the pieces of the continuous-time state $x_k(t)$ and $e_k(t)$ are introduced as:

$$\begin{aligned} x_k[k,i](v) &:= x_k(kT_s + iT_f + v) \\ e_k[k,i](v) &:= e_k(kT_s + iT_f + v) \end{aligned}, \;\; v \in [0, T_f] \tag{33}$$

where $e_x(t) = x(t) - x(\infty) = [e_k(t)^T, e_c(t)^T]^T$

Lemma 3: $e_k[k,i](v)$ is a function of error-vector $e_x[k]$ as follows:

$$e_k[k,i](v) = He^{\hat{A}v}\overline{S_i}e_x[k], \quad v \in [0, T_f] \tag{34}$$

where

$$\hat{A} = \begin{bmatrix} \hat{A}_{01} & O \\ O & \hat{A}_{02} \end{bmatrix}, \hat{A}_{01} = \begin{bmatrix} A_{01} & B_{01} \\ O & O \end{bmatrix}, \hat{A}_{02} = \begin{bmatrix} A_{02} & B_{02} \\ O & O \end{bmatrix}$$

$$\overline{S_i} = \begin{bmatrix} S_{vi} \\ S_{pi} \end{bmatrix} \times \begin{bmatrix} I_{n_v}, O \\ O_{n_p \times n_v}, I_{n_p}, O_{n_p \times (n_{vc}+n_{pc})} \\ C_u \end{bmatrix}, H = \begin{bmatrix} [I_{n_v}, O], O \\ O, [I_{n_p}, O] \end{bmatrix}$$

$$S_{vi} = \begin{bmatrix} A_1^i, & O_{n_v \times n_p}, & \sum_{j=0}^{i-1} A_1^j B_1, & O_{n_v \times m} \\ O_{1\times n_v}, & O_{1\times n_p}, & \Delta_{i+1\,(1\times m)}, & O_{1\times m} \end{bmatrix}, \quad \Delta_{i+1} = [0,\ldots,\underset{i}{0}, \underset{i+1}{1}, \underset{i+2}{0},\ldots.0]$$

$$S_{pi} = \begin{bmatrix} O_{n_p \times n_v} & A_2^i & O_{n_p \times m} & [A_2^{i-1}B_2, A_2^{i-2}B_2, \ldots, B_2, 0\ldots 0] \\ O_{1\times n_v} & O_{1\times n_p} & O_{1\times m} & \Delta_{i+1\,(1\times m)} \end{bmatrix}$$

The proof is omitted due to the space limitation.

Noting that

$$y[k,i](v) = y_v[k,i](v) + y_p[k,i](v) = C_{01}x_v[k,i](v) + C_{02}x_p[k,i](v)$$
$$= \hat{C}_0 x_k[k,i](v), \quad \hat{C}_0 = [C_{01} \;\; C_{02}]$$
$$y[k,i](v) - y[\infty] = \hat{C}_0 e_k[k,i](v) \tag{35}$$

and using Lemma 3 , we obtain

$$J_1 = \int_0^\infty e_y(t)^2\,dt = \sum_{k=0}^{\infty}\sum_{i=0}^{m-1}\{\int_0^{T_f}[y[k,i](v) - y[\infty]]dv\}$$
$$= \sum_{k=0}^{\infty}\sum_{i=0}^{m-1}\{\int_0^{T_f} e_x^T[k,i](v)Q_c e_x[k,i](v)dv\}$$
$$= \sum_{k=0}^{\infty}\sum_{i=0}^{m-1}\{e_x^T[k]\overline{S_i}^T(\int_0^{T_f} e^{\hat{A}^T v}(H^T Q_c H)e^{\hat{A}v}dv)\overline{S_i}e_x[k]\}$$
$$= \sum_{k=0}^{\infty}\sum_{i=0}^{m-1}\{e_x^T[k]\overline{S_i}^T\hat{Q}_c\overline{S_i}e_x[k]\} = \sum_{k=0}^{\infty} e_x[k]^T Q_1 e_x[k]$$

where $Q_1 = \sum_{i=0}^{m-1}\overline{S}_i^T\hat{Q}_c\overline{S}_i$, $\hat{Q}_c = \int_0^{T_f} e^{\hat{A}^T v}(H^T Q_c H)e^{\hat{A}v}dv$, $Q_c = \hat{C}_0^T\hat{C}_0$

Thus, we conclude that the performance index J in (32) can also be transformed into the standard quadratic form as: $J = \sum_{k=0}^{\infty} e_x[k]^T Q\, e_x[k]$.

