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# Stabilization of Networked Control Systems under Feedback-based Communication

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***Abstract*— We study the stabilization of a networked control system (NCS) in which multiple sensors and actuators of a physical plant share a communication medium to exchange information with a remote controller. The plant's sensors and actuators are allowed only limited access to the controller at any one time, in a way that is decided on-line using a feedback-based communication policy. Our NCS model departs from those in previous formulations in that the controller and plant handle communication disruptions by "ignoring" (in a sense that will be made precise) sensors and actuators that are not actively communicating. We present an algorithm that provides a complete and straightforward method for simultaneously determining stabilizing gains and communication policies and avoids the computational complexity and limitations associated with some previously proposed models. We introduce three feedback-based scheduling policies that quadratically stabilize the closed-loop NCS while achieving various objectives related to the system's rate of convergence, the priorities of different sensors and actuators, and the avoidance of chattering.**

## I. Introduction

A control system is called a *networked control system* (NCS) if its feedback loop(s) are closed via a shared communication medium. In a NCS, the medium's limited capacity must be allocated to all the actuators, sensors, and controllers in the system. As a consequence, various communication constraints such as delays [1], [2], [3], data rate limitations [4], [5], [6], quantization effects [7], [8], [9], and medium access constraints [10], [11], [12], [13] all become potential problems whose effects on closed-loop performance and controller design must be understood and dealt with.

The focus of this paper is the stabilization of NCS under medium access constraints. More precisely, we consider a NCS in which multiple sensors (outputs) and actuators (inputs) of a physical plant are connected to a remote controller via a shared communication medium. The medium has limited number of channels so that at any one time only some of the sensors and actuators can exchange information with the controller, while others must wait. In contrast to traditional control systems, the control of NCS involves choosing not only the controller but also a medium access strategy. Traditionally, the design of each of these two components has been studied separately by selecting one while assuming the other is given.

In [10], the temporal order of the medium access for the sensors and actuators was described by a *"communication sequence"*. Given a periodic communication sequence, the problem of designing a constant feedback controller that stabilizes a linear plant is NP-hard [14], [11]. Under the assumption that the feedback controller has been designed in advance (for good performance in the absence of communication constraints), two classes of medium access strategies have been proposed. These are termed *static access scheduling* and *dynamic (or feedback-based) access scheduling*. Under static scheduling, the order of medium access for the sensors and actuators is designed off-line and remains fixed over time. Static scheduling can be implemented via MAC (Medium Access Control) protocols such as polling, token-passing, and TDMA. The existence and design of periodic communication sequences (a subclass of static scheduling protocols) that stabilize a NCS was studied in [15]. In [13], the schedulablity of NCS under static scheduling is verified by the rate monotonic (RM) rule.

Static scheduling may be less robust when the plant is subject to unpredictable disturbances, because the controller may not be able to respond quickly to a sensor or actuator that requires immediate attention. Moreover, a global timer is needed to synchronize all the sensors and actuators. These restrictions have fueled research on dynamic access scheduling in which the access to the medium is determined in real-time based on a feedback-based arbitration policy. Examples of dynamic scheduling policies include MEF-TOD [12], [16] and CLS-$\epsilon$ [17]. Dynamic scheduling can be implemented via random access MAC protocols such as CSMA/CR (Carrier Sense, Multiple Access, with Collision Resolution).

It should be noted that most of the medium access strategies proposed in previous works have focused only on NCS whose dynamics are "block-diagonal", in the sense that they consist of collections of sub-systems that are uncoupled in the absence of communication constraints, as is the case with [17], or are attached to very conservative stability criteria [16]. Furthermore, most previous works assume zero order holding (ZOH) at the receiver side of a communication medium: when an actuator or sensor fails to access the medium the value stored in a ZOH will be fed to the plant or controller. In this work, we forgo the use of ZOH and let *zero* be fed into the plant or controller when an actuator or sensor fails to access the medium. We show that

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this protocol leads to a simpler but more powerful model for NCS which enables one to jointly design a stabilizing feedback gain and a dynamic access scheduling policy, and to investigate their interactions. The design method presented in this paper is based on switched system theory [18] and can be used with a more general class of NCS, which have "fully coupled" dynamics.

The remainder of this paper is structured as follows: In Section II, we describe a switch system-based model for NCS under static feedback. In Section III, we present an algorithm that allows one to design a static feedback controller that guarantees the stabilizability of a NCS. In Section IV, we introduce three feedback-based medium access scheduling policies that quadratically stabilize a NCS while achieving different design objectives related to the system's rate of convergence, the priorities of different sensors and actuators, and the avoidance of chattering. Section V contains simulations that illustrate our design approach.

