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# Improving the Rotational and Transient Performance of Magnetic Bearings by the $\mathcal{H}_\infty$ DIA Control

Toru Namerikawa and Hiroki Seto

***Abstract*— This paper deals with an application of $\mathcal{H}_\infty$ control attenuating initial-state uncertainties to the magnetic bearing and examines the $\mathcal{H}_\infty$ control problem, which treats a mixed Disturbance and an Initial state uncertainty Attenuation (DIA) control. The mixed $\mathcal{H}_\infty$ DIA problem supplies $\mathcal{H}_\infty$ controls with good transients and assures $\mathcal{H}_\infty$ controls of robustness against initial-state uncertainty.**

**On the other hand, active magnetic bearings allow contract-free suspension of rotors and they are used for various industrial purposes. We derive a mathematical model of the magnetic bearing which has complicated rotor dynamics and nonlinear magnetic property.**

**Then we apply this proposed $\mathcal{H}_\infty$ DIA control for the magnetic bearing, and design a robust $\mathcal{H}_\infty$ controller both for exogenous disturbances and for initial state uncertainties of the plant.**

**Experimental results show that the proposed robust control approach is effective for improving rotational performance, transient response and robust performance.**

## I. INTRODUCTION

$\mathcal{H}_\infty$ control problem has been proven an effective robust control design methodology and it has been applied to a variety of industrial products. On the other hand, recent precision control industries and manufacturing technologies require not only robust stability of the control systems but also transient performance for reference signals. One of the major approach for this problem is a two-degree of freedom robust control, but this approach generally has a coupling problem of feedforward and feedback control design. An $H_2/H_\infty$ control approach[1] seems to be effective, but it is not easy to design such controller for MIMO complex systems.

A mixed Disturbance and an Initial-state uncertainty Attenuation (DIA) control is expected to provide a good transient characteristic as compared with conventional $\mathcal{H}_\infty$ control[2], [3]. Recently, hybrid/switching control are actively studied, this method might be one of the most reasonable approach to implement them. In this paper, we apply the proposed $\mathcal{H}_\infty$ DIA control to the magnetic bearing, and designed a robust $\mathcal{H}_\infty$ controller both for exogenous disturbances and for initial state uncertainties of the plant.

Active magnetic bearings are used to support and maneuver a levitated object, often rotating, via magnetic force[4], [5]. Because magnetic bearings support rotors without physical contacts, they have many advantages, e.g. frictionless operation, less frictional wear, low vibration, quietness, high rotational speed, usefulness in special environments, and low maintenance. On the other hand, disadvantages of magnetic bearings include the expense of the equipment, the necessity of countermeasures in case of a power failure, and instability in their control systems. However, there are many real-world applications which utilize the advantages outlined above. Examples of these applications are : turbo-molecular pumps, high-speed spindles for machine tools, flywheels for energy storage[4], reaction wheels for artificial satellites, gas turbine engines, blood pumps[6], and fluid pumps, etc. [5], [7].

Toru Namerikawa and Hiroki Seto are with Department of Mechanical Engineering, Nagaoka University of Technology, 1603-1 Kamitomioka, Nagaoka 940-2188 Japan toru@nagaokaut.ac.jp

In this paper, we apply the $\mathcal{H}_\infty$ control attenuating initial-state uncertainties to the magnetic bearing. First we derive a mathematical model of magnetic bearing systems considering rotor dynamics and nonlinearities of magnetic force. Then we set the generalized plant which contains design parameter for uncertainty and control performance.

Experimental results show that the proposed robust control approach is effective for a mixed disturbance and an initial-state uncertainty attenuation and for improving rotational performance, transient response and robust performance.

## II. $\mathcal{H}_\infty$ DIA CONTROL

Consider the linear time-invariant system which is defined on the time interval $[0, \infty)$.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u, \quad x(0) = x_0 \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \end{aligned} \tag{1}$$

where $x \in R^n$ is the state and $x_0 = x(0)$ is the initial state; $u \in R^r$ is the control input; $y \in R^m$ is the observed output; $z \in R^q$ is the controlled output; $w \in R^p$ is the disturbance. The disturbance $w(t)$ is a square integrable function defined on $[0, \infty)$. $A$, $B_1$, $B_2$, $C_1$, $C_2$, $D_{12}$ and $D_{21}$ are constant matrices of appropriate dimensions and satisfies that

