2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

The Third Generation Wind Structural Benchmark: A Nash Cumulant Robust Approach

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Abstract— **The use of cost-cumulant-based controllers to mitigate vibrations, caused in civil engineering structures by winds and seismic events, has led to performances [8]-[14], [16] which compare well with other control paradigms, when applied to civil engineering benchmarks. In this paper, we extend the cost-cumulant control concept to include a provision for explicit consideration of robustness considerations. Structured uncertainty is associated with one player, while control is associated with another player. Each of the players has a distinct cost, which has cumulants generated by a wind or earthquake process. Design is carried out by a Nash-type algorithm. The method is applied to the Third Generation Wind Structural Benchmark, and it seems to be a natural fit for the civil engineering applications which entail both environmental disturbances and modelling errors.**

I. Introduction

In recent years, the American Society of Civil Engineers has put into place a set of structural benchmark control problems. These include buildings of various heights and methods of construction, on the one hand, and bridges on the other hand. Both seismic and wind excitations have been considered; and various types of protective mechanisms have been incorporated.

Taken all together, this family of benchmark problems provides a rich selection of evaluation possibilities for control engineers interested in vibration reduction. The structures, as a group, may be classified as lightly damped. This means that controllers can add damping, in an efficient way, with devices of acceptable size and cost. Indeed this family is a classic instance of the utility of feedback control engineering, in which a relatively small action can produce a relatively large-and beneficial- effect.

Whether seismic events or wind events, the benchmark problems have natural stochastic processes associated with them. In some cases the events are actual records of earthquakes which have been observed in different parts of the world. It is also true that some local building codes even specify such actual records as events for which protection is to be provided. It therefore is most appropriate to employ control design methods which make use of stochastic measures of performance, but which provide a certain degree of flexibility to handle the wide range of RMS and peak response criteria of interest to civil engineers.

Moreover, for the multiple degrees of freedom involved in these structures, it is not easy to obtain accurate data on the stiffness and damping coefficients of the various modes. Even if they could be known in theory, it would be difficult to ensure their values in an actual construction. Accordingly, the family of benchmarks presents a natural application for robust control design methods. In a benchmark, for example, this might be addressed with the specifications of $\pm 15\%$ change in stiffness parameters.

In this paper we consider the Third Generation Wind Structural Benchmark. To incorporate the stochastic aspect, we employ the cost-cumulant control paradigms [8]-[14], [16] which have proven competitive with other methods in recent years. To incorporate the robust design aspect, we make use of multiobjective optimization. One player, the controller, has the first performance goal; and a second player, the structural model error, has the second performance goal. Both goals are expressed in terms of cost cumulants, and design solutions are obtained by Nash methods.

The combination of cost-cumulant methods with multiobjective methods seems most natural for the civil benchmarks under seismic or wind excitation and with genuine stiffness and damping uncertainties.

The results obtained in this paper support such a conclusion, by providing a robustness option with little or no reduction in performance, in fact sometimes an improvement in performance.

II. Wind Benchmark Problem Statement

The third generation benchmark problem for wind-excited buildings considers a 76-story concrete building proposed for Melbourne, Australia. This problem is discussed in [18]. The building model has been subjected to wind tunnel tests. The data from these tests have resulted in the wind forces for use in this benchmark problem. For control purposes there is an active tuned mass damper on the top floor of the building.

Due to the large computation tasks that are involved for a 77 DOF building, a reduced order model is used. The evaluation model will be given by

$$\dot{x} = Ax + Bu + Ew \tag{1}$$

where $x = [\bar{x}', \dot{\bar{x}}']'$. The quantity $\bar{x}$ is a column vector given as the displacements of the 3rd, 6th, 10th, 13th, 16th, 20th, 23rd, 26th, 30th, 33rd, 36th, 40th, 43rd, 46th, 50th, 53rd, 56th, 60th, 63rd, 66th, 70th, 73rd, and 76th floors, in that order, as well as x_m which is the displacement of the mass damper. The matrices A, B, and E are of the size 48×48, 48×1, and 48×77 respectively. Along with the evaluation

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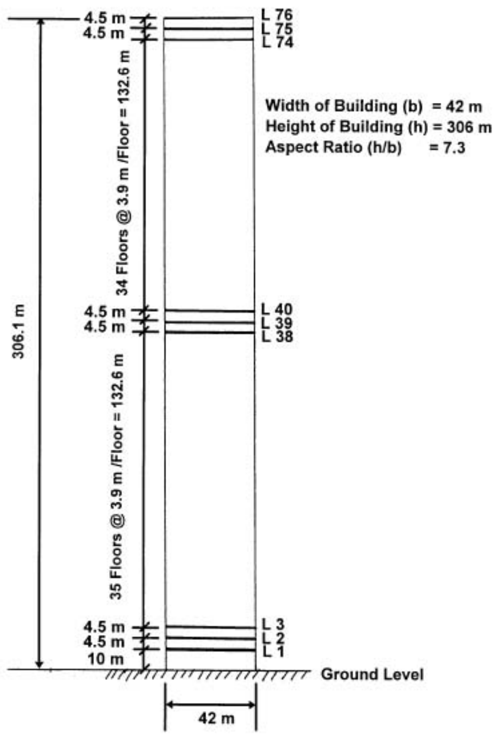

