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LMI-based H_∞ Fuzzy Equalizer Design for Discrete-time Nonlinear Channels

Sheng-Yi Lin, Te-Jen Su and Gwo-Jia Jong
Department of Electronic Engineering
National Kaohsiung University of Applied Sciences
Kaohsiung 807, Taiwan, ROC
Tel: (07) 3814526 ext. 5606
Fax: (07) 3811182
Email: sutj@cc.kuas.edu.tw

Abstract

This paper investigates the problems of nonlinear channel equalization based on LMI-fuzzy methodology. According to Takagi-Sugeno (T-S) fuzzy modeling concept, the discrete-time nonlinear channel can be constructed by the piecewise linear subsystems. The FIR fuzzy equalizer design for nonlinear channel is transformed into a standard linear matrix inequality (LMI) optimization problem, and the coefficients of the fuzzy equalizer are obtained by solving LMIs. Thus, the result is very simple and the computation is numerically tractable. Finally, simulation result is given to demonstrate the effectiveness of the proposed methodology.

Keywords: H_∞ ; FIR; Equalizer; LMI; T-S fuzzy model.

Ⅰ. Introduction

In modern digital communication systems, there have been several methods proposed to treat the problem of the channel equalization. The equalizer design problem via two-block H_∞ optimization technique is presented in [1]. The risk-sensitive FIR equalizer has been formulated as the constrained analytic centering problem, which is another type of convex problem [2,3]. The maximum likelihood Viterbi algorithm and the conventional decision feedback equalizer are used for the Bayesian decision feedback equalizer [4]. However, most of them mentioned above are firmly based on linear filter algorithms. In general, equalization is a nonlinear classification problem and it is more difficult to model and control the nonlinear systems.

Recently, fuzzy techniques have been applied in various fields such as control systems, communication systems, signal processing, and so on. Among various fuzzy modeling themes, Takagi-Sugeno (T-S) model based fuzzy control approach has been rapidly and successfully developing in nonlinear control frameworks [5]. The T-S fuzzy model offers an effective way to represent the nonlinear systems with a set of fuzzy If-Then rules, each of the rules can be represented as a local linear state equation and the overall fuzzy system is achieved by fuzzy “blending” of the these linear state equation.

We propose an LMI-fuzzy based approach to design the FIR fuzzy equalizer for nonlinear discrete-time channels from H_∞ perspective, and employ the state-space description in conjunction with bounded real lemma [3,6] to calculate the optimal γ value and the coefficients of the FIR fuzzy equalizer, where the γ value, a tolerance level, can be regarded as an indication of the quality of the filter. Moreover, the effect of the γ value for different equalizer lengths is discussed. The FIR fuzzy equalizer design presented in this paper involves only solving a set of LMIs. Thus, the result is very simple and the computation is numerically tractable.

Two primary contributions of this paper are (i) deal with the nonlinear system by T-S fuzzy model and (ii) eliminating the effect of the external disturbance as much as possibly.

The notation used in this paper is fairly standard. $M > 0$ ($M < 0$) means that the matrix M is symmetric and positive (negative) definite, M^T represents transpose of M. I_k stands for the identity matrix with dimension *k*. $\|\cdot\|_\infty$ denotes the infinity norm of a discrete-time stable proper transfer function matrix. $\prod$ stands for minimum operation.

The rest of this paper is organized as follows. In Section II, the system model is described and a T-S fuzzy model is used to construct the nonlinear channel. Furthermore, the state-space representation for the error transfer function is presented. Converting the problems to the task of finding an optimal γ value by solving LMIs is described in Section III. In Section IV, a numerical example is proposed and simulation results are demonstrated as well. Finally, conclusion is drawn in Section V.