- $(A, B_1)$ is stabilizable and $(A, C_1)$ is detectable
- $(A, B_2)$ is controllable and $(A, C_2)$ is observable
- $D_{12}^T D_{12} \in R^{r\times r}$ is nonsingular
- $D_{21} D_{21}^T \in R^{m\times m}$ is nonsingular

For system (1), every admissible control $u(t)$ is given by linear time-invariant system of the form

$$\begin{aligned} \dot{\zeta} &= A_k \zeta + B_k y \\ u &= C_k \zeta + D_k y \quad \zeta(0) = 0 \end{aligned} \tag{2}$$

which makes the closed-loop system given internally stable, where $\zeta(t)$ is the state of the controller of a finite dimension;

$A_k$, $B_k$, $C_k$ and $D_k$ are constant matrices of appropriate dimensions. For the system and the class of admissible controls described above, consider a mixed-attenuation problem state as below.

*Problem 1:* $\mathcal{H}_\infty$ **DIA control problem**
Find an admissible control attenuating disturbances and initial state uncertainties in the way that, for given $N > 0$, $z$ satisfies

$$\|z\|_2^2 < \|w\|_2^2 + x_0^T N^{-1} x_0 \tag{3}$$

for all $w \in L^2[0, \infty)$ and all $x_0 \in R^n$, s.t., $(w, x_0) \neq 0$. Such an admissible control is called the **D**isturbance and **I**nitial state uncertainty **A**ttenuation (DIA) control.

The existence condition of the $\mathcal{H}_\infty$DIA controller is given by the following theorem.

*Theorem 1:* Assume that there exists a positive definite constant matrix $X > 0$ and $Y > 0$ such that

1) $$\begin{bmatrix} \begin{matrix}(A - B_2(D_{12}^T D_{12})^{-1} \\ \times D_{12}^T C_1)X \\ +X(A - B_2 \\ \times (D_{12}^T D_{12})^{-1} D_{12}^T C_1)^T \\ -B_2(D_{12}^T D_{12})^{-1} B_2^T \\ +B_1 B_1^T\end{matrix} & X C_1^T \\ C_1 X & \begin{matrix} D_{12}(D_{12}^T D_{12})^{-1} D_{12}^T \\ -I \end{matrix} \end{bmatrix} < 0$$

2) $$\begin{bmatrix} \begin{matrix} Y(A - B_1 D_{21}^T \\ \times (D_{21} D_{21}^T)^{-1} C_2) \\ (A - B_1 D_{21}^T \\ \times (D_{21} D_{21}^T)^{-1} C_2)^T Y \\ -C_2^T (D_{21} D_{21}^T)^{-1} C_2 \\ +C_1^T C_1 \end{matrix} & Y B_1 \\ B_1^T Y & \begin{matrix} D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} \\ -I \end{matrix} \end{bmatrix} < 0$$

3) $$\begin{bmatrix} X & I \\ I & Y \end{bmatrix} > 0$$

Then, the $\mathcal{H}_\infty$ controller is a DIA control if and only if the following condition 4) is satisfied.

4) $\quad Z^{-1} + N^{-1} - Y > 0$

where $Z$ is the maximal solution of the Linear Matrix Inequality

$$\begin{aligned} &(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\quad +(B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) Y) Z \\ &+Z(A - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} C_2 \\ &\quad +(B_1 B_1^T - B_1 D_{21}^T (D_{21} D_{21}^T)^{-1} D_{21} B_1^T) Y)^T \\ &-(B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1} (C_2 Y^{-1} + D_{21} B_1^T) L)^T \\ &\quad \times (B_1^T - D_{21}^T (D_{21} D_{21}^T)^{-1} (C_2 Y^{-1} + D_{21} B_1^T) L) \\ &< 0 \end{aligned}$$

with $L := XY(XY - I)^{-1}$.

## III. SYSTEM DESCRIPTION AND MODELING

The experimental setup of the magnetic suspension system[8] is shown in Fig.1 and rotor coordinate is defined in Fig.2.

