Adaptive Tracking Control of On-Line Path Planners: Velocity Fields and Navigation Functions[1]

M. L. McIntyre,† W. E. Dixon,‡ D. M. Dawson,† and B. Xian⋆

†Department of Electrical & Computer Engineering, Clemson University, Clemson, SC 29634-0915

‡Department of Mechanical & Aerospace Engineering, University of Florida, Gainesville, FL 32611-6250

⋆Department of Mechanical Engineering and Material Sciences, Duke University, Durham, NC 27708

E-mail: mmcinty@ces.clemson.edu, wdixon@ufl.edu, darren.dawson@ces.clemson.edu, xbin@duke.edu

Abstract: Traditionally, robot control research has focused on the position tracking problem where the objective is to force the robot's end-effector to follow an a priori known desired time dependent trajectory. Motivated by task objectives that are more effectively described by on-line, state-dependent trajectories, two adaptive tracking controllers are developed in this paper that accommodate on-line path planning objectives. An example adaptive controller is first modified to achieve velocity field tracking in the presence of parametric uncertainty in the robot dynamics. An adaptive navigation function based controller is then designed that targets the trajectory planning problem where the task objective can be described as the desire to move from an initial condition to a goal configuration while avoiding known obstacles.

1 Introduction

Traditionally, robot control researchers have focused on the position tracking problem where the objective is to force the robot to follow a desired time dependent trajectory. Since the objective is encoded in terms of a time dependent trajectory, the robot may be forced to follow an unknown course to catch up with the desired trajectory in the presence of a large initial error. For example, several researchers have reported the so called radial reduction phenomena (e.g., [19], [21]) in which the actual path followed has a smaller radius than the specified trajectory. In light of this phenomena, the control objective for many robotic tasks are more appropriately encoded as a contour following problem in which the objective is to force the robot to follow a state-dependent function that describes the contour. One example of a control strategy aimed at the contour following problem is velocity field control (VFC) where the desired contour is describe by a velocity tangent vector [20]. The advantages of the VFC approach can be summarized as follows.[1] (1) The velocity field error more effectively penalizes the robot for leaving the desired contour. (2) The control task can be specified invariant of the task execution speed. (3) Task coordination and synchronization is more explicit for contour following.

The ability for a velocity field to encode certain contour following tasks has recently prompted researchers to investigate VFC for various applications. For example, Li and Horowitz utilized a passive VFC approach to control robot manipulators for contour following applications in [20], and more recently, Dee and Li used VFC to achieve passive bilateral teleoperation of robot manipulators in [17]. The authors of [19] utilized a passive VFC approach to develop a force controller for robot manipulator contour following applications. Yamakita et al. investigated the application of passive VFC to cooperative mobile robots and cooperative robot manipulators in [29] and [30], respectively. Typically, VFC is based on a nonlinear control approach where exact model knowledge of the system dynamics are required. Motivated by the desire to account for uncertainty in the robot dynamics, Cervantes et al. developed a robust VFC in [4]. Specifically, in [4] a proportional-integral controller was developed that achieved semiglobal practical stabilization of the velocity field tracking errors despite uncertainty in the robot dynamics. From a review of VFC literature, it can also be determined that previous research efforts have focused on ensuring the robot tracks the velocity field, but no development has been provided to ensure the link position remains bounded. The result in [4] acknowledged the issue of boundedness of the robot position; however, the issue is simply addressed by an assumption that the following norm

$$\left\| q(0) + \int_0^t \vartheta(q(\sigma))d\sigma \right\| \tag{1}$$

yields globally bounded trajectories, where $q(t)$ denotes

[1]This research was supported in part by U.S. NSF Grant DMI-9457967, ONR Grant N00014-99-1-0589, a DOC Grant, and an ARO Automotive Center Grant at Clemson University, and in part by AFOSR contract number F49620-03-1-0381 at the University of Florida.

[1]See [4], [19], and [20] for a more thorough discussion of the advantages and differences of VFC with respect to traditional trajectory tracking control.

the position, and $\vartheta(\cdot)$ denotes the velocity field.