Ⅱ. System model description

The general structure of the error transfer function corresponding to the nonlinear channel equalization problem is illustrated in Fig.1 [7], where b_i is the transmitted digital information sequence, v_i is the unknown noise, e_i is the error between the equalizer output and the delay of the desired transmitted sequence, $H_{NL}(z)$ is the discrete equivalent of the nonlinear time-invariant communication channel, $K(z)$ is the equalizer to be designed and $L(z) = z^{-d}$ is the delay. The discrete data sequence b_i passes through the nonlinear time-invariant channel $H_{NL}(z)$, the observation sequence y_i is then formed by the addition of an unknown measurement disturbance v_i with the output of the nonlinear communication channel $H_{NL}(z)$.

The nonlinear channel $H_{NL}(z)$, contaminated with the noise at the output, is described by the following piecewise linear T-S fuzzy model：

Model Rule j:

IF z_1 is M_{j1} and … and z_p is M_{jp},

THEN $s_{i+1} = A_{cj}s_i + B_{cj}u_i$, (1)

$y_i = C_{cj}s_i + D_{cj}u_i$. (2)

$j = 1,2,\cdots,L.$

Where M_{jp} are the fuzzy sets and L is the number of model rules; s_i is the state vector, $u_i = [b_i \quad v_i]^T$ is the input vector and y_i is the measured output; A_{cj}, B_{cj}, C_{cj} and D_{cj} are constant matrices; z_1 ,…, z_p are premise variables that may be functions of the state variables, external disturbances, and/or time.

The final outputs of the fuzzy systems (1) and (2) are inferred as follows:

$$s_{i+1} = \frac{\sum_{j=1}^{L} w_j(z)\{A_{cj}s_i + B_{cj}u_i\}}{\sum_{j=1}^{L} w_j(z)}$$

$$= \sum_{j=1}^{L} h_j(z)\{A_{cj}s_i + B_{cj}u_i\} = A_c s_i + B_c u_i, \quad (3)$$

$$y_i = \frac{\sum_{j=1}^{L} w_j(z)\{C_{cj}s_i + D_{cj}u_i\}}{\sum_{j=1}^{L} w_j(z)}$$

$$= \sum_{j=1}^{L} h_j(z)\{C_{cj}s_i + D_{cj}u_i\} = C_c s_i + D_c u_i. \quad (4)$$

where $z = [z_1 \quad z_2 \quad \cdots \quad z_p]$, $w_j(z) = \prod_{k=1}^{p} M_{jk}(z_k)$,

$$h_j(z) = \frac{w_j(z)}{\sum_{j=1}^{L} w_j(z)},$$

$$A_c = \frac{\sum_{j=1}^{L} w_j(z)A_{cj}}{\sum_{j=1}^{L} w_j(z)}, \qquad B_c = \frac{\sum_{j=1}^{L} w_j(z)B_{cj}}{\sum_{j=1}^{L} w_j(z)},$$

$$C_c = \frac{\sum_{j=1}^{L} w_j(z)C_{cj}}{\sum_{j=1}^{L} w_j(z)} \text{ and } D_c = \frac{\sum_{j=1}^{L} w_j(z)D_{cj}}{\sum_{j=1}^{L} w_j(z)}. \quad (5)$$

The term $M_{jk}(z_k)$ is the grade of membership function of z_k in M_{jk}. It is easy to find that

$w_j(z) \geq 0, \qquad j = 1,2,\cdots,L$

and $\sum_{j=1}^{L} w_j(z) > 0$. (6)

Therefore, $h_j(z) \geq 0$ for $j = 1,2,\cdots,L$ and $\sum_{j=1}^{L} h_j(z) = 1$. (7)

The delay operator $L(z)= z^{-d}$ is represented by

$\ell_{i+1} = A_d \ell_i + B_d u_i$, (8)

$z_i = C_d \ell_i$. (9)

where $d > 0$, ℓ_i is the state vector, $u_i = [b_i \quad v_i]^T$ is the input vector and $A_d = \begin{bmatrix} 0 & 0 \\ I_{(d-1)\cdot(d-1)} & 0 \end{bmatrix}$, $B_d = \begin{bmatrix} I & 0 \\ 0 & 0 \end{bmatrix}$, $C_d = [0 \quad I]$.