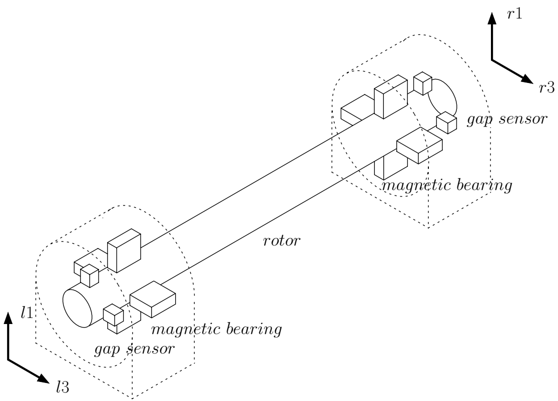


Fig. 1. Magnetic Bearing

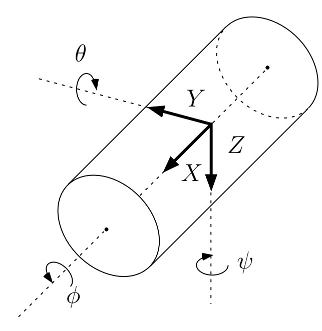


Fig. 2. Rotor

The controlled plant is a 4-axis controlled type active magnetic bearing with symmetrical structure. The axial motion is not controlled actively. The electromagnets are located in the horizontal and the vertical direction of both sides of the rotor. Moreover, hall-device-type gap sensors are located in the both sides of the vertical and horizontal direction. The physical model parameters of the system are given in Table I.

The equation of the motion of the rotor in $Y$ and $Z$ directions in Fig.2 has been derived as follows[5].

$$\begin{aligned} m\ddot{y}_s &= -f_{l3} - v_{ml3} - f_{r3} - v_{mr3} && (4) \\ m\ddot{z}_s &= mg - f_{l1} - v_{ml1} - f_{r1} - v_{mr1} && (5) \\ J_y\ddot{\theta} &= -J_x p\dot{\psi} + l_m(f_{l1} + v_{ml1} - f_{r1} - v_{mr1}) && (6) \\ J_y\ddot{\psi} &= -J_x p\dot{\theta} + l_m(-f_{l3} - v_{ml3} + f_{r3} + v_{mr3}) && (7) \end{aligned}$$

where $y_s(t)$ and $z_s(t)$ are displacements of $Y$ direction and $Z$ direction respectively; $\theta(t)$ and $\psi(t)$ are angles about $Y$ direction and $Z$ direction respectively; $m$ is mass of the rotor; $g$ is gravity; $l_m$ is distance between center and electromagnet; $J_x$ and $J_y$ are Moments of Inertia about $X$ axis and $Y$ axis respectively; $p$ is rotation rate of the rotor;

$f_j$s are electromagnetic force; and $v_{mj}$s are exogenous disturbance. Here the subscript '$j$' shows the each four directions:$\{l1,\ r1,\ l3,\ r3\}$ in Fig.1.

The position variables $y_s$ and $z_s$ and the rotational variables $\theta$ and $\psi$ can be transformed by using gap lengths: $\{g_{l1}, g_{r1}, g_{l3}, g_{r3}\}$ which are small deviations from the equilibrium point as follows.

$$y_s = -(g_{l3} + g_{r3})/2 \tag{8}$$

$$z_s = -(g_{l1} + g_{r1})/2 \tag{9}$$

$$\theta = (g_{l1} - g_{r1})/2l_m \tag{10}$$

$$\psi = (-g_{l3} + g_{r3})/2l_m \tag{11}$$

Next, attractive force of electromagnets is given as followed.

$$f_j = k\frac{(i_j + 0.5)^2}{(g_j - 0.0004)^2} - k\frac{(i_j - 0.5)^2}{(g_j + 0.0004)^2} \tag{12}$$

The electric circuit equations are given as followed.

$$L\frac{di_j(t)}{dt} + R(I_j + i_j(t)) = E_j + e_j(t) + v_{Lj}(t) \tag{13}$$

where $i_j(t)$ is a deviation form steady current; $e_j(t)$ is a deviation form steady voltage; $v_{Lj}$ is noise.