In addition to VFC, some task objectives are motivated by the need follow a trajectory to a desired goal configuration while avoiding known obstacles in the configuration space. For this class of problems, it is more important for the robot to follow an obstacle free path to the desired goal point than it is to meet a time-based requirement. Numerous researchers have investigated algorithms to address this motion control problem. A comprehensive summary of techniques that address the classic geometric problem of constructing a collision-free path and traditional path planning algorithms is provided in Section 9, "Literature Landmarks", of Chapter 1 of [15]. Since the pioneering work by Khatib in [10], it is clear that the construction and use of potential functions has continued to be one of the mainstream approaches to robotic task execution among known obstacles. In short, potential functions produce a repulsive potential field around the boundary of the robot task-space and obstacles and an attractive potential field at the goal configuration. A comprehensive overview of research directed at potential functions is provided in [15]. One criticism of the potential function approach is that local minima can occur that can cause the robot to "get stuck" without reaching the goal position. Several researchers have proposed approaches to address the local minima issue (e.g., see [1], [2], [5], [11], [28]). One approach to address the local minima issue was provided by Koditschek in [12] for holonomic systems (see also [13] and [24]) that is based on a special kind of potential function, coined a navigation function, that has a refined mathematical structure which guarantees a unique minimum exists. By leveraging from previous results directed at classic (holonomic) systems, more recent research has focused on the development of potential function-based approaches for nonholonomic systems. For a review of this literature see [3], [6], [7], [8], [14], [16], [22], [24], [26], and [27].

The aim of this paper is to illustrate how an example adaptive controller (e.g., the benchmark adaptive tracking controller presented in [25]) can be modified to incorporate trajectory planning techniques with the controller. To this end, two adaptive controllers are developed. The first controller focuses on the VFC problem. Specifically, the benchmark adaptive controller given in [25] is modified to yield VFC in the presence of parametric uncertainty. The contribution of the development is that velocity field tracking is achieved by incorporating a norm squared gradient term in the control design that is used to prove the link positions are bounded through a Lyapunov-analysis rather than by an assumption. In lieu of the assumption in (1), the VFC development is based on the selection of a velocity field that is first order differentiable, and that a first order differentiable, nonnegative function $V(q) \in \mathbb{R}$ exists such that the following inequality holds

$$\frac{\partial V(q)}{\partial q}\vartheta(q) \leq -\gamma_3(\|q\|) + \zeta_0 \qquad (2)$$

where $\frac{\partial V(q)}{\partial q}$ denotes the partial derivative of $V(q)$ with respect to $q(t)$, $\gamma_3(\cdot) \in \mathbb{R}$ is a class $\mathcal{K}$ function[2], and $\zeta_0 \in \mathbb{R}$ is a nonnegative constant. That is, in lieu of the assumption in (1) this paper introduces a stability-based condition on the velocity field. It is interesting to note that the velocity field described in the experimental results provided in [4] can be shown to satisfy the stability-based condition in (2) *(see [23] for proof)*. As an extension to the VFC problem, a navigation function is incorporated with the benchmark adaptive controller in [25] to track a reference trajectory that yields a collision free path to a constant goal point in an obstacle cluttered environment with known obstacles.

This paper is organized as follows. In Section 2, the dynamic model for a robot manipulator is provided. In Section 3, the VFC development is presented, including a two-part stability analysis. The first analysis proves that if a velocity field tracking signal is square integrable then the link position is globally uniformly bounded (GUB). The second analysis proves that the velocity field tracking signal is square integrable, all the system states are bounded, and that the velocity field tracking error converges to zero despite parametric uncertainty in the dynamic model. In Section 4, a navigation function based trajectory planning and control development is presented, along with the stability analysis. This analysis proves that a backstepping signal is square integrable, all the system states are bounded, and that the robot manipulator will track an obstacle free path to a goal point, despite parametric uncertainty in the dynamic model. Concluding remarks are provided in Section 5. For experimental results for both controllers see [23].

2 System Model

The mathematical model for an n-DOF robotic manipulator is assumed to have the following form

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) = \tau. \qquad (3)$$

In (3), $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration, respectively, $M(q) \in \mathbb{R}^{n\times n}$ represents the positive-definite, symmetric inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ represents the centripetal-Coriolis terms, $G(q) \in \mathbb{R}^n$ represents the known gravitational vector, and $\tau(t) \in \mathbb{R}^n$ represents the torque input vector. We will assume that $q(t)$ and $\dot{q}(t)$ are measurable.

[2] A continuous function $\alpha : [0, \alpha) \to [0, \infty)$ is said to belong to class $\mathcal{K}$ if it is strictly increasing and $\alpha(0) = 0$ [9].