## II. MODELING NCS

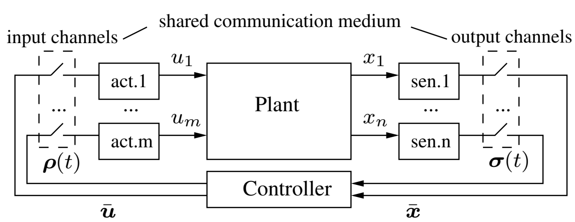


Fig. 1. A Networked Control System with $m$ inputs and $n$ outputs.

Consider the NCS depicted in Fig. 1 and suppose that the plant is a controllable LTI system:

$$\dot{\boldsymbol{x}}(t) = A\boldsymbol{x}(t) + B\boldsymbol{u}(t); \quad \boldsymbol{x} \in \mathbb{R}^n, \boldsymbol{u} \in \mathbb{R}^m \tag{1}$$

where $\boldsymbol{x} = [x_1, \cdots, x_n]^T$ and $\boldsymbol{u} = [u_1, \cdots, u_m]^T$ are the states and inputs of the plant. For now we will assume state feedback, meaning that all of the $n$ states $x_i$ are available for measurement by a corresponding sensor. The communication medium connecting the sensors and the controller has $w_\sigma$ ($1 \le w_\sigma < n$) channels (called the *output channels*). At any one time, only $w_\sigma$ of the $n$ sensors can access these channels to communicate with the controller while others have to wait. At the input side, each of the plant's inputs is connected to an actuator. The $m$ actuators share $w_\rho$ ($1 \le w_\rho < m$) *input channels* to communicate with the controller, only $w_\rho$ of the $m$ actuators can access the input channels at any one time.

### A. The effect of medium access constraints

For $i = 1, \cdots, n$, let the binary-valued function $\sigma_i(t)$ denote the medium access status of sensor $i$ at time $t$, i.e. $\sigma_i(t) : \mathbb{R} \mapsto \{0, 1\}$, where 1 means "accessing" and 0 means "not accessing". The medium access status of the $n$ sensors over time can be represented by the "*$n$-to-$w_\sigma$" communication sequence*" ([11], [17])

$$\boldsymbol{\sigma}(t) = [\sigma_1(t), \cdots, \sigma_n(t)]^T$$

*Definition 1:* An M-to-N communication sequence is a map $\boldsymbol{\sigma}(t) : \mathbb{R} \mapsto \{0, 1\}^M$, satisfying $\|\boldsymbol{\sigma}(t)\|^2 = N$, $\forall t$.

Any given output, say $x_i(t)$, is available to the controller only when its sensor is accessing the communication medium, i.e. $\sigma_i(t) = 1$. When sensor $i$ cannot access the communication medium ($\sigma_i(t) = 0$), we assume that a *zero* value will be used by the controller for that sensor to generate the control signals, while the actual output $x_i(t)$ will be ignored due to its being unavailable. Let $\bar{x}_i(t)$ denote the output signal used by the controller at time $t$, based on the above protocol; then, we can write $\bar{x}_i(t) = \sigma_i(t) \cdot x_i(t)$; $\forall i, t$.

We define the *matrix form of a communication sequence* $\boldsymbol{\eta}(t)$ to be

$$M_\eta(t) \triangleq diag(\boldsymbol{\eta}(t))$$

If we let $\bar{\boldsymbol{x}} = [\bar{x}_1, \bar{x}_2, \cdots, \bar{x}_n]^T$, we see that the signal available to the controller is not the state $x$ but rather

$$\bar{\boldsymbol{x}}(t) = M_\sigma(t) \cdot \boldsymbol{x}(t) \tag{2}$$

Similarly, whenever actuator $j$ loses its access to the communication medium, the actual control signal generated at the controller for that actuator will be unavailable to and hence ignored by the plant. Instead, we let the plant use a *zero* value for $u_j$ until actuator $j$ regains access. We represent the medium access status of the plant's $m$ actuators by an $m$-to-$w_\rho$ communication sequence (see Definition 1), $\boldsymbol{\rho}(t)$. Let $\bar{\boldsymbol{u}} = [\bar{u}_1, \cdots \bar{u}_m]^T$ denote the actual signal generated by the controller and let $\boldsymbol{u}(t)$ denote the input signals as viewed from the plant. The two are related by

$$\boldsymbol{u}(t) = M_\rho(t)\bar{\boldsymbol{u}}(t) \tag{3}$$

Finally, let the controller in Fig.1 be given by the feedback law

$$\bar{\boldsymbol{u}}(t) = K \cdot \bar{\boldsymbol{x}}(t) \tag{4}$$

From (1)-(4), we see that the closed-loop dynamics of the NCS are

$$\dot{\boldsymbol{x}}(t) = (A + BM_\rho(t)KM_\sigma(t))\boldsymbol{x}(t) \tag{5}$$

