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State-Periodic Adaptive Compensation of Cogging and Coulomb Friction in Permanent Magnet Linear Motors

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***Abstract*— This paper focuses on the state-periodic adaptive compensation of cogging and Coulomb friction for permanent magnet linear motors (PMLM) executing a task repeatedly. The cogging force is considered as a position dependent disturbance and the considered Coulomb friction is non-Lipschitz at zero velocity. The key idea of our disturbance compensation method is to use one trajectory-period past information along the state axis to update the current adaptation law. The new method consists of three different steps: Firstly, in the first repetitive trajectory, an adaptive compensator is designed to guarantee the ℓ_2-stability of the overall system; secondly, from the second repetitive trajectory and onwards, a trajectory-periodic adaptive compensator is designed to stabilize the system; and finally, to make use of the stored past state-dependent cogging information, a search process is utilized for adapting the current cogging coefficient. The validity of our adaptive cogging and friction compensator is illustrated through a simulation example.**

***Index Terms*— Cogging force, Coulomb friction force, state-dependent disturbance, adaptive control, trajectory-periodic adaptation.**

I. Introduction

Permanent magnet (PM) motors are the most popularly used electromechanical devices for accurate speed and position control of the linear system or rotary system. In parallel with the popularity of PM motors, the nonlinear torques inherent to PM motors have been addressed in numerous literatures [1], [2], [3]. In particular, in [4], load torques, friction effects, and cogging torques are addressed as inherent torques of the permanent magnet stepper motors; and in [5], [6], friction, cogging and reluctance forces are modelled for iron-core permanent magnet linear motors. As explained in [7], the cogging forces are due to the interaction between the permanent magnets and the steel teeth of the primary section; and the friction force is a velocity-dependent nonlinear disturbance, which is inherent to most of the electromechanical systems.

In permanent-magnet linear motors (PMLMs), nonlinear mechanical disturbances such as back-lash are greatly reduced; while the cogging forces are considered as the main disturbance [3], [5]. However, static friction force such as Coulomb friction is still a dominant basic disturbance and should be compensated for accurate speed and position control of PMLMs. Thus, in this paper, we focus on the cogging force and the Coulomb friction. These disturbances are compensated by the trajectory-periodic adaptation based on Lyapunov stability analysis on the time-axis.

Cogging forces are position dependent periodic disturbances due to the slotted nature of the primary core [3], [6], and generally it is modelled as Fourier expansion [1], [2]. However, in control strategies, it has been modelled as a simple sinusoidal signal such as:

$$F_{cogging} = A\sin(\omega x + \varphi) \tag{1}$$

and the unknown parameters such as A, ω and φ have been compensated by certain parameter adaptation scheme [4], [5], [8]. However, this approach does not represent high order terms in the Fourier series, hence it cannot compensate the cogging force completely. In this paper, we do not assume any model such as (1); instead, it is considered that the cogging force could be any kind of Fourier expansion such as:

$$F_{cogging} = \sum_{i=1}^{\infty} A_i \sin(\omega_i x + \varphi_i) \tag{2}$$

In order to compensate cogging force of (2), it is suggested to make use of the periodicity of cogging disturbance on the repetitive trajectory. Note that cogging force waveform is periodic over a pole-pitch in PMLMs [2].

In control community, Coulomb friction force has been studied widely in [9], [10]; and many efforts have been devoted in late $80's$ and early $90's$ to compensating friction force [11], [12], [13], [14], [15]. After these early works, several adaptive friction compensation controllers have been suggested [16], [17], [18], [19]. We can see that the friction compensation is still considered as a hot topic/

The paper is organized as follows: In Section II, a new adaptive state-dependent cogging and friction compensator is designed based on Lyapunov stability analysis. Simulation tests are performed in Section III. Conclusions are given in Section IV.

II. State-Dependent Adaptive Cogging Compensation

In this section, the state-dependent periodic adaptive cogging and friction compensator is designed. The cogging force of (2) can be written as: $-a(x)$, where $a(x)$ is the function of x. Coulomb friction is modelled as:

$$F_{fric} = -b\text{sgn}(v), \tag{3}$$

where friction is discontinuous at zero velocity. In this paper, to present our ideas clearly, without loss of generality, the following simplified servo control problem is considered:

$$\dot{x}(t) = v(t) \tag{4}$$
$$\dot{v}(t) = -a(x) - b\text{sgn}(v) + u, \tag{5}$$

where x is the position; $a(x)$ is the unknown position-dependent cogging disturbance; b is the friction coefficient; v is the velocity; and u is the control input.