The state-space model for the FIR equalizer $K(z) = k_0 + k_1 z^{-1} + \ldots + k_{R-1} z^{-(R-1)}$ with order R-1 has the following state-space structure:

Model Rule j:

IF z_1 is M_{j1} and … and z_p is M_{jp},

THEN $\omega_{i+1} = A_{ej}\omega_i + B_{ej}y_i$, (10)

$\hat{z}_i = C_{ej}\omega_i + D_{ej}y_i$. (11)

$j = 1,2,\cdots,L.$

where M_{jp} are the fuzzy sets and L is the number of model rules; ω_i is the state vector, y_i is the measured output, $\hat{z}_i$ is the output of the fuzzy equalizer, $z_1,\ldots,z_p$ are premise variables that may be functions of the state variables, external disturbances, and/or time. $A_{ej} = \begin{bmatrix} 0 & 0 \\ I_{(R-2)\cdot(R-2)} & 0 \end{bmatrix}$, $B_{ej} = \begin{bmatrix} I \\ 0_{(R-2)\cdot 1} \end{bmatrix}$, $C_{ej} = [k_{1j} \ldots\ldots\ldots k_{(R-1)j}]$, $D_{ej} = k_{0j}$ and $k_{0j}, k_{1j}, \ldots, k_{(R-1)j}$ are the coefficients of the equalizer which to be designed.

The final outputs of the fuzzy equalizer are inferred as follows:

$$\omega_{i+1}=\frac{\sum_{j=1}^{L}w_j(z)\{A_{ej}\omega_i+B_{ej}y_i\}}{\sum_{j=1}^{L}w_j(z)}$$

$$=\sum_{j=1}^{L}h_j(z)\{A_{ej}\omega_i+B_{ej}y_i\}=A_e\omega_i+B_ey_i, \quad (12)$$

$$\hat{z}_i=\frac{\sum_{j=1}^{L}w_j(z)\{C_{ej}\omega_i+D_{ej}y_i\}}{\sum_{j=1}^{L}w_j(z)}$$

$$=\sum_{j=1}^{L}h_j(z)\{C_{ej}\omega_i+D_{ej}y_i\}=C_e\omega_i+D_ey_i. \quad (13)$$

where $z=\lfloor z_1 \quad z_2 \quad \cdots \quad z_p \rfloor$, $w_j(z)=\prod_{k=1}^{p}M_{jk}(z_k)$,

$$h_j(z)=\frac{w_j(z)}{\sum_{j=1}^{L}w_j(z)}, \qquad A_e=\frac{\sum_{j=1}^{L}w_j(z)A_{ej}}{\sum_{j=1}^{L}w_j(z)},$$

$$B_e=\frac{\sum_{j=1}^{L}w_j(z)B_{ej}}{\sum_{j=1}^{L}w_j(z)}, \qquad C_e=\frac{\sum_{j=1}^{L}w_j(z)C_{ej}}{\sum_{j=1}^{L}w_j(z)} \qquad \text{and}$$

$$D_e=\frac{\sum_{j=1}^{L}w_j(z)D_{ej}}{\sum_{j=1}^{L}w_j(z)}. \quad (14)$$

From the above preliminary, we can obtain the state-space model for the error transfer function with mapping input disturbances u_i to the error $e_i=z_i-\hat{z}_i$ as follows:

$$x_{i+1}=Ax_i+Bu_i, \quad (15)$$

$$e_i=Cx_i+Du_i. \quad (16)$$

where $d>0$

and $A=\begin{bmatrix} A_c & 0 & 0 \\ 0 & A_d & 0 \\ B_eC_c & 0 & A_e \end{bmatrix}$, $B=\begin{bmatrix} B_c & B_d & B_eD_c \end{bmatrix}^T$,

$C=\begin{bmatrix} -D_eC_c & C_d & -C_e \end{bmatrix}$, $D=\begin{bmatrix} -D_eD_c \end{bmatrix}$,

$x_i=[s_i \quad \ell_i \quad \omega_i]^T, u_i=\begin{bmatrix} b_i & v_i \end{bmatrix}^T$.

