# A TIME-OPTIMAL UNIFIED SERVO CONTROL METHOD WITH A TWO-DEGREE-OF-FREEDOM STRUCTURE FOR A HARD DISK DRIVE

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Abstract-In this paper, a time-optimal unified servo control (TOUSC) strategy with a Two-Degree-Of-Freedom (2DOF) structure is proposed for a hard disk drive. The feed-forward control signal in the 2DOF dominates in the track-seek phase while the TOUSC takes the main responsibility to maintain the track following performance. This unified design does not switch controllers in the transition from track seeking to track following. By using the saturation voltages of power amplifier in its profile generator, the track-seek performance is improved significantly over the single TOUSC scheme. The unified controller also achieves better disturbance suppression in track-following with superior tracking precision. Simulations show that the proposed 2DOF-based TOUSC provides satisfactory overall performances as a unified servo controller.

**Keywords**-Discrete Time-Optimal Control, Two-Degree-Of-Freedom Control, Unified Servo Controller, Hard Disk Drive Servo.

# I. INTRODUCTION

The computer hard disk drive (HDD) servo design plays a significant role in the industry technology development. It is the key in achieving very high-speed and high-precision positioning control of magnetic heads during read/write (R/W) processes.

To access the data stored in concentric tracks of a disk, two control modes are usually employed by the actuator servo: a track-seek mode and a track following mode. The track-seek control attempts to move the heads from one track to another in minimum time, whereas the track following servo must keep the heads at the center of a selected track as precisely as possible during a read/write process. Among HDD control techniques, the Mode Switching Control (MSC) [1] is widely employed. The MSC uses two separate controllers for the track-seek mode and the track following mode, respectively. Classical control methods, such as the proximate timeoptimal control, are adopted in the track-seek controller while lead-lag compensators or Proportional Integral Derivative (PID) compensators are often used in the track following phase.

With the rapid advance in computer hardware, the HDDs evolve in the trend of smaller sizes and higher recording density. This in turn poses more challenges to the performances of disk servo control schemes. The classical control techniques in MSC are no longer able to

meet the new demands. Recently, a dual-stage actuator HDD [2-4] has been proposed which consists of a primary actuation stage and a secondary micro-actuator. Although the dual-stage systems can improve the HDD performances, it also needs two different controllers and thus introduces new problems, such as the decoupling of the two subsystems in its control design.

The research leading to this paper concentrates on a unified HDD servo algorithm. The feedback control design is based on a novel discrete time-optimal control law first shown in [5]. The detailed mathematical derivation was then shown in [6], and its properties and applications were shown in [7]. This controller, as it is applied to computer HDD problems, is called a timeoptimal unified servo control (TOUSC) [15] because it is a single, fixed, controller that performs the positioning control for both the track-seek and the track following modes. To address unique problems in HDD, such as saturation, overshoot, resonant frequencies, etc., the TOUSC is combined with the nonlinear Two-Degree-of-Freedom (2DOF) control structure [8-12] to form a new control strategy. The smooth motion profile used in the 2DOF scheme helps to prevent controller from exciting the resonant modes of HDD and the use of a feed-forward term helps to reduce overshoot. Overall, it is shown in simulation that the performance of TOUSC is improved significantly in a HDD application.

The paper is organized as follows. The TOUSC control method is briefly outlined in Section II. The 2DOF-based TOUSC method is introduced in Section III. Simulations in a 13kTPI Hard Disk Drive (HDD) model and robustness analysis are described in Section IV. Finally, the conclusion can be found in Section V.

# II. THE TOUSC CONTROL STRATEGY

Before the introduction of the 2DOF-based TOUSC approach, it is necessary to briefly outline the TOUSC [5-7, 15]. The structure of the TOUSC is shown in Figure 1. The control system consists of two components: a state space observer implemented in a unique digital form, known as the current estimator [16], and the TOUSC controller.