The above discussion is summarized in the block diagram of Fig. 2.

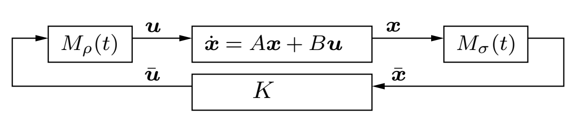


Fig. 2. The closed-loop dynamics of a NCS

*Remark 1:* As we have mentioned, previous works (e.g. [10], [11]), have often assumed that there is a zero-order-hold (ZOH) at the receiving side of a communication medium. The use of ZOH significantly increases the system's complexity because it introduces time-varying delays to the closed-loop dynamics (see, for example, the "extensive form" in [11] and other similar constructions). Moreover, holding delayed input/output for the entire access disruption period may not necessarily improve matters: Consider the scalar system $\dot{x} = x + u$ stabilized by the feedback controller $u = -2x$. Suppose that medium access for $u$ is "cut" off when $x = 1$. Under ZOH, the system will then evolve according to $\dot{x} = x - 2$ afterwards. Depending on the time it takes to re-establish access for the controller, $x$ could become negative, and the control $u = -2$ could result in $|x|$ becoming larger than if $u = 0$ had been used. In this work, we have chosen to "ignore" the inputs or outputs that are not actively communicating. We have shown that the effect of medium access constraints is equivalent to cascading the original plant with a pair of communication sequences (in their matrix form). The resulting closed loop dynamics (5) have lower complexity than previous models and lead to a straightforward and complete solution to the stabilization problem, as we will show next.

### *B. An equivalent switched system*

Without loss of generality, we begin with the case $w_\sigma = w_\rho = 1$. Now $\boldsymbol{\rho}(t)$ is an $m$-to-1 communication sequence and $\boldsymbol{\sigma}(t)$ is an $n$-to-1 communication sequence. By definition, $\boldsymbol{\rho}(t)$ can only have $m$ possible values and $\boldsymbol{\sigma}(t)$ can only have $n$ possible values. Hence the closed loop NCS (5) is essentially a switched system with $m \cdot n$ possible dynamics.

For simplicity, it will be helpful to introduce one additional piece of notation. Let $\boldsymbol{\eta}(t)$ be a $p$-to-1 communication sequence. By definition, $\boldsymbol{\eta}(t)$ takes on values on the set of standard $p$-dimensional basis vectors, $E_p = \{e_p^1, e_p^2, \cdots, e_p^p\}$, where $e_p^1 = [1, 0 \cdots 0]^T$, $e_p^2 = [0, 1, 0 \cdots 0]^T$, $\cdots$ $e_p^p = [0 \cdots 0, 1]^T$. We define the *scalar form of the communication sequence* $\boldsymbol{\eta}(t)$ to be the map $\bar{\eta}(t) : \mathbb{R} \mapsto \{1, \cdots p\}$, such that $\boldsymbol{\eta}(t) = e_p^{\bar{\eta}(t)}, \quad \forall t$. In other words, $\bar{\eta}(t)$ equals $i$ if $\boldsymbol{\eta}(t) = e_p^i$.

Now let the scalar forms of $\boldsymbol{\rho}$ and $\boldsymbol{\sigma}$ be $\bar{\rho}(t)$ and $\bar{\sigma}(t)$, and let $s(t) = [\bar{\rho}(t), \bar{\sigma}(t)]$. Then, the closed loop system (5) is equivalent to the following switched system

$$\dot{\boldsymbol{x}} = \mathcal{A}_{s(t)} \boldsymbol{x} \tag{6}$$

where $s(t)$ defines a switching rule, $s(t) : \mathbb{R} \mapsto \{1, \cdots m\} \times \{1, \cdots n\}$. The matrix $\mathcal{A}_{s(t)}$ takes values on the set

$$\{\mathcal{A}_{ij} : i = 1, \cdots, m;\ j = 1, \cdots, n\}$$

where $\mathcal{A}_{ij}$ denotes the closed-loop dynamics when actuator $i$ and sensor $j$ are accessing the communication medium, i.e. $\boldsymbol{\rho} = e_m^i$ and $\boldsymbol{\sigma} = e_n^j$. From (5) it easy to see that

$$\mathcal{A}_{ij} = A + BK_{ij} \tag{7}$$

where $K_{ij} = diag(e_m^i) \cdot K \cdot diag(e_n^j)$.