The sensors provide the information for the gap lengths $g_j(t)$. Hence the measurement equations can be written as

$$y_j(t) = g_j(t) + w_j \tag{14}$$

where $w_j(t)$ represents the sensor noise as well as the model uncertainties. Thus, summing up the above results (4)-(14), the state-space equations for the system are

TABLE I

MODEL PARAMETER

| Parameter | Symbol | Value |
|---|---|---|
| Mass of the Rotor | $m$ | 0.248[kg] |
| Length of the Rotor | $L_R$ | 0.269[m] |
| Distance between Center and Electromagnet | $l_m$ | 0.1105[m] |
| Moment of Inertia about $X$ | $J_x$ | $5.05 \cdot 10^{-6}$ [kgm$^2$] |
| Moment of Inertia about $Y$ | $J_y$ | $1.59 \cdot 10^{-3}$ [kgm$^2$] |
| Steady Gap | $G$ | $0.4 \times 10^{-3}$[m] |
| Coefficients of $f_j(t)$ | $k$ | $2.8 \times 10^{-7}$ |
| steady Current(vertical) | $I_{l1}, I_{r1}$ | 0.1425[A] |
| steady Current(horizontal) | $I_{l3}, I_{r3}$ | 0[A] |
| Resistance | $R$ | 4[Ω] |
| Inductance | $L$ | $8.8 \times 10^{-4}$[H] |
| Steady Voltage(vertical) | $E_{l1}, E_{r1}$ | 0.57[V] |
| Steady Voltage(horizontal) | $E_{l3}, E_{r3}$ | 0[V] |

$$\begin{aligned}
\begin{bmatrix} \dot{x}_v \\ \dot{x}_h \end{bmatrix} &= \begin{bmatrix} A_v & pA_{vh} \\ -pA_{vh} & A_h \end{bmatrix}\begin{bmatrix} x_v \\ x_h \end{bmatrix} + \begin{bmatrix} B_v & 0 \\ 0 & B_h \end{bmatrix}\begin{bmatrix} u_v \\ u_h \end{bmatrix} + \begin{bmatrix} D_v & 0 \\ 0 & D_h \end{bmatrix}\begin{bmatrix} v_v \\ v_h \end{bmatrix} \\
\begin{bmatrix} y_v \\ y_h \end{bmatrix} &= \begin{bmatrix} C_v & 0 \\ 0 & C_h \end{bmatrix}\begin{bmatrix} x_v \\ x_h \end{bmatrix} + \begin{bmatrix} w_v \\ w_h \end{bmatrix}
\end{aligned} \tag{15}$$

$$\begin{aligned}
x_v &= [g_{l1}\ g_{r1}\ \dot{g}_{l1}\ \dot{g}_{r1}\ i_{l1}\ i_{r1}]^T \\
x_h &= [g_{l3}\ g_{r3}\ \dot{g}_{l3}\ \dot{g}_{r3}\ i_{l3}\ i_{r3}]^T \\
u_v &= [e_{l1}\ e_{r1}]^T, \quad u_h = [e_{l3}\ e_{r3}]^T \\
v_v &= [v_{ml1}\ v_{mr1}\ v_{Ll1}\ v_{Lr1}]^T \\
v_h &= [v_{ml3}\ v_{mr3}\ v_{Ll3}\ v_{Lr3}]^T \\
y_v &= [y_{l1}\ y_{r1}]^T, \quad y_h = [y_{l3}\ y_{r3}]^T \\
w_v &= [w_{l1}\ w_{r1}]^T, \quad w_h = [w_{l3}\ w_{r3}]^T \\
A_v &:= \begin{bmatrix} 0 & I_2 & 0 \\ K_{x1}A_1 & 0 & K_{i1}A_1 \\ 0 & 0 & -(R/L)I_2 \end{bmatrix} \\
A_h &:= \begin{bmatrix} 0 & I_2 & 0 \\ K_{x3}A_1 & 0 & K_{i3}A_1 \\ 0 & 0 & -(R/L)I_2 \end{bmatrix} \\
A_{vh} &:= \begin{bmatrix} 0 & 0 & 0 \\ 0 & A_2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \\
B_v &= B_h := \begin{bmatrix} 0 \\ 0 \\ (1/L)I_2 \end{bmatrix} \\
C_v &= C_h := \begin{bmatrix} I_2 & 0 & 0 \end{bmatrix} \\
D_v &= D_h := \begin{bmatrix} 0 & 0 \\ A_1 & 0 \\ 0 & (1/L)I_2 \end{bmatrix} \\
A_1 &:= \begin{bmatrix} 1/m + l_m^2/J_y & 1/m - l_m^2/J_y \\ 1/m - l_m^2/J_y & 1/m + l_m^2/J_y \end{bmatrix} \\
A_2 &:= \begin{bmatrix} J_x/2J_y & -J_x/2J_y \\ -J_x/2J_y & J_x/2J_y \end{bmatrix}
\end{aligned}$$