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Figure 1 TOUSC control system structure

It was shown in [16] that, in contrast to a conventional state observer in digital form, namely the predicted estimator, the current estimator provides the estimate of the state based on the current measurements, as oppose to the previous ones. Assume that the discrete HDD plant is described by

$$\begin{cases} X(k+1) = \Phi X(k) + \Gamma u(k) \\ y(k) = HX(k) \end{cases}$$
(1)

where *u* and *y* are the input and output, respectively, *X* is the state variable, and  $\Phi$ ,  $\Gamma$  and *H* are constant matrices of appropriate dimensions. The current estimate  $\hat{X}(k)$  is obtained as

$$\hat{X}(k) = \overline{X}(k) + L_{c}[y(k) - H\overline{X}(k)]$$
<sup>(2)</sup>

where  $L_c$  is the observer gain, and  $\overline{X}(k)$  is the predicted estimate based on the model in (1). That is

$$\overline{X}(k) = \Phi \hat{X}(k-1) + \Gamma u(k-1)$$
(3)

It can be seen in equation (2) that estimated state  $\hat{X}(k)$  is updated using the current output of the plant, y(k), instead of the previous one, y(k-1). This proves to be critical in applications where the sampling frequency is limited physically and the associated time delay affects the performance of the control system.

Now, let's introduce the TOUSC algorithm, first proposed by Han and Yuan in [5]. Ignoring friction, the HDD can be approximated as a double integrator with a nominal gain, K. Time optimal control for a double integral plant is well known in continuous time domain. The associated chattering in control signal makes it less attractive in practical use. The contribution of Han and Yuan is that they derived a closed-form solution in discrete time domain which resolves the chattering issue. More details can be found in [6, 7]. Only the computational algorithm is given below.

Let *R* be the control signal saturation value, i.e.,  $|u(k)| \le R$ . With a sampling period of *h*, the TOUSC algorithm can be described as follows:

$$\delta = hKR, \qquad \delta_1 = h^2 KR, \qquad (4)$$

$$x = (x_1, x_2)^T = (-\hat{e}, -\hat{e})^T,$$
 (5)

$$z_1 = x_1 + hx_2,$$
  $z_2 = x_2$  (6)

$$g(z) = \begin{cases} z_2 - sign(z_1) \frac{KR}{2} \left( h - \sqrt{\frac{8|z_1|}{KR}} + h^2 \right), & |z_1| > \delta_1 \\ z_2 + \frac{z_1}{h}, & |z_1| \le \delta_1 \end{cases}$$
(7)

$$u(z_1, z_2) = -R \bullet sat(g(z), hKR) \tag{8}$$

where

$$sat(x,\delta) = \begin{cases} sign(x), & |x| > \delta \\ \frac{x}{\delta}, & |x| \le \delta. \end{cases}$$
(9)

It was observed that TOUSC performs better in the presence of sensor noises if *h* is replaced by  $k_h *h$  in (8), with  $k_h$  is the tuning parameter with the constraint  $k_h \ge 1$ . Intuitively, this expands the linear control region in the controller and makes the control signal smoother at the costs of degradation of the performance [6,7]. For the HDD applications, the challenge is to make  $k_h$  as small as possible while maintaining an acceptable level of smoothness in the control signal. The 2DOF approach described in the following section is motivated to serve this purpose.

# **III. THE 2DOF-BASED TOUSC METHOD**

In order to improve the track-seek performance, a 2DOF-based TOUSC controller is proposed [15], as shown in Figure 2. Along with the TOUSC controller, it has an additional profile generator. The profile generator generates desired position and velocity trajectories and the feed-forward control signal for the track-seek mode.



Figure 2 Structure of nonlinear 2DOF unified servo

This control scheme is denoted as 2DOF because it has the feedback and the feed-forward components. Both are running continuously and no hard switch between different controllers is necessary. In other words, this is a unified controller.

In the track-seek mode, the feed-forward control plays the main role while the TOUSC acts as a fine-tune controller. During the track following phase, the feedforward signal becomes negligible while the TOUSC takes the responsibility of maintaining high track following precision. The use of the feed-forward allows the TOUSC to be tuned more aggressively, which increases the positioning precision and its capability to suppress the disturbances without degrading the stability of the system.

# 3.1 Profile Generator

A position profile for hard disk drive servo is used to provide nearly the fastest (time-optimal) trajectory for the output to follow. It is also made smooth enough so that high-frequency components of the plant are not excited. A compromise is made between the speed and the smoothness in the profile design.