First, before proceeding our main results, the following definitions and assumptions are necessary.

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Definition 2.1: The total passed trajectory is given as:

$$s = \int_0^t \frac{|\mathrm{d}x|}{\mathrm{d}\tau}\mathrm{d}\tau = \int_0^t |v(\tau)| d\tau,$$

where x is the position, and v is the velocity. In [20], it was defined as the curvilinear abscissa associated to the trajectory of the relative motion. In our definition, since s is the summation of absolute position increase along the time axis, s is a monotonous growing signal. Physically, it is the total passed trajectory, hence it has the following property:

$$s(t_1) \geq s(t_2), \text{ iff } \; t_1 \geq t_2.$$

With notation s, the position corresponding to $s(t)$ is denoted as $x(s)$ and the cogging force corresponding to $s(t)$ is denoted as $a(s)$.

Definition 2.2: Since cogging force arises as a result of the mutual attraction between the magnets and cores of the translator, cogging force is periodic with respect to position [5]. So, based on Definition 2.1, the following relationship is derived:

$$a(s) = a(s - s_p), \text{ and } x(s) = x(s - s_p), \tag{6}$$

where s_p is the periodicity of the trajectory.

Definition 2.3: In Assumption 2.2, s_p was defined as periodic trajectory. So, $x(t) - s_p$ is one trajectory past point from $x(t)$ on the s axis. Let us denote the time corresponding to $x(t) - s_p$ with T_t. Then, $t - T_t$ is the time-elapse to complete one periodic trajectory from the time T_t to time t. This time-elapse is called "cycle". Particularly, it is called "trajectory cycle" at time t and denoted as P_t. So, $P_t = t - T_t$. It is called "the search process" to find P_t at time instant t (note: the search process can be performed by interpolation).

Furthermore, time is always monotonically increasing signal, and in controller, the discrete time points are used. So, the monotonically increasing time signal is denoted as: $t_i, i = 0, \cdots, \infty$, where t_0 is the initial time when the motor starts to move. Then, the following relationship is immediate:

$$s(t_{i+1}) \geq s(t_i).$$

From now on, for accurate notation, the position corresponding to time t_i is denoted as: $x(t_i)$ and its total passed trajectory by the time t_i is denoted as: $s(t_i)$. Henceforward, one trajectory past time from the time instant t_i is denoted as T_{t_i}, and its corresponding cycle is denoted as P_{t_i} (i.e, $P_{t_i} = t_i - T_{t_i}$).

Assumption 2.1: Throughout the paper, it is assumed that the current position and current time of PMLMs are measured. Let us denote the current position as $x(t_i)$ at time t_i, where x is the position corresponding to t_i. Then, T_{t_i} is always calculated, hence P_{t_i} is calculated at time instant t_i.

With the above definitions and assumption, the following property is observed.

Property 2.1: The following relationship is derived:

$$x(t_i) = s(t_i) - m s_p, \tag{7}$$

where m is the integer part of the quotient of $s(t_i)/s_p$.

Remark 2.1: As will be shown in the following theorem, the actual state-dependent cogging force $a(s(t_i))$ is not estimated on the state axis. In our adaptation law, $a(t_i)$ is estimated on the time axis. So, to find $a(s(t_i) - s_p)$, the following formula is used:

$$a(s(t_i) - s_p) = a(t_i - P_{t_i}) \tag{8}$$

Here, P_{t_i} is calculated in Assumption 2.1 (recall that P_{t_i} can be used to indicate exactly one-trajectory past position).

From (7) and (8), we also have the following property:

Property 2.2: The current cogging and friction forces are equal to one-trajectory past cogging and friction forces. From the relationship:

$$\begin{aligned} a(s(t_i) - s_p) &= a(x(t_i) + m s_p - s_p) \\ &= a(x(t_i)) \\ &= a(t_i - P_{t_i}) \end{aligned} \tag{9}$$

the following equality is derived: $a(x(t_i)) = a(t_i - P_{t_i})$.

Now, based on the above discussions, the following stability analysis is performed. Our compensation approach is summarized as follows:

- When $s(t_i) < s_p$, the system is controlled to be bounded input bounded output (in ℓ_2-norm).
- When $s(t_i) \geq s_p$, the system is stabilized to follow the desired speed at the desired position. By trajectory periodic adaptation, the unknown external disturbances (the summation of the cogging and friction forces) are estimated.