E. Real Time Computation Efforts

The proposed method significantly saves real time computation by ODOF servo structure and multi-rate scheme. And as is known, the TDOF short seeking control increases the real time computation because feed-forward control is introduced to shape the transient response. In the proposed method, the desired transient characteristics in short-seeking can be obtained by tuning the initial values of the feedback controllers. So, ODOF structure is retained, which means no real time computation for feed-forward control is needed. Furthermore, computation is saved by multi-rate scheme, because the VCM controller is updated at a slow rate.

III. Simulation and Experiment

In this section, the proposed method is applied to a dual actuator HDD and evaluated by simulations and experiments.

A. Experimental Setup

Figure 3 shows an experiment setup which includes a 3.5" HDD, a digital signal processor (TI TMS230C67X DSP), and a laser Doppler vibrometer for measuring the position of the read/write head. All the digital controllers are implemented on the DSP using C codes. The sampling rate is 50kHz, and the multi-rate ratio m is set to 2. Fig. 4 shows the HDD with a VCM as a first-stage actuator and a PZT as a second-stage actuator. The frequency responses of VCM and PZT plant models are shown in Fig. 5. Notice that they have resonance modes at high frequencies.

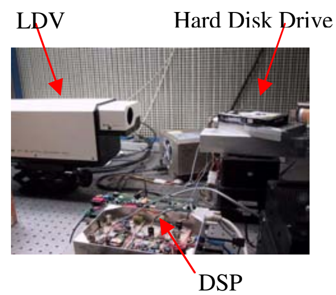

Fig. 3 Experiment Setup

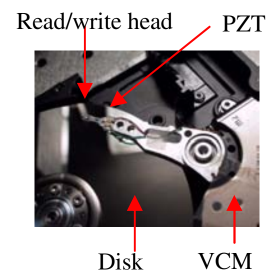

Fig. 4 Dual-actuator HDD

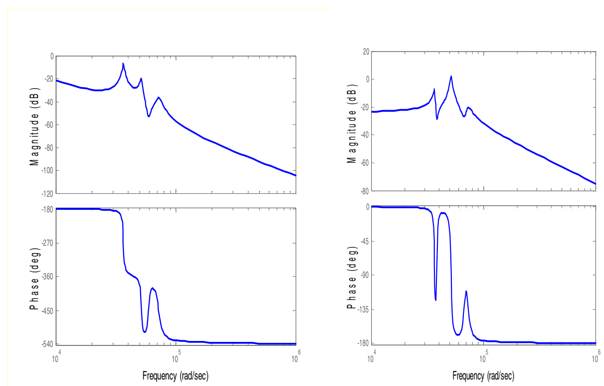

Fig. 5 Frequency responses of VCM and PZT plants

B. Simulation Results

The track following controllers for VCM (C_1) and PZT (C_2) are designed to be a lead-lag compensator and a lead-lag compensator with two notch filters, respectively. Fig. 6 shows the response of the output without initial value adjustment of the controllers for 1-track seeking. The track pitch is 1 μm. The dark line shows the head position, the light line and dotted line are VCM and PZT position respectively. Simulation results of 1-track seeking using proposed method are shown in Fig. 7. From Fig. 7, we can see that a fast and smooth response in short seeking can be achieved by initial value adjustment of the controllers. The design parameters, q_1 *and* q_2, are set to the following values after some trial and errors: $q_1=0.09$ and $q_2=0.005$.

It could be noted from Fig. 7 that the dual-stage seeking control greatly improves the responses. It takes about 0.3 ms to move the head to the desired track so that the head can read or write data on the data track. The PZT actuator first moves towards the target, and then returns to its stroke center to cancel out the movement of the VCM actuator after the head is on track. Fig. 7(c) compares the performance of *Design I* and *Design II,* which confirms that inter-sampling ripples are decreased using *Design II*.