In order to design a profile that is physically attainable, the dynamic model of the HDD plant is used [8]. Assume that a simplified HDD model is illustrated in Figure 3. Note that the power amplifier controller has a saturation voltage,  $V_{max}$ , for its output. Based on the plant model, the structure of the profile generator is illustrated in Figure 4. Besides the double-integrator motion model, it also includes the back electromotive force, the VCM model and dynamics of the power amplifier. Additionally, the DAC saturation block is needed behind the VCM model block to limit the current output and a "Non-negative" saturation block is placed to prevent the velocity from becoming negative, as seen in the figure. The DAC saturation block has  $\pm R$  (A) as the saturation limits while the "Non-negative" block sets zero as its lower limitation.



Figure 3 A simplified hard disk drive model



Figure 4 Structure of the profile generator

To produce the fastest and attainable position profile using the plant model in the profile generator, a maximum voltage,  $V_{max}$ , is first applied to the VCM model for a certain time interval, resulting in maximum acceleration. Then, the voltage is switched to the maximum negative value,  $-V_{max}$ , resulting in maximum deceleration. Because of the "Non-negative" Saturation block, the velocity will reduce to zero and stay there, and the position will reach its steady state value. The length of the time interval during which the  $V_{max}$  is applied determines the steady state value of the position output. For example, in a 13kTPI hard disk drive plant described in the Appendix, it was determined by simulation that to move the position to 10,000 tracks, the duration for  $V_{max}$  should be  $3.829184 \times 10^{-3}$ s, as shown in Figure 5.

To make the profile smooth and avoid exciting the high-frequency components, an additional 1<sup>st</sup> order filter,  $\frac{R_t}{2.5L_{coil} s + R_t}$ , is applied to the velocity signal. To

determine the timing of the *Switch* in Figure 4, off-line simulations are required to obtain a look-up table, which records the relationships between the track distance and

the switch time. Also, the plant model in Figure 4 needs to be converted into its discrete form in implementation.



Figure 5 Inputs and outputs of the profile generator

# 3.2 The Feed-Forward Signal and Velocity Profile

In addition to the position profile,  $Y_{sp}$ , the profile generator also provides the feed-forward current control signal and velocity reference,  $V_{sp}$ . The velocity reference is essential to the TOUSC closed-loop control. It provides the approximate derivative of the position error signal:

$$\dot{e}(t) = V_{sp}(t) - \hat{v}(t),$$
 (10)

The feed-forward control signal is used to improve the track-seek performance. As we will see in the simulation, it can shorten the seek time significantly. On the other hand, a small overshoot arises during the transition from the track-seek process to the track following process.

The imprecision of this feed-forward controller is possibly responsible for the small overshoot. If the current is switched at improper time, it imposes an obvious input disturbance to the TOUSC at the transition point when the track-seek phase ends. So, the TOUSC has to make significant adjustment to correct the disturbance after the transition, while it plays a very small role before the transition since the current feed-forward control is dominant. The sudden role change in the TOUSC may lead to the undesired overshoot. One solution is to make TOUSC play a more significant role even in the trackseek mode by adjusting the feedforward gain  $K_{ff}$ . The value of  $K_{ff}$  is experimentally determined to yield the fastest and smoothest transient response. Simulations in the next section will show the effectiveness of the feedforward gain.

# **IV. SIMULATION AND ANALYSIS**

The proposed 2DOF-based TOUSC method is applied to a 13kTPI hard disk drive model in simulation in this section with a sampling rate of 15 kHz. The parameters of the disk drive model are listed in the Appendix. The D/A output has a saturation of  $\pm 1.9A$ , which corresponds to the parameter *R* in TOUSC, and the power amplifier's maximum output voltage is  $\pm 12.0$  V.