The dynamic model in (3), exhibits the following properties that are utilized in the subsequent control development and stability analysis.

Property 1: The inertia matrix can be upper and lower bounded by the following inequalities [18]

$$m_1 \|\xi\|^2 \leq \xi^T M(q)\xi \leq m_2(q) \|\xi\|^2 \quad \forall \xi \in \mathbb{R}^n \quad (4)$$

where m_1 is a positive constant, $m_2(\cdot)$ is a positive function, and $\|\cdot\|$ denotes the Euclidean norm.

Property 2: The inertia and centripetal-Coriolis matrices satisfy the following relationship [18]

$$\xi^T \left(\frac{1}{2}\dot{M}(q) - V_m(q,\dot{q})\right)\xi = 0 \quad \forall \xi \in \mathbb{R}^n \quad (5)$$

where $\dot{M}(q)$ represents the time derivative of the inertia matrix.

Property 3: The robot dynamics given in (3) can be linearly parameterized as follows [18]

$$Y(q,\dot{q},\ddot{q})\theta \triangleq M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) \quad (6)$$

where $\theta \in \mathbb{R}^p$ contains constant system parameters, and $Y(q,\dot{q},\ddot{q}) \in \mathbb{R}^{n\times p}$ denotes a regression matrix composed of $q(t)$, $\dot{q}(t)$, and $\ddot{q}(t)$.

3 Adaptive VFC

3.1 Control Objective

As described previously, many robotic tasks can be effectively encapsulated as a velocity field. That is, the velocity field control objective can be described as commanding the robot manipulator to track a velocity field that is defined as a function of the current link position. To quantify this objective, a velocity field tracking error, denoted by $\eta_1(t) \in \mathbb{R}^n$, is defined as follows

$$\eta_1(t) \triangleq \dot{q}(t) - \vartheta(q) \quad (7)$$

where $\vartheta(\cdot) \in \mathbb{R}^n$ denotes the velocity field. To achieve the control objective, the subsequent development is based on the assumption that $q(t)$ and $\dot{q}(t)$ are measurable, and that $\vartheta(q)$ and its partial derivative $\frac{\partial\vartheta(q)}{\partial q} \in \mathbb{R}^n$, are assumed to be bounded provided $q(t) \in \mathcal{L}_\infty$.

3.2 Benchmark Control Modification

To develop the open-loop error dynamics for $\eta_1(t)$, we take the time derivative of (7) and premultiply the resulting expression by the inertia matrix as follows

$$\begin{aligned} M(q)\dot{\eta}_1 \quad = \quad & \tau - V_m(q,\dot{q})\dot{q} - G(q) + V_m(q,\dot{q})\vartheta(q) \\ & -V_m(q,\dot{q})\vartheta(q) - M(q)\frac{\partial\vartheta(q)}{\partial q}\dot{q} \end{aligned} \quad (8)$$

where (3) was utilized. From (7), the expression in (8) can be rewritten as follows

$$M(q)\dot{\eta}_1 = -V_m(q,\dot{q})\eta_1 - Y_1(q,\dot{q})\theta + \tau \quad (9)$$

where θ was introduced in (6) and $Y_1(q,\dot{q}) \in \mathbb{R}^{n\times p}$ denotes a measurable regression matrix that is defined as follows

$$Y_1(q,\dot{q})\theta \triangleq M(q)\frac{\partial\vartheta(q)}{\partial q}\dot{q} + V_m(q,\dot{q})\vartheta(q) + G(q). \quad (10)$$

Based on the open-loop error system in (9), a number of control designs could be utilized to ensure velocity field tracking (i.e., $\|\eta_1(t)\| \to 0$) given the assumption in (1). Motivated by the desire to eliminate the assumption in (1), a norm squared gradient term is incorporated in an adaptive controller introduced in [25] as follows

$$\tau(t) \triangleq -\left(K + \left\|\frac{\partial V(q)}{\partial q}\right\|^2 I_n\right)\eta_1 + Y_1(q,\dot{q})\hat{\theta}_1 \quad (11)$$

where $K \in \mathbb{R}^{n\times n}$ is a constant, positive definite diagonal matrix, $I_n \in \mathbb{R}^{n\times n}$ is the standard $n\times n$ identity matrix, and $\frac{\partial V(q)}{\partial q}$ was introduced in (2). In (11), $\hat{\theta}(t) \in \mathbb{R}^p$ denotes a parameter estimate that is generated by the following gradient update law