We note that although we have assumed $w_\sigma = w_\rho = 1$, all of the discussion in this paper applies to any number of channels. For example, when the communication medium has $w_\rho$ ($1 < w_\rho < m$) input channels and $w_\sigma$ ($1 < w_\sigma < m$) output channels, then $\boldsymbol{\rho}(t)$ and $\boldsymbol{\sigma}(t)$ will have $\binom{m}{w_\rho}$ and $\binom{n}{w_\sigma}$ possible values, respectively. The closed loop system will then switch between $\binom{m}{w_\rho} \cdot \binom{n}{w_\sigma}$ possible dynamics. The required modifications are straightforward and will not be given here.

## III. STABLE CONVEX COMBINATIONS

As demonstrated in [19], the switched system (6) is stabilizable under a feedback-based switching rule $s(t)$ if there exist positive real numbers $\alpha_{ij}$, $i = 1, \cdots, m$, $j = 1, \cdots, n$, satisfying

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ij} = 1 \tag{8}$$

such that the convex combination of $\mathcal{A}_{ij}$'s

$$\mathcal{A} \triangleq \sum_{i=1}^{m} \sum_{j=1}^{n} \alpha_{ij} \mathcal{A}_{ij} \tag{9}$$

is stable. If $\mathcal{A}$ is stable, it is well known that there exist positive definite matrices $P, Q$ such that

$$\mathcal{A}^T P + P\mathcal{A} = -Q \tag{10}$$

Then for all $\boldsymbol{x}(t) \neq 0$

$$\boldsymbol{x}^T(t)(\mathcal{A}^T P + P\mathcal{A})\boldsymbol{x}(t) = -\boldsymbol{x}^T(t) Q \boldsymbol{x}(t) < 0$$

An important fact in the proof of the stable convex combination result is that the last equation can be rewritten as

$$\sum_{i,j} \alpha_{ij} \boldsymbol{x}^T(t)(\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij})\boldsymbol{x}(t) = -\boldsymbol{x}^T(t) Q \boldsymbol{x}(t) < 0$$

for all $\boldsymbol{x}(t) \neq 0$. Because $\alpha_{ij} > 0$ it follows that for all $\boldsymbol{x}(t) \neq 0$ there always exist indices $i(x) \in \{1, \cdots m\}$ and $j(x) \in \{1, \cdots n\}$ such that

$$\boldsymbol{x}^T(t)(\mathcal{A}_{i(x)j(x)}^T P + P\mathcal{A}_{i(x)j(x)})\boldsymbol{x}(t) < 0$$

Hence if the switched system is switched according to $s(t) = [i(x(t)), j(x(t))]$ the Lyapunov function $V(t) = \boldsymbol{x}^T(t) P \boldsymbol{x}(t)$ will always be decreasing.

Here the stabilizability of the switched system (6) relies on the existence of a stable convex combination (9). However, if the $A_{ij}$'s are given and the number of possible dynamics (in our case, $m \cdot n$) is greater than two, identifying such a stable convex combination (if one exists) is NP-hard [20]. Fortunately, in a NCS we also have the freedom to design the controller in order to obtain a stable convex combination (9).

From (7), we see that $\mathcal{A}$ can be expressed as

$$\mathcal{A} \triangleq \sum_{i,j} \alpha_{ij} \mathcal{A}_{ij} = A + B\mathcal{K} \tag{11}$$

where

$$\mathcal{K} = \begin{bmatrix} \alpha_{11}k_{11} & \alpha_{12}k_{12} & \cdots & \alpha_{1n}k_{1n} \\ \alpha_{21}k_{21} & \alpha_{22}k_{22} & \cdots & \alpha_{2n}k_{2n} \\ & \cdots & \cdots & \\ \alpha_{m1}k_{m1} & \alpha_{m2}k_{m2} & \cdots & \alpha_{mn}k_{mn} \end{bmatrix} \tag{12}$$

and $k_{ij}$ is the $(i,j)$ entry of $K$. Now a feedback gain $K$ that guarantees a stable convex combination (9) can be found by the following

**Algorithm:**

1) Choose $m \cdot n$ positive numbers $\alpha_{ij}$'s such that (8) is satisfied.
2) Choose a set of desired (stable) eigenvalues for $\mathcal{A}$.
3) Solve the pole-placement problem for $\mathcal{K}$, such that $\mathcal{A} = A + B\mathcal{K}$ has the desired eigenvalues, provided that $(A, B)$ is controllable.
4) Solve for $K = [k_{ij}]_{m \times n}$ from (12).