Fig. 1. Elevation View

model there are regulated output z and output y equations given by

$$\begin{aligned} z &= C_z x + D_z u + F_z w \\ y &= C_y x + D_y u + F_y w + v \end{aligned} \tag{2}$$

where $\tilde{x} = [x_1, x_{30}, x_{50}, x_{55}, x_{60}, x_{65}, x_{70}, x_{75}, x_{76}, x_m]'$, $z = [\tilde{x}', \dot{\tilde{x}}', \ddot{\tilde{x}}']'$, and $y = [\dot{\tilde{x}}', \ddot{\tilde{x}}']'$. The matrices C_z, D_z, F_z, C_y, D_y, and F_y are appropriately dimensioned. The elevation view of this building can be seen in Fig. 1.

To evaluate the performance of each control method there are twelve performance criteria. The criteria are based on the results of the simulation of the evaluation model with control and 900 sec of wind data. The first criterion measures the effect of the controller on the maximum floor acceleration. That is, the criterion is given by

$$J_1 = \frac{\max(\sigma_{\ddot{x}_1}, \sigma_{\ddot{x}_{30}}, \sigma_{\ddot{x}_{50}}, \sigma_{\ddot{x}_{55}}, \sigma_{\ddot{x}_{60}}, \sigma_{\ddot{x}_{65}}, \sigma_{\ddot{x}_{70}}, \sigma_{\ddot{x}_{75}})}{\sigma_{\ddot{x}_{75o}}}$$

where σ_{x_i} is the RMS acceleration of the ith floor and $\sigma_{\ddot{x}_{75o}} = 9.142\,\mathrm{cm/s}$ is the RMS uncontrolled acceleration of the 75th floor. The second performance criterion is given by

$$J_2 = \frac{1}{6}\sum_i \frac{\sigma_{\ddot{x}_i}}{\sigma_{\ddot{x}_{io}}}$$

for $i = 50, 55, 60, 65, 70, 75$ and where $\sigma_{\ddot{x}_{io}}$ is the uncontrolled RMS acceleration of the ith floor. These two performance criteria have not included the top floor, floor 76. In the next two performance criteria this floor is included. They are given by

$$\begin{aligned} J_3 &= \frac{\sigma_{x_{76}}}{\sigma_{x_{76o}}} \\ J_4 &= \frac{1}{7}\sum_i \frac{\sigma_{x_i}}{\sigma_{x_{io}}} \end{aligned}$$

for $i = 50, 55, 60, 65, 70, 75, 76$ and $\sigma_{x_{76o}} = 10.137$ cm, the uncontrolled displacement of the 76th floor. The previous performance criteria dealt with the performance of the building. While this is the main objective, one cannot focus on these without some constraints on the control and actuator. The actuator's physical constraints are that the RMS control force, σ_u, must not be greater than 100 kN and that the RMS actuator stroke, σ_{x_m}, must not be greater than 30 cm. While these constraints are physical constraints, there are also criteria designed to determine the control effort. These criteria are given by

$$\begin{aligned} J_5 &= \frac{\sigma_{x_m}}{\sigma_{x_{76o}}} \\ J_6 &= \left[\frac{1}{T}\int_0^T (\dot{x}_m(t)u(t))^2 dt\right]^{1/2} \end{aligned}$$

where T is the total time of integration.

With the RMS performance taken into account, we now give the performance criteria for the peak response. The first four criteria are given by

$$\begin{aligned} J_7 &= \frac{\max(\ddot{x}_{p1}, \ddot{x}_{p30}, \ddot{x}_{p50}, \ddot{x}_{p55}, \ddot{x}_{p60}, \ddot{x}_{p65}, \ddot{x}_{p70}, \ddot{x}_{p75})}{\ddot{x}_{p75o}} \\ J_8 &= \frac{1}{6}\sum_i \frac{\ddot{x}_{pi}}{\ddot{x}_{pio}} \\ J_9 &= \frac{x_{p76}}{x_{p76o}} \\ J_{10} &= \frac{1}{7}\sum_j \frac{x_{pj}}{x_{pjo}} \end{aligned}$$

for $i = 50, 55, 60, 65, 70, 75$, $j = i, 76$. Also x_{pi}, x_{pio} are the controlled and uncontrolled peak displacements of the ith floor respectively, and $\ddot{x}_{pi}, \ddot{x}_{pio}$ are respectively the controlled and uncontrolled peak accelerations of the ith floor. Similar to the RMS case, the actuator constraints are $\max_t |u(t)| \leq 300$ kN, $\max_t |x_m(t)| \leq 95$ cm. Furthermore the control effort is measured by

$$\begin{aligned} J_{11} &= \frac{x_{pm}}{x_{p76o}} \\ J_{12} &= \max_t |\dot{x}_m(t)u(t)| \end{aligned}$$

where x_{pm} is the peak actuator displacement.