$$\dot{\hat{\theta}}_1(t) = -\Gamma_1 Y_1^T(q,\dot{q})\eta_1 \quad (12)$$

where $\Gamma_1 \in \mathbb{R}^{p\times p}$ is a constant, positive definite diagonal matrix. After substituting (11) into (9), the following closed-loop error system can be obtained

$$\begin{aligned} M(q)\dot{\eta}_1 \quad = \quad & -V_m(q,\dot{q})\eta_1 - Y_1(q,\dot{q})\tilde{\theta}_1 \\ & -\left(K + \left\|\frac{\partial V(q)}{\partial q}\right\|^2 I_n\right)\eta_1 \end{aligned} \quad (13)$$

where the parameter estimation error signal $\tilde{\theta}_1(t) \in \mathbb{R}^p$ is defined as follows

$$\tilde{\theta}_1(t) \triangleq \theta - \hat{\theta}_1. \quad (14)$$

Remark 1 *While the control development is based on a modification of the adaptive controller introduced in [25], the norm squared gradient term could also be incorporated in other benchmark controllers to yield similar results (e.g., sliding mode controllers).*

3.3 Stability Analysis

To facilitate the subsequent stability analysis, the following preliminary theorem is utilized.

Theorem 1 *Let $\bar{V}(t) \in \mathbb{R}$ denote the following non-negative, continuous differentiable function*

$$\bar{V}(t) \triangleq V(q) + P(t) \quad (15)$$

where $V(q) \in \mathbb{R}$ denotes a nonnegative, continuous differentiable function that satisfies (2) and the following inequalities

$$0 \leq \gamma_1(\|q\|) \leq V(q) \leq \gamma_2(\|q\|) \tag{16}$$

where $\gamma_1(\cdot)$, $\gamma_2(\cdot)$ are class $\mathcal{K}$ functions, and $P(t) \in \mathbb{R}$ denotes the following nonnegative, continuous differentiable function

$$P(t) \triangleq \gamma - \int_{t_0}^{t} \varepsilon^2(\sigma) d\sigma \tag{17}$$

where $\gamma \in \mathbb{R}$ is a positive constant, and $\varepsilon(t) \in \mathbb{R}$ is defined as follows

$$\varepsilon \triangleq \left\| \frac{\partial V(q)}{\partial q} \right\| \|\eta_1\| . \tag{18}$$

If $\varepsilon(t)$ is a square integrable function, where

$$\int_{t_0}^{t} \varepsilon^2(\sigma) d\sigma \leq \gamma, \tag{19}$$

and if after utilizing (7), the time derivative of $\bar{V}(t)$ satisfies the following inequality

$$\dot{\bar{V}}(t) \leq -\gamma_3(\|q\|) + \xi_0 \tag{20}$$

where $\gamma_3(\cdot)$ is the class $\mathcal{K}$ function introduced in (2), and $\xi_0 \in \mathbb{R}$ denotes a positive constant, then $q(t)$ is global uniformly bounded.

Proof: *See [23] for proof.* ■

Theorem 2 *The adaptive VFC given in (11) and (12) yields global velocity field tracking in the sense that*

$$\|\eta_1(t)\| \to 0 \quad as \quad t \to \infty. \tag{21}$$

Proof: *See [23] for proof.* ■

4 Navigation Function Control Extension

4.1 Control Objective

The objective in this extension is to navigate a robot's end-effector along a collision-free path to a constant goal point, denoted by $q^* \in \mathcal{D}$, where the set $\mathcal{D}$ denotes a free configuration space that is a subset of the whole configuration space with all configurations removed that involve a collision with an obstacle, and $q^* \in \mathbb{R}^n$ denotes the constant goal point in the interior of $\mathcal{D}$. Mathematically, the primary control objective can be stated as the desire to ensure that

$$q(t) \to q^* \text{ as } t \to \infty \tag{22}$$

where the secondary control is to ensure that $q(t) \in \mathcal{D}$. To achieve these two control objectives, we define $\varphi(q) \in \mathbb{R}$ as a function $\varphi(q) : \mathcal{D} \to [0, 1]$ that is assumed to satisfy the following properties:

P1) The function $\varphi(q)$ is a first order and second order differentiable Morse function [13] (i.e., $\frac{\partial}{\partial q}\varphi(q)$and $\frac{\partial}{\partial q}\left(\frac{\partial}{\partial q}\varphi(q)\right)$ exist on $\mathcal{D}$).