Notice that for the same choice of $\mathcal{A}$, different choices of $\alpha_{ij}$'s results in different values of the feedback gain $K$. A larger $\alpha_{ij}$ leads to a smaller $k_{ij}$. This fact gives us additional freedom in the design of $K$. By properly choosing $\alpha_{ij}$'s we can make the controller $K$ meet certain optimization or design criterion, for example, $\max_{i,j} |k_{ij}| < k_m$, where $k_m$ is the highest gain a controller may provide.

## IV. Medium Access policies that guarantee quadratic stability

In this section, we introduce three feedback based switching rules that guarantee the quadratic stability of (6).

*Definition 2:* [20] The system (6) is said to be quadratically stable if there exists a positive definite quadratic function $V(x) = \boldsymbol{x}^T P \boldsymbol{x}$, a positive number $\epsilon$ and a switching rule $s(t)$ such that $\frac{d}{dt}V(\boldsymbol{x}) < -\epsilon \boldsymbol{x}^T \boldsymbol{x}$ for all trajectories $\boldsymbol{x}$ of the system (6).

Notice that the Lyapunov function $V = \boldsymbol{x}^T P \boldsymbol{x}$ is continuous and piecewise differentiable along trajectories of (6). Then between any two consecutive switches

$$\frac{d}{dt}V = \boldsymbol{x}^T(t)(\mathcal{A}_{s(t)}^T P + P\mathcal{A}_{s(t)})\boldsymbol{x}(t) \tag{13}$$

We first introduce the switching rule:

**Weighted Fastest Decay (WFD):**
For all $t$, let $s(t)$ be determined by

$$s(t) = arg \min_{i,j} \alpha_{ij} \boldsymbol{x}^T(t)[\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij}]\boldsymbol{x}(t) \tag{14}$$

*Theorem 1:* If $\mathcal{A}$ is stable, system (6) is quadratically stable under the switching rule WFD.

*Proof:* Since $\mathcal{A}$ is stable, there exist positive definite matrices $P$, $Q$ such that (10) holds. Then

$$\sum_{i=1}^{m}\sum_{j=1}^{n} \alpha_{ij} \boldsymbol{x}^T(t)[\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij}]\boldsymbol{x}(t) = -\boldsymbol{x}^T(t) Q \boldsymbol{x}(t) \tag{15}$$

(14) and (15) imply, for all $t$,

$$\alpha_{s(t)} \boldsymbol{x}^T(t)[\mathcal{A}_{s(t)}^T P + P\mathcal{A}_{s(t)}]\boldsymbol{x}(t) \leq -\frac{\boldsymbol{x}^T(t) Q \boldsymbol{x}(t)}{m \cdot n}$$

Hence

$$\begin{aligned} \dot{V}(t) &= \boldsymbol{x}^T(t)(\mathcal{A}_{s(t)}^T P + P\mathcal{A}_{s(t)})\boldsymbol{x}(t) \\ &\leq -\frac{\boldsymbol{x}^T(t) Q \boldsymbol{x}(t)}{m \cdot n \cdot \alpha_{s(t)}} \leq -\epsilon^* \boldsymbol{x}^T(t)\boldsymbol{x}(t) \end{aligned}$$

where

$$\epsilon^* \triangleq \frac{\lambda_{min}(Q)}{m \cdot n \cdot \alpha_{max}} \tag{16}$$

, $\lambda_{min}(Q)$ denotes the smallest eigenvalue of $Q$, and $\alpha_{max} \triangleq \max_{i,j} \alpha_{ij}$. ■

From (14), we see that each $\alpha_{ij}$ acts as a weight associated with the dynamics $\mathcal{A}_{ij}$. A greater $\alpha_{ij}$ will result in a greater chance that (6) is switched to $\mathcal{A}_{ij}$. Note that $\mathcal{A}_{ij}$ corresponds to the dynamics when $u_i$ and $x_j$ are accessing the communication medium. Hence the choice of $\alpha_{ij}$'s actually assigns "priorities" for every input and output of the NCS. In the previous section, the choice of $\alpha_{ij}$'s also affects the feedback gain, $K$. A modification of the WFD rule, which decouples controller design and medium access weighting, can be made by replacing $\alpha_{ij}$ in (14) with a different set of weights, for example, $w_{ij}$ $(i = 1 \cdots m,\ j = 1 \cdots n)$. It can be shown that this modification still gives quadratic stability of (6).