where $I_2 \in R^{2\times 2}$ is unit matrix, and $K_{x1} = K_{xl1} = K_{xr1}$, $K_{x3} = K_{xl3} = K_{xr3}$, $K_{i1} = K_{il1} = K_{ir1}$, $K_{i3} = K_{il3} = K_{ir3}$ in (12), and $p$ is the rotor speed. Here $p$ is equal to 0 and we do not consider a rotation of the rotor in this paper.

The equation (15) can is also expressed simply as

$$\begin{aligned}
\dot{x}_g &= A_g x_g + B_g u_g + D_g v_0 \\
y_g &= C_g x_g + w_0
\end{aligned} \tag{16}$$

where $x_g := [x_v^T\ x_h^T]^T$, $u_g := [u_v^T\ u_h^T]^T$, $v_0 := \begin{bmatrix} v_v^T & v_h^T \end{bmatrix}^T$, $w_0 = \begin{bmatrix} w_v^T & w_h^T \end{bmatrix}^T$ and $A_g$, $B_g$, $C_g$, $D_g$ are constant matrices of appropriate dimensions.

## IV. CONTROL SYSTEM DESIGN

Let us construct a generalized plant for the magnetic bearing control system.

First, consider the system disturbance $v_0$. Since $v_0$ mainly acts on the plant in a low frequency range in practice, it is helpful to introduce a frequency weighting factor. Hence let $v_0$ be of the form

$$\begin{aligned} v_0 &= W_v(s)w_2 \quad (17)\\ W_v(s) &= \begin{bmatrix} I_2 & 0 \\ I_2 & 0 \\ 0 & I_2 \\ 0 & I_2 \end{bmatrix} W_{v0}(s) \\ W_{v0}(s) &= C_{v0}\left(sI_4 - A_{v0}\right)^{-1} B_{v0} \end{aligned}$$

where $W_v(s)$ is a frequency weighting whose gain is relatively large in a low frequency range, and $w_2$ is a $(1,2)$ element of $w$ . These values, as yet unspecified, can be regarded as free design parameters.

Let us consider the system disturbance $w_0$ for the output. The disturbance $w_0$ shows an uncertain influence caused via unmodeled dynamics, and define

$$\begin{aligned} w_0 &= W_w(s)w_1 \quad (18)\\ W_w(s) &= I_4 W_{w0}(s) \\ W_{w0}(s) &= C_{w0}\left(sI_4 - A_{w0}\right)^{-1} B_{w0} \end{aligned}$$

where $W_w(s)$ is a frequency weighting function and $w_1$ is a $(1,1)$ element of $w$. Note that $I_4$ is unit matrix in $R^{4\times 4}$.

Next we consider the variables which we want to regulate. In this case, since our main concern is in the stabilization of the rotor, the gap and the corresponding velocity are chosen; i.e.,

$$\begin{aligned} z_g &= F_g x_g, \quad (19)\\ F_g &= \begin{bmatrix} I_2 & 0 & 0 & 0 & 0 & 0 \\ 0 & I_2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_2 & 0 & 0 \\ 0 & 0 & 0 & 0 & I_2 & 0 \end{bmatrix} \\ z_1 &= \Theta z_g,\ \ \Theta = \mathrm{diag}\begin{bmatrix} \theta_1 & \theta_2 & \theta_1 & \theta_2 \end{bmatrix} \quad (20) \end{aligned}$$

where $\Theta$ is a weighting matrix on the regulated variables $z_g$, and $z_1$ is a $(1,1)$ element of $z$. This value $\Theta$, as yet unspecified, are also free design parameters.

Furthermore the control input $u_g$ should be also regulated, and we define

$$z_2 = \rho u_g \quad (21)$$

where $\rho$ is a weighting scalar, and $z_2$ is a $(1,2)$ element of $z$. Finally, let $x := \begin{bmatrix} x_g^T & x_v^T & x_w^T \end{bmatrix}^T$, where $x_v$ denotes the state of the function $W_v(s)$, $x_w$ denotes the state of the function $W_w(s)$, and $w := \begin{bmatrix} w_1^T & w_2^T \end{bmatrix}^T$, $z := \begin{bmatrix} z_1^T & z_2^T \end{bmatrix}^T$, then we can construct the generalized plant as in Fig.3 with an unspecified controller $K$.