One then wants to design a controller and to test that controller with the preceding performance criteria. In the design of the controller there are several constraints. One is that the designer may choose only 6 outputs for the design. Thus one must choose y_r from 6 elements in y, so that y_r is a vector of at most dimension 6. Furthermore the control compensator order must not exceed 12.

III. Controller Design Problem Statement

In this problem the control shall be developed for a class of nonlinear systems given by

$$dx(t) = f(t,x(t),u(t),w(t))dt + \sigma(t,x(t))d\xi(t) \quad (3)$$

where $x(t_0) = x_0$ is a random variable independent of ξ, ξ is d-dimensional Brownian motion on the probability space $(\Omega, \mathscr{F}, P)$, $x(t) \in \mathbb{R}^n$ is the state, $u(t) \in \mathscr{U}$ is the control, $w(t) \in \mathscr{W}$ is the disturbance, and $t \in T = [t_0, t_f]$. Let $Q_0 = (t_0, t_f) \times \mathbb{R}^n$ and $\bar{Q}_0$ be its closure, that is $\bar{Q}_0 = T \times \mathbb{R}^n$. Assume the functions f and σ are Borel measurable and are of class $C^1(\bar{Q}_0 \times \mathscr{U} \times \mathscr{W})$ and $C^1(\bar{Q}_0)$ respectively. Thus the functions f and σ have continuous partial derivatives of first order. It shall be assumed that the functions f and σ satisfy both a linear growth condition and a Lipschitz condition. Also it will be assumed that the strategies u, w will be of the form $u(t) = \mu(t,x(t))$ and $w(t) = \nu(t,x(t))$, so that they will be feedback strategies. Furthermore μ and ν shall also satisfy linear growth and Lipschitz conditions.

The game that will be used to solve this control problem will involve two cost functions given by

$$J_1(t,x,u,w) = \int_t^{t_f} L_1(\tau,x,u,w)d\tau + \psi_1(x_f) \quad (4)$$

$$J_2(t,x,u,w) = \int_t^{t_f} L_2(\tau,x,u,w)d\tau + \psi_2(x_f) \quad (5)$$

with L_1, L_2 running costs and ψ_1, ψ_2 terminal costs that both satisfy polynomial growth conditions. These cost functions are used to give the performance indices of both players in the forms

$$\phi_1(t,x,u,w) = E\{J_1(t,x,u,w)\} + \gamma Var\{J_1(t,x,u,w)\} \quad (6)$$

where ϕ_1 is for the control u and

$$\phi_2(t,x,u,w) = E\{J_2(t,x,u,w)\} \quad (7)$$

with ϕ_2 for the disturbance w. In the performance index for the control there is a parameter γ. This is the parameter that determines how much the control should weigh the value of the variance of its cost. With this preliminary discussion on the game in place, there should be a definition of what exact game is used. In this case it will be a Nash game. Since both players are assumed to be using feedback strategies, we shall let the information patterns be the class of all feedback strategies, $\mathscr{U}_F$ for the control and $\mathscr{W}_F$ for the disturbance.

Definition 1: The pair (μ^*, ν^*) is a Nash equilibrium solution if it satisfies the inequalities

$$\phi_1(0,x,\mu^*,\nu^*) \le \phi_1(0,x,\mu,\nu^*)$$

$$\phi_2(0,x,\mu^*,\nu^*) \le \phi_2(0,x,\mu^*,\nu)$$

$\forall \mu \in \mathscr{U}_F$ and $\forall \nu \in \mathscr{W}_F$.

Now let $V_1(t,x;\mu,\nu) = E_{tx}\{J_1(t,x,u,w)\}$ and $V_2(t,x;\mu,\nu) = E_{tx}\{J_1^2(t,x,u,w)\}$ be the first and second moments of the cost function $J_1(t,x,u,w)$.

Definition 2: A function $M : \bar{Q}_0 \to \mathbb{R}^+$ is an admissible mean cost function if there exists an admissible strategy μ such that $M(t,x) = V_1(t,x;\mu,\nu^*)$ for $t \in T, x \in \mathbb{R}^n$.

From now on we shall assume that M is an admissible mean cost function.

Definition 3: M defines a class of admissible strategies $\mathscr{U}_M$ such that $\mu \in \mathscr{U}_M$ if and only if the strategy μ is admissible and satisfies Definition 2.

Definition 4: An MCV control strategy $\mu^* \in \mathscr{U}_M$ is one that minimizes the second moment, i.e. $V_2(t,x,\mu^*,\nu^*) = V_2(t,x) \le V_2(t,x,\mu,\nu^*)$ for $t \in T, x \in \mathbb{R}^n, \nu^* \in \mathscr{W}_F$, where $\mu \in \mathscr{U}_M$. Furthermore the variance is found through $V(t,x) = V_2(t,x) - M^2(t,x)$.