As suggested in [19], the following switching rule ensures maximum instantaneous decay of $V$:

**Fastest Decay (FD):** For all $t$, let $s(t)$ be determined by

$$s(t) = arg \min_{i,j} \boldsymbol{x}^T(t)[\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij}]\boldsymbol{x}(t) \tag{17}$$

*Theorem 2:* If $\mathcal{A}$ is stable, system (6) is quadratically stable under the switching rule FD.

*Proof:* Under the FD rule, at any time $t$, system (6) is switched to the set of dynamics that gives the fastest decay of $V(t)$. The instantaneous value of $\dot{V}$ is thus no greater than when (6) is under any other switching rules, including WFD. Hence, for all $t$, $\dot{V}(t) \leq -\epsilon^* \boldsymbol{x}^T(t)\boldsymbol{x}(t)$. ■

Although Theorems 2 and 1 provide switching rules that guarantee quadratic stability the switching rate is not bounded under the FD or WFD rule. High-speed switching is often impractical and may lead to chattering. One way to bound the switching rate is to introduce a minimum *dwell time* [18] $\tau > 0$ and restrict the interval between any two consecutive switches to be no smaller than $\tau$. The switching rule we introduce next guarantees a dwell time between switchings. The idea is to let the system evolve with one set of dynamics until the decay rate of $V$ is less than a certain threshold.

**Guaranteed Dwell-time (GD):**
Let $\epsilon_0$ be a number satisfying $0 < \epsilon_0 < \epsilon^*$

1) Denote the current switch time by $t_0$, choose $s(t_0)$ according to (17)
2) Let $s(t) = s(t_0)$ for all $t \in [t_0, t_1)$, where $t_1$ is the next switch time determined by

$$t_1 = \inf_{t>t_0} \boldsymbol{x}^T(t)(\mathcal{A}_{s(t_0)}^T P + P\mathcal{A}_{s(t_0)})\boldsymbol{x}(t) \geq -\epsilon_0 \boldsymbol{x}^T(t)\boldsymbol{x}(t)$$

3) Repeat from step 1 for $t_1$

*Theorem 3:* If $\mathcal{A}$ is stable, system (6) is quadratically stable under the switching rule GD. Moreover there exists a number $\tau > 0$, such that the dwell time between any consecutive switches is no greater than $\tau$.

*Proof:* The quadratic stability of (9) is obvious because according to GD, $\dot{V} < -\epsilon_0 \boldsymbol{x}^T(t)\boldsymbol{x}(t)$ for all $t$. We only need to prove boundedness of the dwell time. Let $t_0$ and $t_1$ ($t_0 < t_1$) be any two consecutive switching times. For $t \in [t_0, t_1)$, define

$$\phi(t) = -\frac{\boldsymbol{x}^T(t)(\mathcal{A}_{s(t_0)}^T P + P\mathcal{A}_{s(t_0)})\boldsymbol{x}(t)}{\boldsymbol{x}^T(t)\boldsymbol{x}(t)}$$

$\phi(\cdot)$ is continuous and differentiable in $[t_0, t_1)$. Let $\mathcal{A}_{s(t_0)}^T P + P\mathcal{A}_{s(t_0)} = -Q_{s(t_0)}$, then for all $t \in [t_0, t_1)$

$$\dot{\phi}(t) = \frac{\boldsymbol{x}^T R_{s(t_0)}\boldsymbol{x} \cdot \boldsymbol{x}^T\boldsymbol{x} - \boldsymbol{x}^T Q_{s(t_0)}\boldsymbol{x} \cdot \boldsymbol{x}^T S_{s(t_0)}\boldsymbol{x}}{(\boldsymbol{x}^T\boldsymbol{x})^2} \quad (18)$$