The state-space formulation of the generalized plant is given as follows.

$$\begin{aligned} \dot{x} &= Ax + B_1 w + B_2 u \\ z &= C_1 x + D_{12} u \\ y &= C_2 x + D_{21} w \quad (22) \end{aligned}$$

where $A$, $B_1$, $B_2$, $C_1$, $C_2$, $D_{12}$ and $D_{21}$ are constant matrices of appropriate dimensions. Since the disturbances $w$ represent the various model uncertainties, the effects of these disturbances on the error vector $z$ should be reduced.

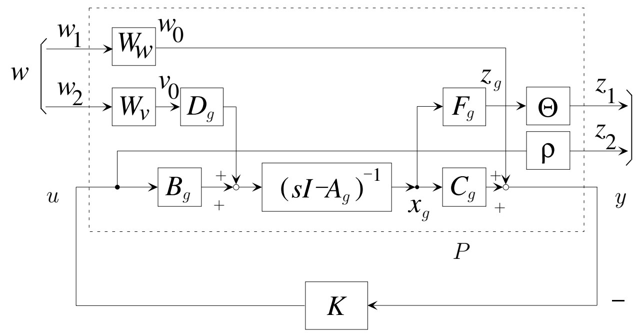


Fig. 3. Generalized Plant

Next our control problem setup is defined as;

**Control problem**:
Find an admissible controller $K(s)$ that attenuates disturbances and initial state uncertainties to achieve DIA condition in (3) for generalized plant (22).

After some iteration in MATLAB environment, design parameters are chosen as follows;

$$\begin{aligned} W_{v0}(s) &= \frac{40000}{s+0.1} \\ W_{w0}(s) &= \frac{1.5(s+10700)(s+2510\pm 4350i)}{(s+53400)(s+0.50\pm 5030i)} \\ \Theta &= diag\begin{bmatrix} \theta_{v1} & \theta_{v2} & \theta_{h1} & \theta_{h2} \end{bmatrix} \\ \theta_{v1} &= diag\begin{bmatrix} 0.4 & 0.4 \end{bmatrix}, \\ \theta_{h1} &= diag\begin{bmatrix} 0.5 & 0.5 \end{bmatrix} \\ \theta_{v2} &= \theta_{h2} = diag\begin{bmatrix} 0.0005 & 0.0005 \end{bmatrix} \\ \rho &= 8.0\cdot 10^{-7} I_4 \end{aligned}$$

$W_{w0}(s)$ represents an uncertainty for the 1st bending mode of the rotor at the resonance frequency 800[Hz].

Direct calculations yield the 24-order $\mathcal{H}_\infty$ DIA central controller $K_{DIA}$ and its frequency response is shown in Fig.4.

The maximum value of the weighting matrix $N$ in the DIA condition (3) is given by

$$N = 3.3176\cdot 10^{-6}\cdot I_{24}. \quad (23)$$

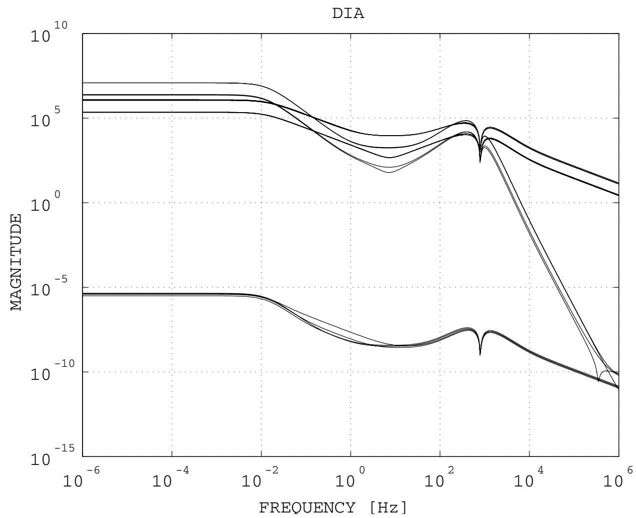


Fig. 4. Frequency Responses of $\mathcal{H}_\infty$DIA Controllers

## V. EVALUATION BY EXPERIMENTS

We conducted control experiments to evaluate properties of the designed $\mathcal{H}_\infty$ DIA controller compared with an integral-type Optimal State Feedback Control with a state observer and a notch filter at 2000[Hz]. We define this controller as "**LQ** Controller".