From eqns. (15) and (16), the equalizer coefficients are only included in the matrices C and D. Therefore, the representation of the error transfer function can be specified as

$$T(z)=C(zI-A)^{-1}B+D. \quad (17)$$

Denoting the disturbance attenuation value γ, the obtained solution guarantees a disturbance rejection capability, which is optimal in the sense of H_∞ norm:

$$\|T(z)\|_\infty<\gamma \quad (18)$$

Consequently, we will show how to cast the H_∞ control problem into LMI framework in the next section

Ⅲ. Problem formulation

Almost all practical systems are subject to external disturbances that can in some situations degrade system performance if their effects are not considered during the design phase. There are many ways to eliminate the effects of the external disturbances in the current literature. Recently, interest has been devoted to the filtering problem with an H_∞ performance criterion. This approach consists of designing a controller that minimizes the H_∞ norm of the transfer function between the controlled output and the external disturbance, or at least guarantees that the H_∞ norm will not exceed a given level $\gamma>0$.

In the previous section, the nonlinear channel has been represented as T-S fuzzy model, based on the fuzzy model; the state equation of the overall system can be obtained. Moreover, the H_∞ equalizer design problem can be formulated as a standard LMI optimization problem via the bounded real lemma. This lemma establishes the equivalence between the following statements:

Theorem 3.1. [3,6] Consider a discrete-time transfer function $T(z)$ of realization $T(z)=C(zI-A)^{-1}B+D$. The following statements are equivalent:

(i). $\|C(zI-A)^{-1}B+D\|_\infty<\gamma$ and A is stable in the discrete-time sense.

(ii). There exists a solution $P>0$ to the LMI:

$$\begin{bmatrix} A^TPA-P & A^TPB & C^T \\ B^TPA & B^TPB-\gamma^2I & D^T \\ C & D & -I \end{bmatrix}<0. \quad (19)$$

Proof: Omitted due to space limit.

Based on Theorem 3.1, both of the internal stability and the H_∞-norm constraint are equivalent to the feasibility of the above matrix inequality (19) for some symmetric matrix $P>0$. Because of the unknown matrix is P and the fuzzy equalizer parameters entering in matrices C and D, the inequality (19) is a standard LMI and the H_∞ constraints can be expressed as a single matrix inequality via the bounded real lemma. To obtain a better performance, the γ value can be reduced to the minimum possible value such that LMI (19) is satisfied. Therefore, the design procedure is summarized as follows.

***Design Procedures*:**

Step 1) Identify the premise variables relevant to the model.
Step 2) Chose the structure of the membership functions.
Step 3) Determine the number of fuzzy rules.
Step 4) Construct the channel fuzzy model.
Step 5) Defuzzification of the channel fuzzy model.
Step 6) Construct the equalizer fuzzy model.
Step 7) Defuzzification of the equalizer fuzzy model.
Step 8) Decide the lengths of the equalizer and delay.
Step 9) Derive the error transfer function of the overall system.
Step 10) Solve the LMI in (19) to obtain the optimal γ value and the coefficients of the fuzzy equalizer.