where $R_{s(t_0)} = \mathcal{A}_{s(t_0)}^T Q_{s(t_0)} + Q_{s(t_0)}\mathcal{A}_{s(t_0)}$ and $S_{s(t_0)} = \mathcal{A}_{s(t_0)}^T + \mathcal{A}_{s(t_0)}$. Now let $\gamma_Q = \max_{i,j} r(Q_{ij})$, $\gamma_R = \max_{i,j} r(\mathcal{A}_{ij}^T Q_{ij} + Q_{ij}\mathcal{A}_{ij})$, and $\gamma_S = \max_{i,j} r(\mathcal{A}_{ij}^T + \mathcal{A}_{ij})$, where $Q_{ij} = -(\mathcal{A}_{ij}^T P + P\mathcal{A}_{ij})$, and for a square matrix $B$, $r(B)$ is its spectral radius, defined as $r(B) \triangleq \max_i(|\lambda_i(B)|)$. Now from (18), it is easy to verify that for all $t \in [t_0, t_1)$, $|\dot{\phi}(t)| \leq \gamma_R + \gamma_Q \cdot \gamma_S$. Hence $|\phi(t_1^-) - \phi(t_0)| \leq (\gamma_R + \gamma_Q \cdot \gamma_S)(t_1 - t_0)$, where $t_1^-$ denotes the instant right before the switch taking place at $t_1$. Notice that $\phi(t_0) \geq \epsilon^*$ because at the beginning of each switch, $s(t)$ is determined by (17). Also $\phi(t_1^-) = \epsilon_0$ according to the GD policy . Hence $|\phi(t_1^-) - \phi(t_0)| \geq \epsilon^* - \epsilon_0$, and

$$t_1 - t_0 \geq \frac{\epsilon^* - \epsilon_0}{\gamma_R + \gamma_Q \cdot \gamma_S} \quad (19)$$

■

## V. Simulations

Let the plant be the unstable batch reactor ([21], p.62)

$$\dot{\boldsymbol{x}} = \begin{bmatrix} 1.38 & -0.2077 & 6.715 & -5.676 \\ -0.5814 & -4.29 & 0 & 0.675 \\ 1.067 & 4.273 & -6.654 & 5.893 \\ 0.048 & 4.273 & 1.343 & -2.104 \end{bmatrix} \boldsymbol{x} + \begin{bmatrix} 0 & 0 \\ 5.67 & 0 \\ 1.136 & -3.146 \\ 1.136 & 0 \end{bmatrix} \boldsymbol{u} \quad ,$$

which has two inputs and four outputs (its states). The batch reactor is being controlled by a static feedback controller $K$ via a shared communication medium which has only one input channel and one output channel. The closed-loop NCS is equivalent to a switched system switching between eight possible linear dynamics $\dot{\boldsymbol{x}} = \mathcal{A}_{ij}\boldsymbol{x}$, ($i = 1, 2$, $j = 1, 2, 3, 4$). We first choose the $\alpha_{ij}$'s to be

$$[\alpha_{ij}] = \boldsymbol{\alpha}^\dagger = \begin{bmatrix} 1/6 & 1/6 & 1/6 & 1/6 \\ 1/12 & 1/12 & 1/12 & 1/12 \end{bmatrix}$$

and the desired eigenvalues of $\mathcal{A}$ (9) to be $[-5, -6, -4, -3]$. We solve the pole-placement problem (11) for $\mathcal{K}$, and then solve for $K$ from (12). The feedback gain $K$ corresponds to $\boldsymbol{\alpha}^\dagger$ is

$$K = K^\dagger = \begin{bmatrix} 0.5463 & -3.1950 & -0.8567 & -2.2001 \\ 23.0186 & 4.3389 & 10.4101 & -2.6616 \end{bmatrix}$$

We choose $Q = I$ and solve for $P$ from (10). Figure 3(a) is the evolution of the state $\boldsymbol{x}$ under the FD rule and $K^\dagger$. According to (16), $\epsilon^* = 3/4$. Figure 3(b) is the evolution of the state $\boldsymbol{x}$ under the GD switching rule and $K^\dagger$, where $\epsilon_0$ was chosen to be 0.1.