P2) The function $\varphi(q)$ obtains its maximum value on the boundary of $\mathcal{D}$.

P3) The function $\varphi(q)$ has a unique global minimum at $q(t) = q^*$.

P4) If $\frac{\partial}{\partial q}\varphi(q) = 0$ then $q(t) = q^*$.

Based on (22) and the above definition, an auxiliary tracking error signal, denoted by $\eta_2(t) \in \mathbb{R}^n$, can be defined as follows to quantify the control objective

$$\eta_2(t) \triangleq \dot{q}(t) + \nabla\varphi(q) \tag{23}$$

where $\nabla\varphi(q) = \frac{\partial}{\partial q}\varphi(q)$ denotes the gradient vector of $\varphi(q)$ defined as follows

$$\nabla\varphi(q) \triangleq \left[\begin{array}{cccc} \frac{\partial\varphi}{\partial q_1} & \frac{\partial\varphi}{\partial q_2} & \dots & \frac{\partial\varphi}{\partial q_n} \end{array} \right]^T . \tag{24}$$

Remark 2 *As discussed in [24], the construction of the function $\varphi(q)$, coined a navigation function, that satisfies all of the above properties for a general obstacle avoidance problem is nontrivial. Indeed, for a typical obstacle avoidance, it does not seem possible to construct $\varphi(q)$ such that $\frac{\partial}{\partial q}\varphi(q) = 0$ only at $q(t) = q^*$. That is, as discussed in [24], the appearance of interior saddle points (i.e., unstable equilibria) seems to be unavoidable; however, these unstable equilibria may have minimal impact in practice. That is, $\varphi(q)$ can be constructed as shown in [24] such that only a "few" initial conditions will result in convergence to the unstable equilibria.*

4.2 Benchmark Control Modification

To develop the open-loop error dynamics for $\eta_2(t)$, we take the time derivative of (23) and premultiply the resulting expression by the inertia matrix as follows

$$M\dot{\eta}_2 = -V_m(q, \dot{q})\eta_2 + Y_2(q, \dot{q})\theta + \tau \tag{25}$$

where (3) and (23) were utilized. In (25), the linear parameterization $Y_2(q, \dot{q})\theta$ is defined as follows

$$Y_2(q, \dot{q})\theta \triangleq M(q)f(q, \dot{q}) + V_m(q, \dot{q}) \nabla \varphi(q) - G(q) \tag{26}$$

where $Y_2(q, \dot{q}) \in \mathbb{R}^{n \times m}$ denotes a measurable regression matrix, $\theta \in \mathbb{R}^m$ was introduced in (6), and the auxiliary signal $f(q,\dot{q}) \in \mathbb{R}^n$ is defined as

$$f(q,\dot{q}) \triangleq \frac{d}{dt}\left(\nabla\varphi(q)\right) = H(q)\dot{q} \tag{27}$$

where the Hessian matrix $H(q) \in \mathbb{R}^{n\times n}$ is defined as follows

$$H(q) \triangleq \begin{bmatrix} \frac{\partial^2\varphi}{\partial q_1^2} & \frac{\partial^2\varphi}{\partial q_1 \partial q_2} & \cdots & \frac{\partial^2\varphi}{\partial q_1 \partial q_n} \\ \frac{\partial^2\varphi}{\partial q_2 \partial q_1} & \frac{\partial^2\varphi}{\partial q_2^2} & \cdots & \frac{\partial^2\varphi}{\partial q_2 \partial q_n} \\ \cdots & \cdots & \cdots & \cdots \\ \frac{\partial^2\varphi}{\partial q_n \partial q_1} & \cdots & \cdots & \frac{\partial^2\varphi}{\partial q_n^2} \end{bmatrix}. \tag{28}$$