Ⅳ. An example

In this section, we will illustrate an example to demonstrate the effectiveness of the proposed methodology. It is assumed that the channel output is observed in additive white Gaussian noise with zero mean and power spectral density $\sigma^2 = 1$. This channel has two non-minimum phase zeros, the delay of this system is chosen as $d = 2$. The following discrete-time nonlinear channel model is considered:

$$\begin{aligned} y(k) &= s(k) + 0.33562 s(k-1) + 0.1 s^2(k-1) \\ &+ 4.6276 s(k-2) + 0.2 s(k-1) s^2(k-2) \\ &-0.14487 s(k-3) + 1.6837 s(k-4) \end{aligned} \quad (20)$$

It is assumed that $s_1, s_2 \in [-1, \ \ 1]$, for the nonlinear terms, define $z_1(k) = s_1$ and $z_2(k) = s_1 \times s_2$. To minimize the design effort and complexity, four rules are used to construct the nonlinear system (20), and the membership functions of $z_1(k)$ and $z_2(k)$ are simply defined using triangular types in Fig.2 and Fig.3. By using these fuzzy sets and rules, the nonlinear system is represented by the following piecewise linear T-S fuzzy model.

Rule 1:

If $z_1(k)$ is "Positive" and $z_2(k)$ is "Big"

Then $s_{i+1} = A_{c1} s_i + B_{c1} u_i$

$$y_i = C_{c1} s_i + D_{c1} u_i \quad (21)$$

Rule 2:

If $z_1(k)$ is "Positive" and $z_2(k)$ is "Small"

Then $s_{i+1} = A_{c2} s_i + B_{c2} u_i$

$$y_i = C_{c2} s_i + D_{c2} u_i \quad (22)$$

Rule 3:

If $z_1(k)$ is "Negative" and $z_2(k)$ is "Big"

Then $s_{i+1} = A_{c3} s_i + B_{c3} u_i$

$$y_i = C_{c3} s_i + D_{c3} u_i \quad (23)$$

Rule 4:

If $z_1(k)$ is "Negative" and $z_2(k)$ is "Small"

Then $s_{i+1} = A_{c4} s_i + B_{c4} u_i$

$$y_i = C_{c4} s_i + D_{c4} u_i \quad (24)$$

Where $A_{c1} = A_{c2} = A_{c3} = A_{c4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,

$B_{c1} = B_{c2} = B_{c3} = B_{c4} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix}$,

$C_{c1} = [0.43562 \quad 4.8276 \quad -0.14487 \quad 1.6837]$,
$C_{c2} = [0.43562 \quad 4.4276 \quad -0.14487 \quad 1.6837]$,
$C_{c3} = [0.23562 \quad 4.8276 \quad -0.14487 \quad 1.6837]$,
$C_{c4} = [0.23562 \quad 4.4276 \quad -0.14487 \quad 1.6837]$,
$D_{c1} = D_{c2} = D_{c3} = D_{c4} = [1 \quad 1]$.

Consider the operate point at $s_1 = -0.2$ and $s_2 = 0.2$, the defuzzification of the channel is carried out as

$$s_{i+1} = \frac{\sum_{j=1}^{4} w_j(z(k)) \{A_{cj} s_i + B_{cj} u_i\}}{\sum_{j=1}^{4} w_j(z(k))} = A_c s_i + B_c u_i, \quad (25)$$

Where $A_c = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$, and $B_c = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$.

$$y_i = \frac{\sum_{j=1}^{4} w_j(z(k)) \{C_{cj} s_i + D_{cj} u_i\}}{\sum_{j=1}^{4} w_j(z(k))} = C_c s_i + D_c u_i \quad (26)$$

Where $C_c = [0.31562 \quad 4.6276 \quad -0.14487 \quad 1.6837]$, $D_c = [1 \quad 1]$.