To conclude this preliminary discussion, we define the backward evolution operator to be

$$\begin{aligned}\mathscr{O}^{\mu,\nu} =& \frac{\partial}{\partial t} + f'(t,x,u,w)\frac{\partial}{\partial x} \\ &+ \frac{1}{2}tr\left(\sigma(t,x)W(t)\sigma'(t,x)\frac{\partial^2}{\partial x^2}\right)\end{aligned} \quad (8)$$

where $E\{d\xi(t)d\xi'(t)\} = W(t)$, superscript $'$ denotes transpose, and tr refers to the trace operator.

IV. Control Algorithm Development

A. Nonlinear System, Non-quadratic Costs Development

We begin this section by giving several lemmas that characterize the control's Nash equilibrium strategy. The first lemma will help by providing a necessary condition for the mean of the cost function.

Lemma 1: Let $M \in C_p^{1,2}(\bar{Q}_0)$ be an admissible mean cost function and μ be an admissible control strategy such that it satisfies Definition 2. Under these assumptions the admissible mean cost function M satisfies

$$\mathscr{O}^{\mu,\nu^*}M(t,x) + L_1(t,x,\mu,\nu^*) = 0 \quad (9)$$

where $M(t_f,x_f) = \psi_1(x_f)$.

Next we have the following Verification Lemma for the mean of the cost function. It provides sufficient conditions for the mean value function. Here the set Q is to be an open subset of Q_0.

Lemma 2 (Verification Lemma): Let $M \in C_p^{1,2}(Q) \cap C(\bar{Q})$ be a solution to

$$\mathscr{O}^{\mu,\nu^*}M(t,x) + L_1(t,x,\mu,\nu^*) = 0 \quad (10)$$

with boundary condition $M(t_f,x_f) = \psi_1(x_f)$. Then $M(t,x) = V_1(t,x;\mu,\nu^*)$ for all $\mu \in \mathscr{U}_M$.

Now that we have the results for the mean of the cost, we have the following Verification Lemma for the second moment of the cost.

Lemma 3 (Verification Lemma): Let $V_2 \in C_p^{1,2}(Q) \cap C(\bar{Q})$ be a nonnegative solution to the following partial differential equation

$$\min_{\mu \in \mathscr{U}_M}\left\{\mathscr{O}^{\mu,\nu^*}V_2(t,x) + 2M(t,x)L_1(t,x,\mu,\nu^*)\right\} = 0 \quad (11)$$

with boundary condition $V_2(t_f,x_f) = \psi_1^2(x_f)$. Then $V_2(t,x) \leq V_2(t,x;\mu,\nu^*)$ for every $\mu \in \mathscr{U}_M$, and $(t,x) \in \bar{Q}_0$. If μ also satisfies

$$\begin{aligned}\min_{\tilde{\mu}\in\mathscr{U}_M} &\left\{ \mathscr{O}^{\tilde{\mu},\nu^*} V_2(t,x) + 2M(t,x)L_1(t,x,\tilde{\mu},\nu^*) \right\} \\ &= \mathscr{O}^{\mu,\nu^*} V_2(t,x) + 2M(t,x)L_1(t,x,\mu,\nu^*)\end{aligned} \tag{12}$$

for all $(t,x) \in \bar{Q}_0$, then $V_2(t,x) = V_2(t,x;\mu,\nu^*)$.

With the aid of these lemmas, we can begin the discussion for the Nash equilibrium solution. The following theorem provides sufficient conditions for the Nash equilibrium solution.

Theorem 1: Consider the two player game described by (3), (6), and (7). Let M be an admissible mean cost function, $M \in C_p^{1,2}(Q) \cap C(\bar{Q})$, with an associated $\mathscr{U}_M$. Also consider the function $V \in C_p^{1,2}(Q) \cap C(\bar{Q})$ that is a solution to

$$\begin{aligned}\min_{\mu\in\mathscr{U}_M} &\left\{ \frac{\partial V}{\partial t}(t,x) + f'(t,x,\mu,\nu^*)\frac{\partial V}{\partial x}(t,x) \right. \\ &+ \frac{1}{2}tr\left(\sigma(t,x)W(t)\sigma'(t,x)\frac{\partial^2 V}{\partial x^2}(t,x)\right) \\ &\left. + \left|\frac{\partial M}{\partial x}(t,x)\right|^2_{\sigma(t,x)W(t)\sigma'(t,x)} \right\} = 0\end{aligned} \tag{13}$$

with $V(t_f,x_f) = 0$ and the function $P \in C_p^{1,2}(Q) \cap C(\bar{Q})$ that satisfies

$$\begin{aligned}\min_{\nu\in\mathscr{W}_F} &\left\{ \frac{\partial P}{\partial t}(t,x) + f'(t,x,\mu^*,\nu)\frac{\partial P}{\partial x}(t,x) \right. \\ &+ \frac{1}{2}tr\left(\sigma(t,x)W(t)\sigma'(t,x)\frac{\partial^2 P}{\partial x^2}(t,x)\right) \\ &\left. + L_2(t,x,\mu^*,\nu) \right\} = 0\end{aligned} \tag{14}$$

with $P(t_f,x_f) = \psi_2(x_f)$. If μ^* and ν^* are the minimizing arguments of (13) and (14), then the pair (μ^*,ν^*) constitutes a Nash equilibrium solution.