C. *Experimental results*

A representative experimental result of 1-track seeking using the proposed method (*Design II*) is shown in Fig. 8. Fig. 9 is a zoom-in version of Fig. 8. Table I summarizes the experimental short seeking performance, and we can see the performance is excellent: the settling time is *0.3ms* and 3σ of *PES* after settling is *1.98%* of the track pitch.

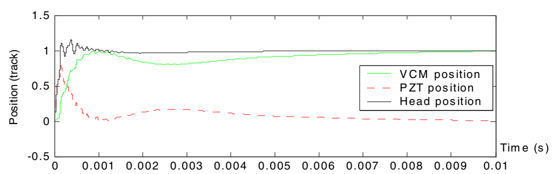

Fig. 6 1-track seeking without IVA for dual-actuator HDDs

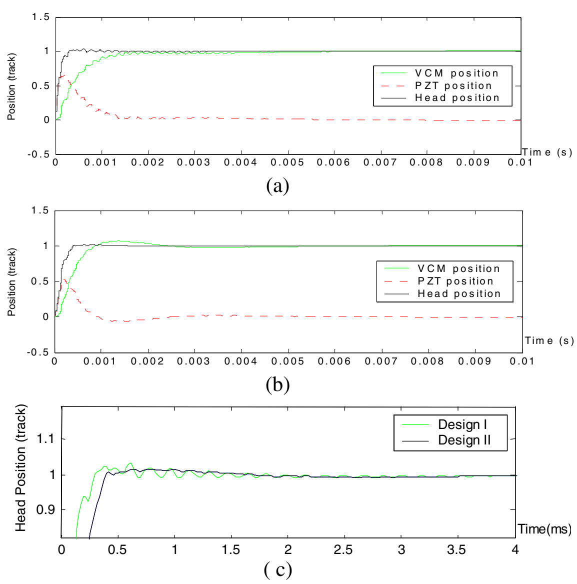

Fig. 7 1-track seek with IVA for dual-actuator HDDs
(a) Design I; (b) Design II; (c) Head position around target track.

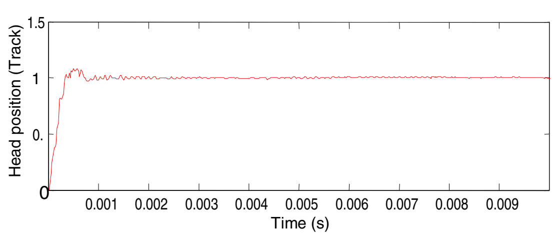

Fig. 8 Experimental result of 1-track seeking using the proposed method

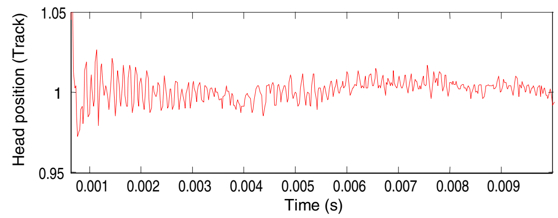

Fig. 9 Experimental result of 1-track seeking (zoom-in version of Fig. 8)

TABLE I
EXPERIMENTAL SHORT-SEEKING PERFORMANCE (*DESIGN II*)

Performance Measures	**Data**
10% Settling Time (ms)	0.30
3σ of PES after settling (Track)	0.0198

IV. CONCLUSIONS

This paper has proposed a short seeking control for dual-actuator HDD based on IVA of the multi-rate track-following controller for reduced real time computation. The multi-rate short seeking control problem was formulated as an optimization problem by converting the multi-rate control system to an equivalent MIMO single-rate system and finding the optimal initial values of the controller for the equivalent MIMO system. By incorporating the inter-sampling error and the smoothness of the control input in the performance index, IVA of the multi-rate track-following controller achieves smooth and fast short seeking with a small computation load. Simulation and experimental results confirmed the effectiveness of the proposed method.

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