The following notations are used:

$$e_x(t_i) = x(t_i) - \hat{x}(t_i); \; e_v = v(t_i) - v_d(t_i);$$

$$e_a(s(t_i)) = a(s(t_i)) - \hat{a}(s(t_i)); e_b(t_i) = b - \hat{b}(t_i),$$

where $\hat{a}(s(t_i)) = \hat{a}(t_i)$ (note: t_i is the current time corresponding to the current total passed trajectory $s(t_i)$). Here, let us change $e_a(s(t_i)) = a(s(t_i)) - \hat{a}(s(t_i))$ into time domain such as:

$$\begin{aligned} e_a(s(t_i)) &= a(s(t_i)) - \hat{a}(s(t_i)) \\ &= a(t_i) - \hat{a}(t_i) \\ &= e_a(t_i). \end{aligned} \tag{10}$$

In the same way, the following relationships are true:

$$e_b(s(t_i)) = e_b(t_i); \; x(s(t_i)) = x(t_i); \; x_d(s(t_i)) = x_d(t_i);$$

$$v_d(s(t_i)) = v(s(t_i)); \; v(s(t_i)) = v(t_i);$$

The control objective is to track or servo the given desired position $x_d(t_i)$ and the corresponding desired velocity $v_d(t_i)$ with tracking error as small as possible. In practice, it is reasonable to assume that $x_d(t_i)$, $v_d(t_i)$ and $\dot{v}_d(t_i)$ are all bounded. From now on, based on relationship: $a(x(t_i)) = a(t_i - P_{t_i}) = a(t_i)$, $a(x(t_i))$ is equalized to $a(t_i)$ as done in (10); and let us omit subscript i from t_i and P_{t_i}. So, $a(x)$ is replaced by $a(t)$ in the following theorems.

The feedback controller is designed as: When $s \geq s_p$,

$$u = \hat{a}(t) + \hat{b}\mathrm{sgn}(v(t)) + \dot{v}_d(t) - \alpha S(t) - \lambda e_v(t), \tag{11}$$

and when $s < s_p$,

$$u = \hat{a}(t) + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t), \tag{12}$$

with

$$S(t) := e_v(t) + \lambda e_x(t), \tag{13}$$

where α and λ are positive gains; $\hat{a}(t)$ is an estimated cogging force from an adaptation mechanism to be specified later; $\hat{b}$ is the estimated friction coefficient; $\dot{v}_d(t)$ is the desired acceleration; and $e_x(t) = x(t) - x_d(t)$ is the position tracking error. Also remind that $e_x(s(t)) = e_x(t)$; and $S(s(t)) = S(t)$

Our adaptation law is designed as follows:

$$\hat{a}(t)=\begin{cases}\hat{a}(t-P_t)-KS(t) & \text{if} \quad s\geq s_p\\ z-g(v) & \text{if} \quad s<s_p\end{cases} \tag{14}$$

$$\dot{\hat{b}}(t)=\begin{cases}-S(t)\mathrm{sgn}(v) & \text{if} \quad s\geq s_p\\ 0 & \text{if} \quad s<s_p\end{cases} \tag{15}$$

where $\hat{a}(t-P_t)=\hat{a}(t^s-P_t)=\hat{a}(s-s_p)$ (note: P_t is the trajectory cycle defined in Definition 2.3); P_1 is the first trajectory cycle specified in Definition 2.4; K is a positive design parameter (it is called the periodic adaptation gain); z will be defined in the following paragraph; and $g(v)$ is a tuning function to be selected later based on certain guidelines.

Definition 2.4: The first trajectory cycle P_1 is the elapsed time to complete the one repetitive trajectory from the initial starting time t_0. In other words, P_1 is the time corresponding to the total passed trajectory when $s(t_i)=s_p$.

In our analysis part, the following tuning function is required for $g(v)$:

$$0<g'(v)<\infty, \tag{16}$$

where $g'(\cdot)=\frac{\partial g(\cdot)}{\partial \cdot}$; and the following tuning mechanism is required for z:

$$\dot{z}=g'(v)[\dot{v}_d-\eta e_x-\lambda e_v]-e_v \tag{17}$$

Consider two cases: 1) when $0\leq t<P_1$ $(0\leq s\leq s_p)$ and 2) when $t\geq P_1$ $(s\geq s_p)$. The key idea is that, for case 1), it is required to show the finite time boundedness of equilibrium points. For case 2), it is necessary to show the stability or asymptotic stability of equilibrium points in the sense of Lyapunov. Let us investigate the case 2) first. Our major results are summarized in the following theorems with Remark 2.2.