B. Application to Linear Quadratic Systems

We consider next the case in which the system given is linear and is described by

$$\begin{aligned}dx(t) &= (A(t)x(t) + B(t)u(t) + D(t)w(t))dt + E(t)d\xi(t) \\ z_1(t) &= H_1(t)x(t) + G_1(t)u(t) \\ z_2(t) &= H_2(t)x(t) + G_2(t)u(t)\end{aligned}$$

where $x(t_0) = x_0$ and z_1, z_2 are the regulated outputs of the system. It also will be assumed that $H_i'H_i = Q_i$, $G_i'H_i = 0$, and $G_i'G_i = R_i$ for $i = 1,2$, where Q_i is positive semidefinite and R_i is positive definite. Furthermore the costs are assumed to be quadratic, and are given by

$$\begin{aligned}J_1 &= \int_{t_0}^{t_f} z_1'(t)z_1(t)dt \\ J_2 &= \int_{t_0}^{t_f} (\delta^2 w'(t)w(t) - z_2'(t)z_2(t))dt\end{aligned}$$

where $Q_f^1 = Q_f^2 = 0$.

Notice that minimizing the performance index of the disturbance will then be imposing a constraint on the input-output properties of the relation from the disturbance w to the regulated output z_2. To see this, consider that for a performance index $E\{J_2\} \geq 0$ we have

$$E\left\{\int_{t_0}^{t_f} (\delta^2 w'(t)w(t) - z_2'(t)z_2(t))dt\right\} \geq 0;$$

but this is the same as

$$\begin{aligned}E\left\{\int_{t_0}^{t_f} ||z_2(t)||^2 dt\right\} &\leq \delta^2 E\left\{\int_{t_0}^{t_f} ||w(t)||^2 dt\right\} \\ \int_{t_0}^{t_f} E\left\{||z_2(t)||^2\right\} dt &\leq \delta^2 \int_{t_0}^{t_f} E\left\{||w(t)||^2\right\} dt \\ \|z_2\|^2_{[t_0,t_f]} &\leq \delta^2 \|w\|^2_{[t_0,t_f]}\end{aligned}$$

where $||\cdot||_{[t_0,t_f]}$ is the 2-norm as defined in [4]. This inequality is a constraint on the "input-output" properties of the system, in fact it is a constraint on the induced norm of the system $\|G_{z_2w}\|_{\infty,[t_0,t_f]} \leq \delta$.

Let us place a notation for the quadratic costs:

$$\begin{aligned}M(t,x) &= x'\mathscr{M}(t)x + m(t) \\ V(t,x) &= x'\mathscr{V}(t)x + v(t) \\ P(t,x) &= x'\mathscr{P}(t)x + p(t)\end{aligned}$$

where $\mathscr{M}, \mathscr{V}, \mathscr{P}$ are matrix functions of time and m, v, p are scalar functions of time. We can state the following theorem.

Theorem 2: Consider the stochastic game in which the system is linear and the costs are quadratic. Suppose that $\mathscr{M}(t), \mathscr{V}(t), \mathscr{P}(t)$ are unique solutions to the coupled Riccati equations

$$\begin{aligned}&\dot{\mathscr{M}} + A'\mathscr{M} + \mathscr{M}A + Q_1 - \mathscr{M}BR_1^{-1}B'\mathscr{M} \\ &- \frac{1}{\delta^2}\mathscr{P}DD'\mathscr{M} - \frac{1}{\delta^2}\mathscr{M}DD'\mathscr{P} \\ &+ \gamma^2\mathscr{V}BR_1^{-1}B'\mathscr{V} = 0\end{aligned} \tag{15}$$

where $\mathscr{M}(t_f) = Q_f^1$,

$$\begin{aligned}&\dot{\mathscr{V}} + A'\mathscr{V} + \mathscr{V}A - \gamma\mathscr{M}BR_1^{-1}B'\mathscr{V} - \gamma\mathscr{V}BR_1^{-1}B'\mathscr{M} \\ &- \frac{1}{\delta^2}\mathscr{P}DD'\mathscr{V} - \frac{1}{\delta^2}\mathscr{V}DD'\mathscr{P} - 2\gamma\mathscr{V}BR_1^{-1}B'\mathscr{V} \\ &+ 4\mathscr{M}EWE'\mathscr{M} = 0\end{aligned} \tag{16}$$