Using the simplified rigid-body model at the 15 kHz sampling rate, the parameters of the current estimator were obtained as

$$\Psi = \begin{bmatrix} 1 & 0.8104 \\ 0 & 1 \end{bmatrix}, \Phi = \begin{bmatrix} 1 & 1.6207 \\ 0 & 1 \end{bmatrix}, \Gamma = \begin{bmatrix} 1.5953 \\ 1.9686 \end{bmatrix}$$
(11)

and  $L_c$  was chosen as  $L_c = [0.79 \ 0.25]$ , which is determined by tuning in simulations.

Simulation studies focused on three aspects: effectiveness of  $K_{ff}$ , track-seek performance and track following performance.

#### 4.1 Impact of the Feed-Forward Gain

As mentioned before, the feed-forward gain,  $K_{ff}$ , is adjusted to overcome the overshoot caused by the change in the feed-forward control signal. The larger the  $K_{ff}$  from its default value of one, the more action the TOUSC will produce in the track-seek process. Accordingly, smoother transition from the track-seek to the track following is obtained.

Figure 6 shows both the feed-forward control signal  $u_2$ and the TOUSC output  $u_1$  during the track-seek process. The larger the  $K_{ff}$ , the stronger the TOUSC control is during the seek process. The corresponding position responses are shown in Figure 7. Smoother position outputs have been achieved at the transition with relatively larger  $K_{ff}$ . The transient performances are shown in Table I with different  $K_{ff}$ . It is evident that a suitable  $K_{ff}$ helps to decrease the overshoot in the 2DOF-based TOUSC scheme.

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K <sub>ff</sub>	Overshoot (tracks)	Seek Time (ms)	Settling Time (0.02 track) (ms)	Steady State Error (tracks)	
1.00	0.23	7.4	8.0	0.008	
1.09	0.02	7.1	7.3	0.008	
1.20	0.008	7.6	8.0	0.005	







TOUSC with different  $K_{ff}$ 

#### 4.2 Improvement of the Track-Seek Performance

To show the improvement of the seek-time, the proposed 2DOF-based TOUSC scheme is compared with the TOUSC method in simulation. The structures and parameters of both controllers have already been described above. For a 10,000-track distance, the response curves of both cases are shown in Figure 8. Notice that  $K_{ff}$  is set to 1.09 and  $k_h$  to 2.5 in the proposed method and that  $k_h$  was 3.5 in the TOUSC.



Figure 8 Simulation curves for 10,000-track seek

According to the simulation, the seek time of proposed method improves to 7.2ms from 8.1ms in the TOUSC case. Similarly, a series of simulation were conducted for different seek distances. The resulting curves are plotted in Figure 9. Using time-optimal profiles and feed-forward control signal, the proposed method evidently produces shorter seek time than the TOUSC while still maintaining satisfactory track settling performance.



Figure 9 Comparison of seek times in simulations

#### 4.3 Improvement of Track Following Performance

By using the feed-forward control and the smooth profile, the TOUSC is allowed to be made more aggressive. Here  $k_h$  is reduced from 3.5 to 2.5, which makes it closer to the ideal case of  $k_h = 1$ . Notice that a smaller  $k_h$  leads to better robustness to external disturbances and higher position control precision.

To test the track following performance, the outputs of feed-forward signal and the profiles are set to zero and white noises are inserted as the torque and position disturbances. Three typical cases were studied in simulation on the basis of how the radial vibration disturbance was applied:

<u>Case 1</u>: a white noise position disturbance with a power of  $1.0 \times 10^{-6}$  and a current disturbance with a power of  $1.25 \times 10^{-10}$  were injected to the servo systems.

<u>Case 2</u>: the simulation was repeated by injecting an additional 60 Hz vibration position disturbance with 5-track amplitude.

<u>Case 3</u>: same as Case 2, except the sinusoidal position disturbance is of 120Hz instead of 60Hz. (120Hz corresponds to 7,200RPM, a typical disk rotation speed). The position and current disturbances in this case are shown in Figure 10 and the position outputs from TOUSC and 2DOF-based TOUSC are shown in Figure 11.

To compare the performance of the original TOUSC and the proposed 2DOF-based TOUSC, the RMS error is used as the performance index, defined as

$$e_{rms} = \sqrt{\frac{\sum_{N} e(k)^2}{N}}$$
(12)

where e(k) is the difference between the desired position and the actual one. The simulation results are listed in Table II. It is evident that the proposed control algorithm has better robustness than the TOUSC as a track following controller.