Remark 2.2: From the relationship (10), it can be said that if $e_a(t)\to 0$ as $t\to\infty$, then $e_a(s)\to 0$ as $s\to\infty$. Thus, in what follows, the stability analysis of $a(x)$ is performed on the time axis.

Theorem 2.1: When $t\geq P_1$ $(s\geq s_p)$, the control law (11) and the periodic adaptation law (14) and (15) guarantee the stability of the equilibrium points $e_x(t)$, $e_v(t)$, $e_a(t)$, and $e_b(t)$ as $t\to\infty$ $(s\to\infty)$.

Proof: Consider the following Lyapunov-like function at $s(t)$, whose corresponding time is t:

$$V(t)=\frac{1}{2}\left(e_b(t)\mathrm{sgn}(v)\right)^2+\frac{1}{2}S^2(t)+\frac{1}{2K}\int_{t-P_t}^{t}e_a^2(\tau)\mathrm{d}\tau,$$

where P_t is calculated by the search process as commented in Definition 2.3. Then, from (18), the difference of the positive Lyapunov-like functions at two discrete time points (note: time difference is P_t) can be calculated as:

$$\begin{aligned}\triangle V(t) &= V(t)-V(t-P_t)\\ &= \frac{1}{2}\left(e_b(t)\mathrm{sgn}(v(t))\right)^2\\ &\quad -\frac{1}{2}\left(e_b(t-P_t)\mathrm{sgn}(v(t-P_t))\right)^2\\ &\quad +\frac{1}{2}S^2(t)-\frac{1}{2}S^2(t-P_t)\\ &\quad +\frac{1}{2K}\int_{t-P_t}^{t}[e_a^2(\tau)-e_a^2(\tau-P_t)]\mathrm{d}\tau\\ &= \int_{t-P_t}^{t}e_b(\tau)\mathrm{sgn}(v(\tau))\dot{e}_b(\tau)\mathrm{sgn}(v(\tau))\\ &\quad +S(t)\dot{S}(t)\mathrm{d}\tau\\ &\quad +\frac{1}{2K}\int_{t-P_t}^{t}[e_a^2(\tau)-e_a^2(\tau-P_t)]\mathrm{d}\tau\\ &= \int_{t-P_t}^{t}e_b(\tau)\dot{e}_b(\tau)+S(t)\dot{S}(\tau)\mathrm{d}\tau\\ &\quad +\frac{1}{2K}\int_{t-P_t}^{t}[e_a^2(\tau)-e_a^2(\tau-P_t)]\mathrm{d}\tau\end{aligned} \tag{18}$$

To simplify our presentation, let the first integral term on the right-hand side be denoted by A and the second integral term by B. That is

$$A:=\int_{t-P_t}^{t}e_b(\tau)\dot{e}_b(\tau)+S(\tau)\dot{S(\tau)}\mathrm{d}\tau,$$

$$B:=\frac{1}{2K}\int_{t-P_t}^{t}[e_a^2(\tau)-e_a^2(\tau-P_t)]\mathrm{d}\tau.$$

Here, from $a(s-s_p)=a(t-P_t)$ in Remark 2.1, the following equalities are satisfied:

$$a(s-s_p)=a(t-P_t)=a(t)=a(s)$$

Then, by several algebraic calculations and using $a(t-P_t)=a(t)$, B can be changed as

$$\begin{aligned}B &= \frac{1}{2K}\int_{t-P_t}^{t}\{[a(\tau)-\hat{a}(\tau)]^2-[a(\tau-P_t)\\ &\quad -\hat{a}(\tau-P_t)]^2\}\mathrm{d}\tau\\ &= \frac{1}{2K}\int_{t-P_t}^{t}[\hat{a}(\tau-P_t)-\hat{a}(\tau)][2\{a(\tau)-\hat{a}(\tau)\}\\ &\quad +\{\hat{a}(\tau)-\hat{a}(\tau-P_t)\}]d\tau\\ &= \frac{1}{2K}\int_{t-P_t}^{t}\beta(\tau)[2\{a(\tau)-\hat{a}(\tau)\}-\beta(\tau)]\mathrm{d}\tau,\end{aligned} \tag{19}$$