with $\mathscr{V}(t_f) = 0$, and

$$\begin{aligned}&\dot{\mathscr{P}} + A'\mathscr{P} + \mathscr{P}A - (\mathscr{M} + \gamma\mathscr{V})BR_1^{-1}B'\mathscr{P} \\ &- \mathscr{P}BR_1^{-1}B'(\mathscr{M} + \gamma\mathscr{V}) - \frac{1}{\delta^2}\mathscr{P}DD'\mathscr{P} \\ &- Q_2 - \mathscr{M}BR_1^{-1}R_2R_1^{-1}B'\mathscr{M} \\ &- \gamma\mathscr{M}BR_1^{-1}R_2R_1^{-1}B'\mathscr{V} - \gamma\mathscr{V}BR_1^{-1}R_2R_1^{-1}B'\mathscr{M} \\ &- \gamma^2\mathscr{V}BR_1^{-1}R_2R_1^{-1}B'\mathscr{V} = 0\end{aligned} \tag{17}$$

with $\mathscr{P}(t_f) = Q_f^2$. Then the Nash equilibrium solution $(\mu^*(t,x), \nu^*(t,x))$ is given by

$$\begin{aligned}\mu^*(t,x(t)) &= -R_1^{-1}(t)B'(t)[\mathscr{M}(t) + \gamma\mathscr{V}(t)]x(t)\\ \nu^*(t,x(t)) &= -\frac{1}{\delta^2}(t)D'(t)\mathscr{P}(t)x(t).\end{aligned} \quad (18)$$

$M(t,x)$, $V(t,x)$, and $P(t,x)$ can be computed with the aid of

$$\begin{aligned}\dot{m}(t) &= -tr(E(t)W(t)E'(t)\mathscr{M}(t))\\ \dot{v}(t) &= -tr(E(t)W(t)E'(t)\mathscr{V}(t))\\ \dot{p}(t) &= -tr(E(t)W(t)E'(t)\mathscr{P}(t))\end{aligned}$$

where $m(t_f) = 0, v(t_f) = 0, p(t_f) = 0$.

V. Wind Benchmark Results

With the control algorithm now in place, this method of design is applied to the third generation benchmark for wind-excited structures. Following [18], a 12 state reduced ordered model is used for the control design. This design model is

$$\begin{aligned}\dot{x}_r &= A_r x_r + B_r u + E_r\xi\\ y_r &= C_{yr}x_r + D_{yr}u + F_{yr}\xi\\ z_r &= C_{zr}x_r + D_{zr}u + F_{zr}\xi + v_r\end{aligned} \quad (19)$$

where $x_r = [x_{16}, x_{30}, x_{46}, x_{60}, x_{76}, x_m, \dot{x}_{16}, \dot{x}_{30}, \dot{x}_{46}, \dot{x}_{60}, \dot{x}_{76}, \dot{x}_m]'$, z_r is the same as the z for the evaluation model, and $y_r = [\ddot{x}_{50}, \ddot{x}_{76}, \ddot{x}_m]'$. Also the disturbance ξ is the wind excitation and v_r is sensor noise, and furthermore the two are uncorrelated. From the baseline LQG controller designed in [18], a cost function

$$J = \int_0^{t_f} ((C_{yr}x_r + D_{yr}u)'Q(C_{yr}x_r + D_{yr}u) + u'Ru)dt \quad (20)$$

will be used for J_1. From (20) we have

$$z_1 = \begin{bmatrix} HC_{yr} \\ 0 \end{bmatrix} x_r + \begin{bmatrix} HD_{yr} \\ R \end{bmatrix} u$$

as the regulated output for the control, where $H'H = Q$.

To help account for some uncertainty, we will add the disturbance w as shown in Fig. 2. The weighting function is given by $W_{z_2} = 2.14 \times 10^{-4} I$. The design model will now be given as

$$\dot{x}_r = A_r x_r + B_r u + D_r w + E_r\xi$$

where D_r is a 12×12 matrix with the first six columns equal to that of A_r, while the last six columns are zero.

The multiobjective control methodology presented in this paper was then simulated using the benchmark problem. To help assess this control paradigm, it was compared with two other control designs. The first control design was the baseline LQG. The second was the 2CC or MCV discussed in [8]-[13], [14]. For the MCV design, the parameter γ was set to be 8×10^{-8}. The simulated results are displayed in Table I.

First consider the RMS performance criteria J_1-J_6 of the wind benchmark problem. Notice that for the MCV and multiobjective controllers the criteria that measure the performance of the building, J_1-J_4, there is substatial improvement from the LQG case. For the MCV case, there is 11.7%, 11.0%, 4.3%, and 4.3% reduction in these cases, and a reduction of 2.2%, 2.1%, 1.0%, and 1.0% for the multibjective case. In particular, for J_1 and J_2 there is improvement. There are also two criteria that deal with control effort, J_5 and J_6. It would be expected that with the decrease in J_1-J_4, the results for J_5 and J_6 would be larger. While this is the case, it can be seen from the σ_u and σ_{x_m} results that the control effort is still within its bounds.