Furthermore, the FIR fuzzy equalizer with order five is represented by the following piecewise linear T-S fuzzy model:

Rule 1:

If $z_1(k)$ is "Positive" and $z_2(k)$ is "Big"

Then $\omega_{i+1} = A_{e1}\omega_i + B_{e1}y_i$

$$\hat{z}_i = C_{e1}\omega_i + D_{e1}y_i \tag{27}$$

Rule 2:

If $z_1(k)$ is "Positive" and $z_2(k)$ is "Small"

Then $\omega_{i+1} = A_{e2}\omega_i + B_{e2}y_i$

$$\hat{z}_i = C_{e2}\omega_i + D_{e2}y_i \tag{28}$$

Rule 3:

If $z_1(k)$ is "Negative" and $z_2(k)$ is "Big"

Then $\omega_{i+1} = A_{e3}\omega_i + B_{e3}y_i$

$$\hat{z}_i = C_{e3}\omega_i + D_{e3}y_i \tag{29}$$

Rule 4:

If $z_1(k)$ is "Negative" and $z_2(k)$ is "Small"

Then $\omega_{i+1} = A_{e4}\omega_i + B_{e4}y_i$

$$\hat{z}_i = C_{e4}\omega_i + D_{e4}y_i \tag{30}$$

Where $A_{e1} = A_{e2} = A_{e3} = A_{e4} = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \end{bmatrix}$,

$B_{e1} = B_{e2} = B_{e3} = B_{e4} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \end{bmatrix}$,

$C_{e1} = [k_{11} \quad k_{21} \quad k_{31} \quad k_{41}]$, $C_{e2} = [k_{12} \quad k_{22} \quad k_{32} \quad k_{42}]$,

$C_{e3} = [k_{13} \quad k_{23} \quad k_{33} \quad k_{43}]$, $C_{e4} = [k_{14} \quad k_{24} \quad k_{34} \quad k_{44}]$,

$D_{e1} = k_{01}$, $D_{e2} = k_{02}$, $D_{e3} = k_{03}$ and $D_{e4} = k_{04}$.

By applying the procedures in the previous section, the optimal γ value can be derived with 0.4548 and the coefficients of the fuzzy equalizer are obtained in Table 1. For compare the performance with different length of the fuzzy equalizer, the length with three and four of fuzzy equalizer are demonstrated, the optimal γ values are obtained with 0.4919 and 0.4916, respectively. The coefficients of the defuzzification are indicated in Table 2 as well. The bit error rate (BER) simulation is also introduced to provide the true picture of performance of the system, where the BER is defined as the bit error probability with respective to SNR, and herein is relative to the transmitted signal b_i. The BER comparison for the length of fuzzy equalizer with five can be seen in Fig. 4.

From the demonstrated example, better performance will be obtained with increasing the length of the equalizer. The result in Fig.4 shows the better BER performance of the proposed method for T-S fuzzy model. This implies that the T-S fuzzy model can represent a highly nonlinear function relation and the designed equalizer can improve the BER performance.

V. Conclusion

In this paper, the LMI-based nonlinear channel equalization has been studied. The FIR fuzzy equalizer coefficients can be easily obtained by solving a set of LMIs only, it is simple and numerically tractable. A given numerical example has demonstrated the effectiveness of the proposed methodology. The results show that the longer equalizer outperforms the shorter equalizer and the nonlinear channel can be constructed as T-S fuzzy linear model by using a reasonable number of fuzzy rules. In general, the designed equalizer LMI-based fuzzy approach shows the facts of improving BER performance.

References

[1] S. Chen, B. Mulgrew and S. McLaughlin, "Adaptive Bayesian equalizer with decision feedback," IEEE Trans. Signal Processing, vol. 41, issue. 9, pp. 2918-2927, Sept. 1993.

[2] A. T. Erdogan, B. Hassibi and T. Kailath, "Linear H_∞ equalization of communication channels," IEEE Trans. Signal Process, pp. 3227-3231, 2000.

[3] K. -L. Hsiung and L. Lee, "Lyapunov inequality and bounded real lemma for discrete-time descriptor systems," IEEE Proc. Control Theory Appl., vol. 146, no. 4, pp. 327-331, July 1999.

[4] S. C. Peng, "An equalizer design for nonminimum phase channel via a two block H_∞ optimization technique," Signal Processing Elsevier Science, vol. 51, pp.1-13, 1996.