where

$$\beta(\tau):=\hat{a}(\tau-P_t)-\hat{a}(\tau).$$

Furthermore, using

$$\begin{aligned}\dot{e}_x &= \dot{x}-\dot{x}_d=e_v,\\ \dot{e}_v &= \dot{v}-\dot{v}_d\\ &= -a(t)-b\mathrm{sgn}(v)+u-\dot{v}_d\\ &= -a(t)-b\mathrm{sgn}(v)+\hat{a}(t)+\hat{b}\mathrm{sgn}(v)+\dot{v}_d-\alpha S(t)\\ &\quad -\lambda e_v(t)-\dot{v}_d\\ &= -e_a(t)-e_b\mathrm{sgn}(v)-\alpha S(t)-\lambda e_v(t),\end{aligned} \tag{20}$$

where (8), (9), and (10) were used to make $e_a(t)$ from $e_a(s)$, we have

$$\begin{aligned}\dot{S} &= \dot{e}_v+\lambda\dot{e}_x(t)\\ &= -e_a(t)-e_b\mathrm{sgn}(v)-\alpha S(t)-\lambda e_v(t)+\lambda e_v(t)\\ &= -e_a-e_b\mathrm{sgn}(v)-\alpha S(t).\end{aligned} \tag{21}$$

Then, using

$$\dot{e}_b = \dot{b}-\dot{\hat{b}}=-\dot{\hat{b}} \tag{22}$$

A can be expressed as

$$A=\int_{t-P_t}^{t}-e_b(\tau)\dot{\hat{b}}+S(\tau)\left(-e_a(\tau)-e_b(\tau)\mathrm{sgn}(v)-\alpha S(\tau)\right)\mathrm{d}\tau \tag{23}$$

Thus, $\triangle V$ becomes

$$\begin{aligned}\triangle V &= A+B \\ &= \int_{t-P_t}^{t} -e_b(\tau)\dot{\hat{b}} + S(\tau)(-e_a(\tau) - e_b(\tau)\text{sgn}(v) \\ &\quad -\alpha S(\tau))\text{d}\tau + \frac{1}{2K}\int_{t-P_t}^{t} \beta[2\{a(\tau) - \hat{a}(\tau)\} \\ &\quad -\beta(\tau)]\text{d}\tau. \end{aligned} \tag{24}$$

Also, using $e_a(t) = a(t) - \hat{a}(t)$ and $\beta(t) = KS(t)$, $A+B$ is changed as:

$$A+B = \int_{t-P_t}^{t} -e_b(\tau)\dot{\hat{b}} - S(\tau)e_b(\tau)\text{sgn}(v) - \alpha S^2 - \frac{1}{2K}\beta^2\text{d}\tau. \tag{25}$$

Then, using $\dot{\hat{b}} = -S(t)\text{sgn}(v)$ from (15)

$$\begin{aligned} A+B &= \int_{t-P_t}^{t} -\alpha S^2 - \frac{1}{2K}\beta^2\text{d}\tau \\ &= \int_{t-P_t}^{t} -\alpha S^2 - \frac{1}{2}KS^2\text{d}\tau. \end{aligned} \tag{26}$$

Therefore, since $\Delta V(t) \leq 0$, the proof is completed. ■

The above theorem only guarantees the stability property in the sense of Lyapunov. To explore the asymptotical stability, the following notation and lemma are provided. The total external disturbances including cogging force and friction force are denoted as:

$$c(t) = a(t) + b\text{sgn}(v)$$

and its corresponding error is denoted as:

$$\begin{aligned} e_c(t) &= a(t) + b\text{sgn}(v) - \hat{a}(t) - \hat{b}\text{sgn}(v) \\ &= e_a(t) + e_b\text{sgn}(v) \end{aligned} \tag{27}$$

Lemma 2.1: In the following equation with initial state $x(0) = x_0 = 0$

$$y = \dot{x} + \tau x, \quad \tau > 0,$$

$y \to 0$ as $t \to \infty$ if and only if $x \to 0$ as $t \to \infty$.