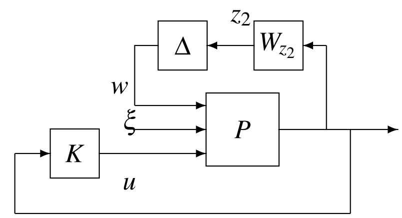

Fig. 2. Block Diagram with Uncertainty

We shall now examine the wind benchmark's peak performance criteria, J_7-J_{12}. Similarly to the RMS criteria, the MCV and multiobjective control methods show improvement in the building performance criteria, J_7-J_{10}. The results show a decrease of 9.2%, 3.0%, 2..9%, and 2.8% respectively for J_7-J_{10}, in the MCV case. For the multiobjective case, we have a decrease of 4.7%, 2.5%, 1.7%, and 1.7% for J_7-J_{10} respectively. Also we see that the results for the control effort for these two control paradigms are larger than the J_{11} and J_{12} of the LQG case. As before, even though there is more control effort being used, it does not exceed the bounds set in the wind benchmark problem.

We have compared the MCV and multiobjective control results with the LQG results, but what about the differences between the results from the MCV and multiobjective

	LQG	MCV	MCC
J_1	0.369	0.326	0.361
J_2	0.417	0.371	0.408
J_3	0.578	0.553	0.572
J_4	0.580	0.555	0.574
J_5	2.271	2.720	2.310
J_6	11.99	19.96	12.82
J_7	0.381	0.346	0.363
J_8	0.432	0.419	0.421
J_9	0.717	0.696	0.705
J_{10}	0.725	0.705	0.713
J_{11}	2.299	2.756	2.279
J_{12}	71.87	122.3	77.62
σ_{x_m}	23.03	27.57	23.42
σ_u	34.07	50.26	38.41
$\max_t \lvert x_m \rvert$	74.27	89.01	73.61
$\max_t \lvert u \rvert$	118.2	194.1	143.9

TABLE I

Benchmark Results ($\Delta K = 0$)

control methods? It should be noted that the multiobjective method presented in this paper is an extension of the MCV control paradigm. It can be seen that the MCV control on the wind benchmark problem performs better than the multiobjective method. This can be seen as a result of the multiobjective method being more robust. Since it is more robust in its design, some performance will suffer.

This added robustness can be seen in Table II. This table shows what happens when the stiffness matrix is changed by -15% and 15%. While both the MCV and MCC control methods perform well for the case of $\Delta K = 15\%$, the case of -15% is not so. The MCV control results show that it performs well, but in doing so greatly exceeds the actuator constraint. Recall that the peak actuator stroke must be within 95 cm. In the MCV case it is not so. The multiobjective control methadology however shows a 5.4%, 5.3%, 2.8%, and 2.7% improvement over LQG for J_1-J_4 and similarly a 6.8% and 3.5% improvement for J_7 and J_8. Actually in this case we can see that the MCC control method performs better than it did for $\Delta K = 0$. Moreover, the MCC results show that it also satisfies the actuator constraints.

VI. Conclusion

In this paper, a multiobjective cumulant method of control has been applied to the third generation benchmark for wind-excited buildings. The benchmark problem has been reviewed. The control methodology has also been examined. An algorithm for the control has been given for the case when the control minimizes the first two cumulants of its cost and the disturbance minimizes the mean of its cost. With the control methodology in place, a controller was designed for the benchmark problem. This controller has then been simulated and its results compared with other known control methodologies. Robustness with respect to the building's stiffness parameters is also examined for a change of $\pm 15\% K$. The multiobjective cumulant control demonstrates that it performs favorably with respect to several other control paradigms.

	$\Delta K = -15\%$			$\Delta K = 15\%$		
	LQG	MCV	MCC	LQG	MCV	MCC
J_1	0.387	0.339	0.366	0.365	0.332	0.359
J_2	0.438	0.387	0.415	0.409	0.376	0.406
J_3	0.711	0.679	0.691	0.487	0.472	0.485
J_4	0.712	0.681	0.693	0.489	0.474	0.487
J_5	2.709	3.299	2.714	1.812	2.252	1.899
J_6	16.61	27.26	17.08	8.463	15.44	9.949
J_7	0.488	0.425	0.455	0.411	0.355	0.398
J_8	0.539	0.499	0.520	0.443	0.434	0.443
J_9	0.770	0.724	0.785	0.607	0.625	0.614
J_{10}	0.779	0.733	0.795	0.614	0.633	0.622
J_{11}	2.836	3.326	2.938	1.852	2.254	1.894
J_{12}	118.3	199.3	129.6	52.68	102.4	66.30
σ_{x_m}	27.46,	33.44	27.52	18.37	22.83	19.25
σ_u	44.32	,64.27	48.50	28.29	43.63	33.35
$\max_t \lvert x_m \rvert$	91.60	107.4	94.89	59.83	72.81	61.17
$\max_t \lvert u \rvert$	164.3	235.3	183.1	105.6	174.4	133.4

TABLE II

Benchmark Results

References

[1] G. J. Balas, "Synthesis of Controllers for the Active Mass Driver System in the Presence of Uncertainty", *Earthquake Engineering and Structural Dynamics*, vol. 27, pp. 1189-1202, 1998.