The objective of this experimental comparison is to evaluate control performance for rotational performance, transient property and robust performance. The experimental results are shown in Figs. 5-10.

### A. Non-Rotational Test

First, the basic control performance for non-rotational rotor for a step-type disturbance rejection is evaluated.

Disturbance responses for a step-type disturbance signal with/without model parameter perturbation are shown in Fig.5 and Fig.8. A 60[g] weight is attached to the center of the rotor as a model perturbation and a step-type force disturbance is added to $-l1$ and $-r1$ directions in Fig.1, where the magnitude of the disturbance is $1/6$ steady-state vertical attractive force.

DIA controller shows good disturbance responses and also good robust performance for step-type disturbance and model perturbation. Compared with LQ control with notch filter, we can see that $\mathcal{H}_\infty$ DIA control has a good robust performance and transient response.

Step responses for a reference signal are evaluated in [9]. LQ control showed a quick response without any overshoot because LQ control utilizes full state information.

### B. Rotational Test

Next the control performance for rotational rotor is evaluated with both controllers. Experimental results are shown in Figs. 6, 7, 9 and 10.

The free-run tests with varying rotational speeds from 6000 [rpm] to 0[rpm] are shown in Figs. 6 and 9. The horizontal axes show times and the rotational speeds of the rotor changed from 6000[rpm] at 0[s] to 0[rpm] at around 26[s]. The vertical axes show the displacement of the left side of the rotor.

Both Figs. 6 and 9 show that the amplitude of the vibration of the rotor changes based on the rotational speed of the rotor.

Compared with Fig.6 with Fig.9, the $\mathcal{H}_\infty$ DIA Controller shows a better rotational performance for varying rotational speed tests.

Figs. 6 and 9 show Lissajous curves of the center of the rotor in 30[s] with both $\mathcal{H}_\infty$ DIA controller and LQ controller. The better control performance of the $\mathcal{H}_\infty$ DIA control shown in the non-rotational tests gives a good rotational performance.

## VI. CONCLUSION

This paper dealt with an application of $\mathcal{H}_\infty$ control attenuating initial-state uncertainties to the magnetic bearing and examined the $\mathcal{H}_\infty$ DIA control problem.

First we derived a mathematical model of magnetic bearing systems considering rotor dynamics and nonlinearities of magnetic force. Then we set the generalized plant which contains design parameter for uncertainty and control performance.

Finally, experimental results for rotational test and non-rotational test showed that the proposed $\mathcal{H}_\infty$ DIA robust control approach is effective for a mixed disturbance and an initial-state uncertainty attenuation and for improving rotational performance, transient response and robust performance.

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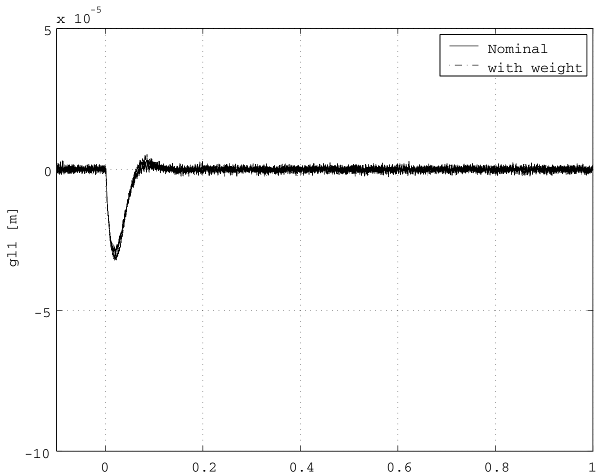


Fig. 5. Disturbance Response of $\mathcal{H}_\infty$DIA Controller with perturbation