Proof: The sufficient condition is immediate because $x = 0$ makes $y = 0$. The necessary condition is proved easily by calculating the solution. When $y = 0$, $x(t)$ is calculated as:

$$x(t) = x_0 + e^{-\tau t}.$$

So, if $x_0 = 0$, as $t \to \infty$, $x(t) \to 0$. ■

Now, let us consider the asymptotically stability condition of the equilibrium points e_x, e_v, and e_c in the following theorem.

Theorem 2.2: If the initial position (x_0) is at the desired initial position ($x_d(0)$), i.e., $e_x(0) = 0$, the control law (12) and the periodic adaptation law (14) guarantee the asymptotically stability of the equilibrium points: e_x, e_v, and e_c as $t \to \infty$ ($t \geq P_1$, or $s \geq s_p$).

Proof: Here, LaSalle's invariant set theorem is used to prove the asymptotical stability. From (26), only $S = 0$ makes $\triangle V = 0$. Using the definition $S = e_v + \lambda e_x$ and relationship $e_v = \dot{e}_x$, we have

$$S = e_v + \lambda e_x = \dot{e}_x + \lambda e_x. \tag{28}$$

So, from Lemma 2.1, if $e_x(0) = 0$, only $e_x = 0$ makes $S = 0$. Also, since $e_x = 0$, we have $e_v = 0$ from $e_v + \lambda e_x = 0$. Therefore, e_x and e_v are asymptotically stable at equilibrium points. Now let us think e_c in what follows. From $\dot{S} = -e_a - e_b\text{sgn}(v) - \alpha S = -e_c - \alpha S$, $\dot{S} = -e_c$ because $S = 0$. Then, by showing that $\dot{S} \to 0$ as $S \to 0$, $e_c = 0$ can be proved. Our approaches are as follows. From the following definition

$$\dot{S} = \lim_{\Delta t \to 0} \frac{S(t+\Delta t) - S(t)}{\Delta t}, \tag{29}$$

we know that as $t \to \infty$, $S(t+\Delta t) \to 0$ and $S(t) \to 0$. However, from our original assumption of the periodicity such as $\Delta t = P_t$, if P_t is not zero, then $\Delta t \neq 0$, while $S(t+\Delta t) - S(t) \to 0$ as $t \to \infty$. Thus, in (29), $\dot{S} \to 0$ as $t \to \infty$, hence if $\text{sgn}(v) \neq 0$, then $-e_c = 0$. However, if $-e_c \neq 0$, $\dot{S} \neq 0$. Then $S(t+\Delta t) - S(t) \neq 0$. This is a contradiction to $S(t+\Delta t) - S(t) = 0$. Therefore, it can be concluded that only $-e_c = 0$ makes $\dot{S} = 0$ and in the sequel, no trajectory can stay except $e_c = 0$ when $S = 0$. Since only $e_x = 0$, $e_v = 0$ and $e_c = 0$ make $S = 0$, from the invariant set theorem, the equilibrium points e_x, e_v, and e_c are asymptotically stable. This completes the proof of this theorem. ■

Remark 2.3: The asymptotical stability of e_c does not guarantee the asymptotical stability of e_a and e_b. In other words, even if the suggested theorem guarantees the asymptotical stability of e_x and e_v, it does not provide the asymptotical stability of e_a and e_b. However, the cogging disturbance and friction disturbance will be compensated altogether successfully by Theorem 2.2.

Now, let us consider the case 1) when $t < P_1$ ($s \leq s_p$) and the overall stability when $t \geq 0$ ($s \geq 0$).

Theorem 2.3: If $|\dot{a}|$ and b are bounded, the equilibrium points of e_x, e_v, e_a, and e_b are stable (or e_c is asymptotically stable) as $t \to \infty$ ($s \to \infty$).

Proof: In this case, let us consider the following Lyapunov function:

$$V(t) = \frac{\eta}{2}e_x^2(t) + \frac{1}{2}e_v^2(t) + \frac{1}{2}e_a^2(t) + \frac{1}{2}e_b^2 \tag{30}$$

Then, the derivative of V is expressed as:

$$\begin{aligned} \dot{V}(t) &= \eta e_x e_v + e_v(-a(t) - b\text{sgn}(v) + u - \dot{v}_d) \\ &\quad + e_a\dot{e}_a + e_b\dot{e}_b \end{aligned} \tag{31}$$