Based on (25) and the subsequent stability analysis, the following adaptive controller introduced in [25] can be utilized

$$\tau \triangleq -k\eta_2 - Y_2(q,\dot{q})\hat{\theta}_2 \tag{29}$$

where $k \in \mathbb{R}$ is a positive constant gain, and $\hat{\theta}_2(t) \in \mathbb{R}^p$ denotes a parameter update law that is generated from the following expression

$$\dot{\hat{\theta}}_2\,(t) \triangleq \Gamma_2 Y_2^T(q,\dot{q})\eta_2 \tag{30}$$

where $\Gamma_2 \in \mathbb{R}^{m\times m}$ is a positive definite, diagonal gain matrix. Note that the trajectory planning is incorporated in the controller through the gradient terms included in (26) and (27). After substituting (29) into (25) the following closed loop error systems can be obtained

$$M\dot{\eta}_2 = -V_m(q,\dot{q})\eta_2 - k\eta_2 + Y_2(q,\dot{q})\tilde{\theta}_2 \tag{31}$$

where $\tilde{\theta}_2(t) \in \mathbb{R}^p$ is defined as follows

$$\tilde{\theta}_2(t) \triangleq \theta - \hat{\theta}_2. \tag{32}$$

4.3 Stability Analysis

Theorem 3 *The adaptive controller given in (29) and (30) ensures that the robot manipulator tracks an obstacle free path to the unique goal configuration in sense that*

$$q(t) \rightarrow q^* \ \text{as } t \rightarrow \infty \tag{33}$$

provided the control gain k introduced in (29) is selected sufficiently large.

Proof: *See [23] for proof.* ■

5 Conclusion

Two trajectory planning and adaptive tracking controllers are presented. The benchmark adaptive tracking controller by Slotine [25] was modified to achieve velocity field tracking in the presence of parametric uncertainty in the robot dynamics. By incorporating a norm squared gradient term to the VFC, the boundedness of all signals can be proven without the typical assumption that bounds the integral of the velocity field. An extension was then provided that also modifies a standard adaptive controller by incorporating a gradient based term. Using standard backstepping techniques, a Lyapunov analysis was used to prove that a navigation function could be incorporated in the control design to ensure the robot remained on an obstacle free path within an expanded configuration space to reach a goal configuration. For experimental results for both controllers see [23].

References

[1] J. Barraquand and J. C. Latombe, "A Monte-Carlo Algorithm for Path Planning with Many Degrees of Freedom," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Cincinnati, Ohio, pp. 584-589, 1990.

[2] J. Barraquand, B. Langlois, and J. C. Latombe, "Numerical Potential Fields Techniques for Robot Path Planning," *IEEE Trans. on Systems, Man, and Cybernetics*, Vol. 22, pp. 224-241, (1992).

[3] A. Bemporad, A. De Luca, and G. Oriolo, "Local Incremental Planning for a Car-Like Robot Navigating Among Obstacles," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Minneapolis, Minnesota, pp. 1205-1211, 1996.

[4] I. Cervantes, R. Kelly, J. Alvarez-Ramirez, and J. Moreno, "A Robust Velocity Field Control," *IEEE Trans. on Control Systems Technology*, Vol. 10, No. 6, pp. 888-894, (2002).

[5] C. I. Connolly, J. B. Burns, and R. Weiss, "Path Planning Using Laplace's Equation," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Cincinnati, Ohio, pp. 2102-2106, 1990.

[6] S. S. Ge and Y. J. Cui, "New Potential Functions for Mobile Robot Path Planning," *IEEE Trans. on Robotics and Automation*, Vol. 16, No. 5, pp. 615-620, (2000).

[7] J. Guldner and V. I. Utkin, "Sliding Mode Control for Gradient Tracking and Robot Navigation Using Artificial Potential Fields," *IEEE Trans. on Robotics and Automation*, Vol. 11, No. 2, pp. 247-254, (1995).

[8] J. Guldner, V. I. Utkin, H. Hashimoto, and F. Harashima, "Tracking Gradients of Artificial Potential Field with Non-Holonomic Mobile Robots," *Proc. of the American Control Conf.*, Seattle, Washington, pp. 2803-2804, 1995.

[9] H. K. Khalil, Nonlinear Systems, Third edition, Prentice Hall, 2002.

[10] O. Khatib, Commande dynamique dans l'espace opérational des robots manipulateurs en présence d'obstacles, Ph.D. Dissertation, École Nationale Supéieure de l'Aeéronatique et de l'Espace (ENSAE), France, 1980.

[11] O. Khatib, "Real-Time Obstacle Avoidance for Manipulators and Mobile Robots," *Inter. Journal of Robotics Research*, Vol. 5, No. 1, pp. 90-99, (1986).

[12] D. E. Koditschek, "Exact Robot Navigation by Means of Potential Functions: Some Topological Considerations," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, Raleigh, North Carolina, pp. 1-6, 1987.