[5] T. Takagi and M. Sugeno, " Fuzzy identification of systems and its applications to modeling and control," IEEE Trans. Syst., Man, Cybern., vol. SMC-15, pp.116-132, 1985.

[6] C. E. de Souza and L. Xie, "On the discrete-time bounded real lemma with application in the characterization of static state feedback H-infin controllers," Systems & Control Letters 18, pp. 61-71, 1992.

[7] A. T. Erdogan, B. Hassibi and T. Kailath, "FIR H_∞ equalization," Signal Processing Elsevier Science, pp. 907-917, 2000.

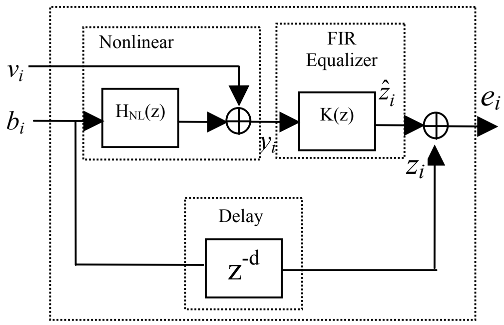

Fig.1. System model

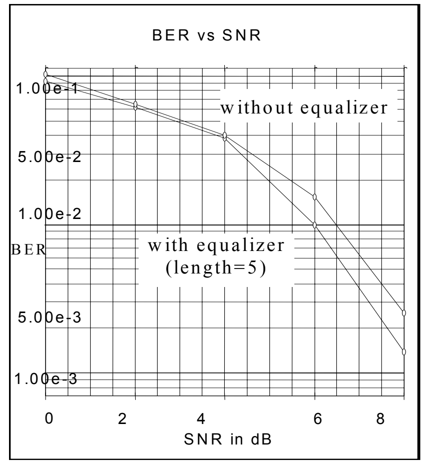

Fig. 4 BER comparison

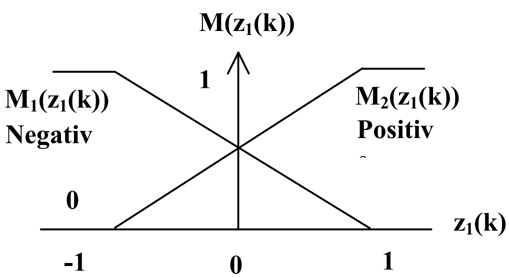

Fig.2 Fuzzy set of z_1(k)

Table 1: The coefficients of fuzzy equalizer

Coefficients	k_{0j}	k_{1j}	k_{2j}	k_{3j}	k_{4j}
Rule1 (j=1)	25784	-2205	18092	-5783	11993
Rule2 (j=2)	-46488	15276	-17217	-4543	-18965
Rule3 (j=3)	-34937	-7404	-27506	7768	2808
Rule4 (j=4)	47374	-1993	25734	-584	2671

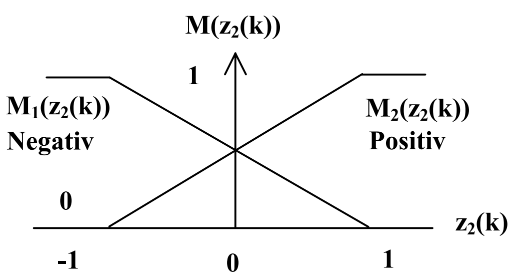

Fig.3 Fuzzy set of z_2(k)

Table 2: *r* values and equalizer coefficients

Equalizer length	3	4	5
optimal *r*	0.4919	0.4916	0.4548
Equalizer coefficients	k0 = 0.2267 k1 = 0.0363 k2 =-0.0934	k0 = 0.2240 k1 = 0.0379 k2 =-0.0937 k3 =-0.0019	k0 = 0.2324 k1 = 0.0354 k2 =-0.0896 k3 =-0.0357 k4 = 0.0460