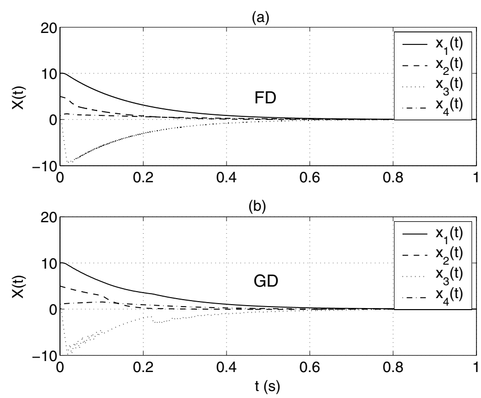


Fig. 3. Trajectories of $\boldsymbol{x}$ under the FD and GD rule ($K = K^\dagger$)

Next we show how the $\alpha_{ij}$'s act as medium access priorities under the WFD rule. Notice that the first row in $[\alpha_{ij}]$ corresponds to the priority of $u_1$, the second row corresponds to the priority of $u_2$. Hence the input $u_1$ is given higher priority than $u_2$ in $\boldsymbol{\alpha}^\dagger$. Now we choose a different set of $\alpha_{ij}$'s,

$$[\alpha_{ij}] = \boldsymbol{\alpha}^\ddagger = \begin{bmatrix} 1/12 & 1/12 & 1/12 & 1/12 \\ 1/6 & 1/6 & 1/6 & 1/6 \end{bmatrix}$$

where $u_2$ is given higher priority. For the same choice of $\mathcal{A}$, the feedback gain $K$ corresponding to $\boldsymbol{\alpha}^\ddagger$ is

$$K = K^\ddagger = \begin{bmatrix} 1.0927 & -6.3900 & -1.7135 & -4.4002 \\ 11.5093 & 2.1694 & 5.2050 & -1.3308] \end{bmatrix}$$

Note that the first row of $K^\ddagger$ (corresponding to the input $u_1$) is a half of the first row of $K^\dagger$, while the second row of $K^\ddagger$ (corresponding to $u_2$) is twice of the second row of $K^\dagger$. This tells us that if an input gets higher medium access priority, less control effort will be needed for that input (This also holds for the FD and GD rule).

We implement the WFD rule in the NCS for the two sets of priorities, $\boldsymbol{\alpha}^{\dagger}$ and $\boldsymbol{\alpha}^{\ddagger}$, with their corresponding feedback gains. Our simulation results show that the NCS is stable under either of the two sets. However, the input communication sequences that stabilize the NCS are very different. As shown in Fig. 4, when $u_1$ is given higher priority (Fig. 4(a)), most of the medium access time is given to the input $u_1$; when $u_2$ is given higher priority (Fig. 4(b)), most of the medium access time is given to the input $u_2$.

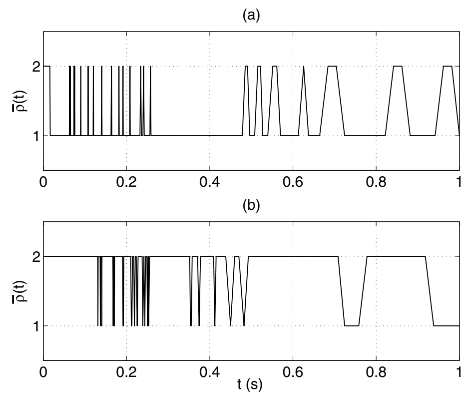


Fig. 4. The input communication sequence $\bar{\rho}(t)$ under the WFD rule. (a): using $\boldsymbol{\alpha}^{\dagger}$ and $K^{\dagger}$; (b): using $\boldsymbol{\alpha}^{\ddagger}$ and $K^{\ddagger}$

## VI. Conclusions and Future work

We have discussed the problem of jointly designing a feedback controller and medium access strategy for stabilizing a linear network-controlled system (NCS). In our NCS model, actuators or sensors that are not actively accessing the communication medium are effectively "ignored" by the plant and controller. In that setting, the complexity of the stabilization problem becomes quite manageable. We presented an algorithm for designing stabilizing feedback gains by solving a standard pole-placement problem. Using these gains, one can then choose from three feedback-based medium access scheduling policies (FD, WFD, and GD) to stabilize the system. The FD policy guarantees fastest decay of a quadratic Lyapunov function; the WFD policy allows one to affect the attention given to different inputs and outputs; the GD policy guarantees a minimum dwell time between any two consecutive switchings, to avoid chattering.

Here, we have assumed that state feedback was available. A straightforward extension is to address the output feedback case, where the plant output is $\boldsymbol{y}(t) = C\boldsymbol{x}(t)$, with $C$ a $p$-by-$n$ matrix ($p < n$). In this case, an observer may be used to reconstruct the state at either the plant or the controller side of the communication medium. We are exploring the construction of such observers under periodic and feedback-based communication.

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