[13] D. E. Koditschek and E. Rimon, "Robot Navigation Functions on Manifolds with Boundary," *Adv. Appl. Math.*, Vol. 11, pp. 412-442, (1990).

[14] K. J. Kyriakopoulos, H. G. Tanner, and N. J. Krikelis, "Navigation of Nonholonomic Vehicles in Complex Environments with Potential Fields and Tracking," *Int. J. Intell. Contr. Syst.*, Vol. 1, No. 4, pp. 487-495, (1996).

[15] J. C. Latombe, Robot Motion Planning, Kluwer Academic Publishers: Boston, Massachusetts, 1991.

[16] J. P. Laumond, P. E. Jacobs, M. Taix, and R. M. Murray, "A Motion Planner for Nonholonomic Mobile Robots," *IEEE Trans. on Robotics and Automation*, Vol. 10, No. 5, pp. 577-593, (1994).

[17] D. Lee and P. Li, "Passive Bilateral Feedforward Control of Linear Dynamically Similar Teleoperated Manipulators," *IEEE Trans. on Robotics and Automation*, Vol. 19, No. 3, pp. 443-456 (2003).

[18] F. Lewis, C. Abdallah, and D. Dawson, Control of Robot Manipulators, New York: MacMillan Publishing Co., 1993.

[19] J. Li and P. Li, "Passive Velocity Field Control (PVFC) Approach to Robot Force Control and Contour Following," *Proc. of the Japan/USA Symposium on Flexible Automation*, Ann Arbor, Michigan, 2000.

[20] P. Li and R. Horowitz, "Passive Velocity Field Control of Mechanical Manipulators," *IEEE Trans. on Robotics and Automation*, Vol. 15, No. 4, pp. 751-763, (1999).

[21] P. Y. Li, "Adaptive Passive Velocity Field Control," *Proc. of the American Controls Conference*, San Diego, California, pp. 774-779, 1999.

[22] A. De Luca and G. Oriolo, "Local Incremental Planning for Nonholonomic Mobile Robots," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, San Diego, California, pp. 104-110, 1994.

[23] M. McIntyre, W. Dixon, D. Dawson, and B. Xian, "Adaptive Tracking Control of On-Line Path Planners: Velocity Fields and Navigation Functions," Clemson University CRB Technical Report, CU/CRB/8/20/04/#1, http://www.ces.clemson.edu/ece/crb/publictn/tr.htm, also submitted to IEEE Trans. on Control System Technology, July 2004.

[24] E. Rimon and D. E. Koditschek, "Exact Robot Navigation Using Artificial Potential Function," *IEEE Trans. on Robotics and Automation*, Vol. 8, No. 5, pp. 501-518, (1992).

[25] J. J. E. Slotine and W. Li, Applied Nonlinear Control, Englewood Cliff, NJ: Prentice Hall, Inc., 1991.

[26] H. G. Tanner and K. J. Kyriakopoulos, "Nonholonomic Motion Planning for Mobile Manipulators," *Proc. of the IEEE Int. Conf. on Robotics and Automation*, San Francisco, California, pp. 1233-1238, 2000.

[27] H. G. Tanner, S. G. Loizou, and K. J. Kyriakopoulos, "Nonholonomic Navigation and Control of Cooperating Mobile Manipulators," *IEEE Trans. on Robotics and Automation,* Vol. 19, No. 1, pp. 53-64, (2003).

[28] R. Volpe and P. Khosla, "Artificial Potential with Elliptical Isopotential Contours for Obstacle Avoidance," *Proc. of the IEEE Conf. on Decision and Control*, Los Angeles, California, pp. 180-185, 1987.

[29] M. Yamakita, T. Yazawa, X. -Z. Zheng, and K. Ito, "An Application of Passive Velocity Field Control to Cooperative Multiple 3-Wheeled Mobile Robots," *Proc. of the IEEE/RJS Int. Conf. on Intelligent Robots and Systems*, Victoria, B. C., Canada, pp. 368-373, 1998.

[30] M. Yamakita, K. Suzuki, X. -Z. Zheng, M. Katayama, and K. Ito, "An Extension of Passive Velocity Field Control to Cooperative Multiple Manipulator Systems," *Proc. of the IEEE/RJS Int. Conf. on Intelligent Robots and Systems*, Grenoble, France, pp. 11-16, 1997.