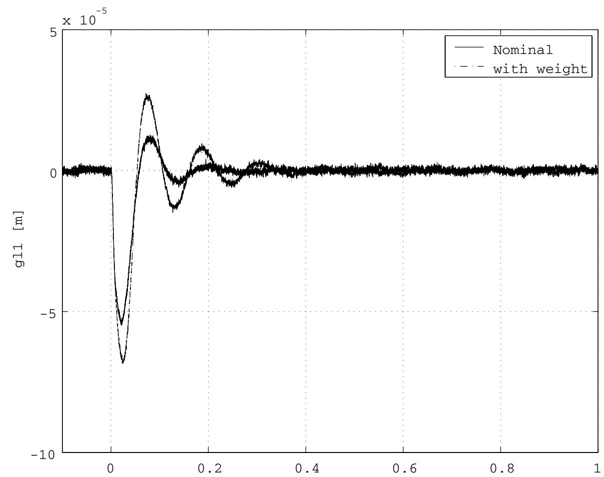


Fig. 8. Disturbance Response of LQ Controller with perturbation

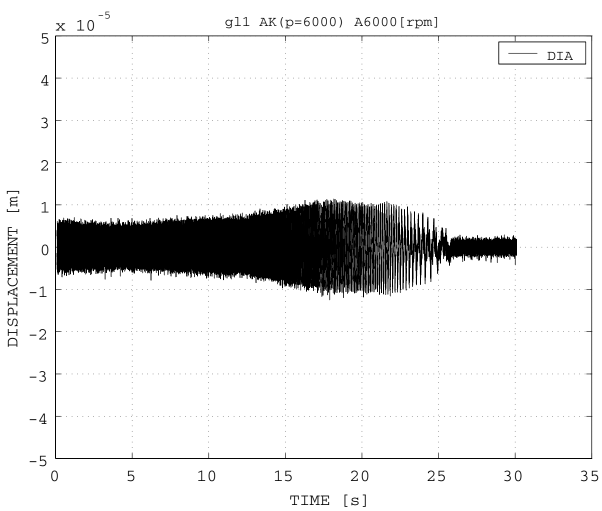


Fig. 6. Displacement of Vertical Axis of $\mathcal{H}_\infty$ DIA Controller

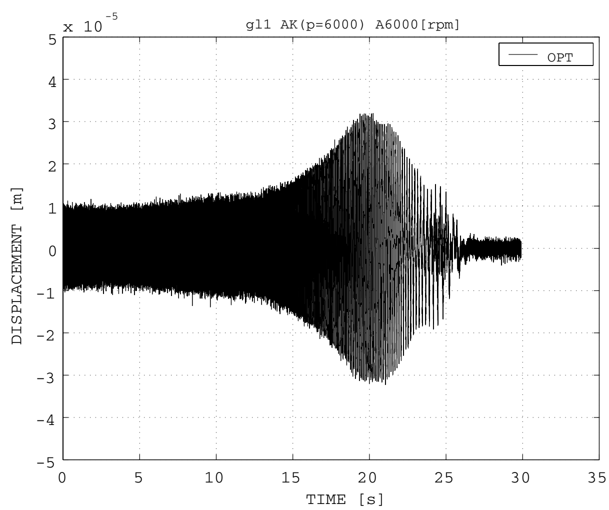


Fig. 9. Displacement of Vertical Axis of LQ Controller

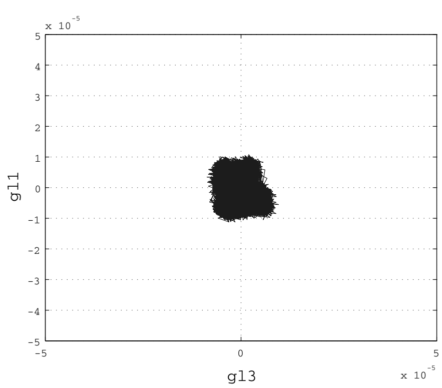


Fig. 7. Lissajous curve of $H_\infty$ DIA Controller

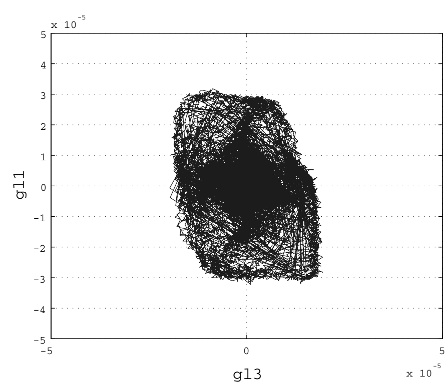


Fig. 10. Lissajous curve of LQ Controller