From (11), (14), (15), and (17), using

$$\dot{e}_b = \dot{b} - \dot{\hat{b}} = 0$$

$$u = \hat{a}(t) + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t)$$

$$\dot{e}_a = \dot{a} - \dot{\hat{a}} = \dot{a} - \dot{z} + g'(v)\dot{v}$$

we have

$$\begin{aligned} \dot{V}(t) = \eta e_x e_v + e_v[-a(t) - b\text{sgn}(v) + \hat{a}(t) + \dot{v}_d(t) \\ -\eta e_x(t) - \lambda e_v(t) - \dot{v}_d] + e_a\left[\dot{a} - \dot{z} + g'(v)\dot{v}\right] \end{aligned} \tag{32}$$

Now, rewriting $\dot{v}$ such as:

$$\begin{aligned} \dot{v} &= -a(x) - b\text{sgn}(v) + u \\ &= -a(x) - b\text{sgn}(v) + \hat{a}(t) + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t) \\ &= -e_a - b\text{sgn}(v) + \dot{v}_d(t) - \eta e_x(t) - \lambda e_v(t) \end{aligned} \tag{33}$$

and from (17), using

$$\dot{z} = g'(v)[\dot{v}_d - \eta e_x - \lambda e_v] - e_v$$

(32) is changed as:

$$\begin{aligned}\dot{V}(t) &= -\lambda e_v^2 - b\mathrm{sgn}(v)e_v - e_a e_v + \\ &\quad e_a[\dot{a} + e_v + g'(v)(-e_a - b\mathrm{sgn}(v))] \\ &= -\lambda e_v^2 - b\mathrm{sgn}(v)e_v \\ &\quad -e_a^2 g'(v) - e_a b g'(v)\mathrm{sgn}(v) + e_a\dot{a}. \end{aligned} \tag{34}$$

Let us investigate $-\lambda e_v^2 - b\mathrm{sgn}(v)e_v$ of (34) and $-e_a^2 g'(v) - e_a b g'(v)\mathrm{sgn}(v) + e_a\dot{a}$ of (34) separately. Then, the following relationship is derived:

$$\begin{aligned}-\lambda e_v^2 - b\mathrm{sgn}(v)e_v &= -\lambda(e_v^2 + \frac{b\mathrm{sgn}(v)}{\lambda}e_v) \\ &= -\lambda(e_v + \frac{b\mathrm{sgn}(v)}{2\lambda})^2 + \frac{b^2}{4\lambda} \end{aligned} \tag{35}$$

and $-e_a^2 g'(v) + (\dot{a} - bg'(v)\mathrm{sgn}(v))e_a$ is changed as

$$-g'(v)\left(e_a + \frac{\dot{a} - bg'(v)\mathrm{sgn}(v)}{2g'(v)}\right)^2 + \frac{(\dot{a} - bg'(v)\mathrm{sgn}(v))^2}{4g'(v)}.$$

Hence, since

$$-\lambda e_v^2 - b\mathrm{sgn}(v)e_v \le \frac{b^2}{4\lambda}$$

and

$$-e_a^2 g'(v) + (\dot{a} - bg'(v)\mathrm{sgn}(v))e_a \le \frac{(\dot{a} - bg'(v)\mathrm{sgn}(v))^2}{4g'(v)},$$

the derivative of Lyapunov function is upper bounded such as:

$$\dot{V}(t) \le \frac{b^2}{4\lambda} + \frac{(\dot{a} - bg'(v)\mathrm{sgn}(v))^2}{4g'(v)}$$

Thus, it concludes that $\dot{V}$ is bounded when $t < P_1$ ($s < s_p$). Consequently, when V is bounded at $t < P_1$, e_x, e_v, e_a, and e_b are also bounded in l_2 vector norm topology at $t < P_1$ ($s < s_p$). Furthermore, when $t \ge P_1$ ($s \ge s_p$), the equilibrium points of e_x, e_v, e_a, and e_b are all (e_c is asymptotically stable with $e_x(0) = 0$) stable from equation (26); so the system (4)-(5) can be (asymptotically with $e_x(0) = 0$) stabilized by the control law (11)-(12) and the adaptation law (14)-(15) as $t \to \infty$. This completes the proof. ■