Figure 11 The track following position outputs (case 3)

**Table II** Comparison of *e<sub>rms</sub>* in track following simulations

	Case 1	Case 2	Case 3
TOUSC	0.0322	0.0628	0.2019
10050	(tracks)	(tracks)	(tracks)
2DOF-based	0.0267	0.0434	0.1272
TOUSC	(tracks)	(tracks)	(tracks)
IMPROVEMENT	14.3%	30.9%	37.0%

#### VI CONCLUSION

A two degree of freedom time optimal control strategy is proposed for the HDD servo system design. Compared to the original single degree of freedom time optimal controller, the proposed method helps to shorten the trackseek time and makes the controller more aggressive in the track following mode. Better robustness in the presence of the position noise and disturbance, as well as torque disturbances, is achieved. The accuracy is improved in both the track seeking and track following phases. Furthermore, no controller switching is needed in the transition from track seeking to track following.

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### Appendix

The model of the VCM actuator of an industrial 13.0kTPI hard disk drive is described by

$$G(s) = \frac{Y(s)}{I_c(s)} = \frac{K_t R_{head}}{J_{total} s^2} + \sum_{i=1}^8 \frac{b_i}{s^2 + 2\xi_i \omega_i s + \omega_i^2}$$

where  $i_c$  is the current command (in ampere) and y is the position output (in meter). The model parameters are listed in the following Table.

	Parameter	Description	Nominal Value
	R <sub>coil</sub>	Coil resistance	5.9Ω
D	L <sub>coil</sub>	Coil inductance	0.368mH
Power	K <sub>pa</sub>	Control gain	$1.0 \times 10^{6}$
Ampimer	e <sub>max</sub>	Saturated voltage	12.0 Volts
	Slewrate	DAC Rate limit	10,000A/s
Actuator	J <sub>total</sub>	Moving inertia	2.54×10 <sup>-6</sup> Kg.m <sup>2</sup>
	K <sub>t</sub>	Torque constant	0.075 N.m/A
	Rhead	Head radius	1.9 Inches
	TPI	Tracks density	13,000 TPI
	Curclip	DAC saturation	±1.9A
	$f_{I}$	1 <sup>st</sup> frequency	4,500 Hz
	$f_2$	2 <sup>nd</sup> frequency	5,400 Hz
	$f_3$	3 <sup>rd</sup> frequency	5,550 Hz
	$f_4$	4 <sup>th</sup> frequency	5,670 Hz
	$f_5$	5 <sup>th</sup> frequency	7,300 Hz
	$f_6$	6 <sup>th</sup> frequency	7,450 Hz
	$f_7$	7 <sup>th</sup> frequency	8,000 Hz
	$f_8$	8 <sup>th</sup> frequency	9,650 Hz
	$b_1$	1 <sup>st</sup> coupling coefficient	1700
	$b_2$	2 <sup>nd</sup> coupling coefficient	260
	<i>b</i> <sub>3</sub>	3 <sup>rd</sup> coupling coefficient	300
	<i>b</i> <sub>4</sub>	4 <sup>th</sup> coupling coefficient	45
Resonance	<i>b</i> <sub>5</sub>	5 <sup>th</sup> coupling 100 coefficient	
	<i>b</i> <sub>6</sub>	6 <sup>th</sup> coupling 105 coefficient	
	<i>b</i> <sub>7</sub>	7 <sup>th</sup> coupling coefficient	105
	$b_8$	8 <sup>th</sup> coupling coefficient	455
	ξı	1 <sup>st</sup> damping ratio	0.018
	ξ2	2 <sup>nd</sup> damping ratio	0.025
	Ē3	3 <sup>rd</sup> damping ratio	0.025
	Ĕı	4 <sup>th</sup> damping ratio	0.001
	E5	5 <sup>th</sup> damping ratio	0.010
	رو مح	6 <sup>th</sup> damping ratio	0.010
	50 E7	7 <sup>th</sup> damping ratio	0.010
	ξ <sub>8</sub>	8 <sup>th</sup> damping ratio	0.013