[2] T. Basar, G. J. Olsder, *Dynamic Noncooperative Game Theory, 2ed.*, SIAM, Philadelphia, 1999.

[3] T. Basar, P. Bernhard, *H^∞-Optimal Control and Related Minimax Design Problems: A Dynamic Game Approach, 2ed.*, Birkhauser, Boston, 1995.

[4] X. Chen, K. Zhou, "Multiobjective H_2/H_∞ Control Design," *SIAM Journal of Control and Optimization*, vol. 40, no. 2, pp. 628-660, 2001.

[5] W. H. Fleming, R. W. Rishel, *Deterministic and Stochastic Optimal Control*, Springer-Verlag, New York, 1975.

[6] E. A. Johnson, P. G. Voulgaris, L. A. Bergman, "Multiobjective Optimal Structural Control of the Notre Dame Building Model Benchmark", *Earthquake Engineering and Structural Dynamics*, vol. 27, pp. 1165-1187, 1998.

[7] D. J. N. Limebeer, B. D. O. Anderson, D. Hendel, "A Nash Game Approach to Mixed H_2/H_∞ Control", *IEEE Transactions on Automatic Control*, vol. 39, no. 1, pp. 69-82, Jan. 1994.

[8] K. D. Pham, M. K. Sain, and S. R. Liberty, "Infinite Horizon Robustly Stable Seismic Protection of Cable-Stayed Bridges Using Cost Cumulants, " *Proceedings of American Control Conference,* pp. 691-696, Boston, Massachusetts, June 30, 2004.

[9] K. D. Pham, G. Jin, M. K. Sain, B. F. Spencer, Jr., and S. R. Liberty, "Generalized LQG Techniques for the Wind Benchmark Problem", *Special Issue of ASCE Journal of Engineering Mechanics on the Structural Control Benchmark Problem*, Vol. 130, No. 4, April 2004.

[10] K. D. Pham, M. K. Sain, and S. R. Liberty, "Cost Cumulant Control: State-Feedback, Finite-Horizon Paradigm with Application to Seismic Protection", Special Issue of *Journal of Optimization Theory and Applications*, Edited by A. Miele, Kluwer Academic/Plenum Publishers, New York, Vol. 115, No. 3, pp. 685-710, December 2002.

[11] K. D. Pham, M. K. Sain, and S. R. Liberty, "Finite Horizon Full-State Feedback kCC Control in Civil Structures Protection", *Stochastic Theory and Adaptive Control, Lecture Notes in Control and Information Sciences*, Proceedings of a Workshop held in Lawrence, Kansas, Edited by B. Pasik-Duncan, Springer-Verlag, Berlin Heidelberg, Germany, Vol. 280, pp. 369-383, September 2002.

[12] K. D. Pham, M. K. Sain, and S. R. Liberty, "Robust Cost-Cumulants Based Algorithm for Second and Third Generation Structural Control Benchmarks", *Proceedings American Control Conference* , pp. 3070-3075, Anchorage, Alaska, May 08-10, 2002.

[13] K. D. Pham, "Statistical Control Paradigms for Structural Vibration Suppression", *Ph. D. Dissertation,* Department of Electrical Engineering, University of Notre Dame, May 2004.

[14] M. K. Sain, C. H. Won, B. F. Spencer Jr., S. R. Liberty, "Cumulants and Risk Sensitive Control: A Cost Mean and Variance Theory with Applications to Seismic Protection of Structures", *Advances in Dynamic Games and Applications, Annals of the International Society of Dynamic Games*, vol. 5, J. A. Filor, V. Gaisgory, K. Mizukami (Eds), Birkhauser, Boston, 2000.

[15] B. F. Spencer Jr., S. J. Dyke, H. S. Deoskar, "Benchmark Problems in Structural Control - Part I: Active Mass Driver System", *Earthquake Engineering and Structural Dynamics*, vol. 27, pp. 1127-1139, 1998.

[16] C. H. Won, "Cost Cumulants in Risk Sensitive and Minimal Cost Variance Control", Ph. D. Dissertation, University of Notre Dame, Notre Dame, IN, July 1994.

[17] J. N. Yang, S. Lin, F. Jabban, "Linear Multiobjective Control Strategies for Wind-Excited Buildings", *Journal of Engineering Mechanics*, 130(14), pp. 471-477, 2004.

[18] J. N. Yang, A. K. Agrawal, B. Samali, J. C. Wu, "Benchmark Problem for Response Control of Wind-Excited Tall Buildings," *Journal of Engineering Mechanics*, 130(14), pp. 437-446, 2004.