III. Simulation Illustrations

For simulation test, let us use the following reference position and velocity signals:

$$\begin{aligned} x_r(t) &= \sin(2\pi f_s t) \\ v_r(t) &= 2\pi f_s \cos(2\pi f_s t) \\ \dot{v}_r(t) &= -(2\pi f_s)^2 \sin(2\pi f_s t) \end{aligned} \tag{36}$$

where $f_s = \frac{1}{Q_s}$, and $Q_s = 2$ sec. The control gains in (11)-(12) were selected as: $\alpha = 10$, $\lambda = 20$ and $\eta = 50$; and $g(v)$ was designed as $10v$ to satisfy (16). In (14), the periodic adaptation gain K was selected as 100. In simulation, the actual cogging force was modelled as:

$$\begin{aligned} F_{cogging} &= 4\sin(2\pi x) + 2\sin(2\pi 2x) + 1\sin(2\pi 3x) \\ &+0.5\sin(2\pi 4x) + 0.25\sin(2\pi 5x) + 0.125\sin(2\pi 6x) \end{aligned} \tag{37}$$

and friction coefficient is $b = 10$. Figure 1 shows the tracking performance. The top-left subplot is the position while the bottom-left is the velocity. The two right subplots are the corresponding errors of the left subplots. From the right subplots, after 2 seconds, the tracking performance was significantly improved by periodic adaptation. The two left subplots of Fig. 2 are the true and estimated cogging and friction forces on the state axis (i.e., the true forces: $a(x)+b\mathrm{sgn}(v)$, and the estimated forces: $\hat{a}(x)+\hat{b}\mathrm{sgn}(v)$). From these two left subplots, we can find that the total passed trajectory from $s = 0$ to $s = 4$ corresponds to the first repetitive trajectory. As shown in the corresponding right subplots, after $s = 4$, the cogging and friction forces were estimated much better than the first repetitive trajectory. In the right subplots, the spikes are due to the sign change of the friction force at zero velocity. Figure 3 shows the control input on the time-axis (left subplot) and on the state-axis (right subplot). From these results, it is found that similar control inputs are required at each repetitive trajectory.

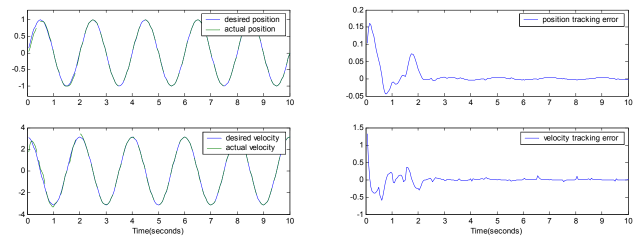

Fig. 1. Top left: desired position and actual position. Bottom left: desired velocity and actual velocity. Top right: position tracking error. Bottom right: velocity tracking error.

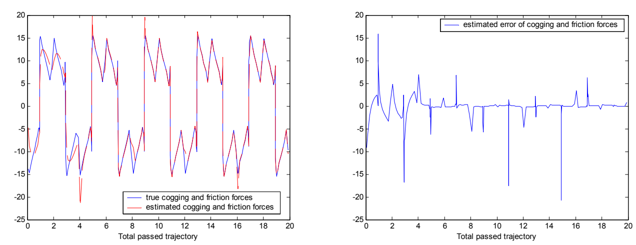

Fig. 2. Left: true/estimated cogging and friction forces on the state-axis(total passed trajectory). Right: estimated cogging and friction force error on the state-axis(total passed trajectory).

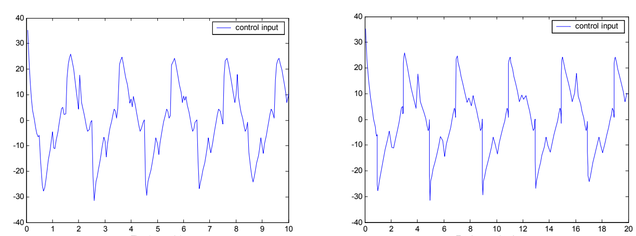

Fig. 3. Left: control input on the time axis. Right: control input on the state-axis.

IV. Conclusion Remarks

In this paper, new cogging and friction force compensation method was suggested for the permanent-magnet linear motors. The key idea of our method was to use the periodicity of the task and the cogging disturbance depending on the position. From one trajectory past information, the current adaptation law was updated. Even though the stability analysis was performed on the time axis, the position-dependent cogging disturbance can be successfully compensated on the state-axis. It is believed that the suggested method can be effectively used in many real applications such as satellite, trail system, factory process control, and etc. We have demonstrated that the state-dependent external disturbance such as cogging can be successfully compensated by making use of the trajectory periodicity of the state-dependent disturbance.

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