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A New Fuzzy H_∞ Filter Design for a Class of Nonlinear Continuous-Time Dynamic Systems

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***Abstract*— This paper studies fuzzy hyperbolic H_∞ filter for signal estimation of nonlinear continuous-time systems with state time delay. The fuzzy hyperbolic model (FHM) can be used to establish models for certain unknown complex systems. Furthermore, the main advantage of using the FHM over the Takagi-Sugeno fuzzy model are that no premise structure identification is needed and no completeness design of premise variables space is needed. Also an FHM is a kind of valid global description and nonlinear model in nature. And it is easy to design a filter based on an FHM since FHM satisfies Lipschitz condition. First, FHM is proposed to represent the state-space model for nonlinear continuous - time systems. Next, we design a stable fuzzy H_∞ filter based on the FHM, which assures asymptotic stability and a prescribed H_∞ index for the filtering error system. A sufficient condition for the existence of such a filter is established through seeking the feasible solutions of a linear matrix inequality (LMI). Simulation example is provided to illustrate the design procedure of the proposed method.**

I. INTRODUCTION

Recently, there have been a lot of interests in the problem of robust H_∞ filtering of systems with uncertain external disturbances and measurement noise [1], [2]. The advantage of using an H_∞ filter over a Kalman filter is that no statistical assumption on the noise signals is needed. The H_∞ filter is designed by minimizing signal estimation error for bounded disturbances and noises of the worst-case. Thus, H_∞ filter is more robust than the Kalman filter. Several robust H_∞ filtering approaches for linear systems have been developed over the past few years [1], [3], [4]. However, it is difficult to design an efficient filter for signal estimation of nonlinear systems.

This paper deals with the fuzzy hyperbolic H_∞ filtering problem for nonlinear continuous-time systems. Recently, there have been many applications of fuzzy systems theory in various fields. Fuzzy system are considered to be universal approximators for complex systems [5], [6], [7]. Parallel to the T-S fuzzy model, a new continuous-time fuzzy model, called the fuzzy hyperbolic model (FHM), has recently been proposed in [8], [9], [10]. the FHM can be used to establish models for certain unknown complex systems. The advantages of using FHM over the T-S model are that no premise structure identification is needed and no completeness design of premise variables space is needed. Furthermore, an FHM is a kind of valid global description and nonlinear model in nature. Thus, H_∞ filter using FHM can obtain the best optimal estimation performance. FHM can be obtained without knowing much information about the real plant, and it can easily be derived from a set of fuzzy rules. The FHM satisfies Lipschitz condition. Also, the FHM can be seen as a neural network model, and we can learn the model parameters using back-propagation (BP) algorithm. In general, it is easy to design a filter based on the FHM.

In this paper, first, an FHM is proposed to represent a class of nonlinear systems. Second, fuzzy H_∞ filtering design based on FHM, called fuzzy hyperbolic H_∞ filter, is addressed, which assures asymptotic stability and a prescribed H_∞ index for the filtering error system. Third, the design of fuzzy hyperbolic H_∞ filter is converted into a feasibility problem of linear matrix inequality (LMI). The LMI feasibility problem can efficiently be solved by the interior point algorithm [11].

II. PRELIMINARIES

In this section we review some necessary preliminaries

Definition 1: Given a plant with n input variables $x = (x_1(t), \cdots, x_n(t))^T$ and n output variables $\dot{x} = (\dot{x}_1(t), \cdots, \dot{x}_n(t))^T$. If each output variable corresponds to a group of fuzzy rules which satisfies the conditions [8], [9]:

(i) For each output variable $\dot{x}_l,\ l = 1, 2, \cdots, n$, the corresponding group of fuzzy rules has the following form:

R^j: IF x_1 is F_{x_1} and x_2 is $F_{x_2}, \cdots$, and x_n is F_{x_n}
THEN $\dot{x}_l = c^{\pm}_{F_{x_1}} + c^{\pm}_{F_{x_2}} + \cdots + c^{\pm}_{F_{xn}},\ l = 1, \cdots, 2^n$

where F_{x_i} $(i = 1, \cdots, n)$ are fuzzy sets of x_i, which include P_{x_i} (positive) and N_{x_i} (negative), and $c^{\pm}_{F_{x_i}} (i = 1, \cdots, n)$ are $2n$ real constants corresponding to F_{x_i};

(ii) The constant terms $c^{\pm}_{F_{x_i}}$ in the THEN-part correspond to F_{x_i} in the IF-part; that is, if the language value of F_{x_i} term in the IF-part is P_{x_i}, $c^{+}_{F_{x_i}}$ must appear in the THEN-part; if the language value of F_{x_i} term in the IF-part is N_{x_i}, $c^{-}_{F_{x_i}}$ must appear in the THEN-part; if there is no F_{x_i} in the IF-part, $c^{\pm}_{F_{x_i}}$ does not appear in the THEN-part.

(iii) There are 2^n fuzzy rules in each rule base; that is, there are a total of 2^n input variable combinations of all the possible P_{x_i} and N_{x_i} in the IF-part.

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We call this group of fuzzy rules "hyperbolic type fuzzy rule base" (HFRB). To describe a plant with n output variables, we will need n HFRBs.

Lemma 1: Given n HFRBs, if we define the membership function of P_{x_i} and N_{x_i} as [8], [9]:

$$\mu_{P_{x_i}}(x_i) = e^{-\frac{1}{2}(x_i-k_i)^2}, \quad \mu_{N_{x_i}}(x_i) = e^{-\frac{1}{2}(x_i+k_i)^2} \quad (1)$$

where $i = 1, \cdots, n$ and k_i are positive constants. Denoting $c^+_{F_{x_i}}$ by $c_{P_{x_i}}$ and $c^-_{F_{x_i}}$ by $c_{N_{x_i}}$, we can derive:

$$\dot{x}_l = f(x) = \sum_{i=1}^{n_l} \frac{c_{P_{x_i}} e^{k_i x_i} + c_{N_{x_i}} e^{-k_i x_i}}{e^{k_i x_i} + e^{-k_i x_i}}$$
$$= \sum_{i=1}^{n_l} p_i + \sum_{i=1}^{n_l} q_i \frac{e^{k_i x_i} - e^{-k_i x_i}}{e^{k_i x_i} + e^{-k_i x_i}} = \sum_{i=1}^{n_l} p_i + \sum_{i=1}^{n_l} q_i \tanh(k_i x_i) \quad (2)$$

where $p_i = \frac{c_{P_{x_i}} + c_{N_{x_i}}}{2}$ and $q_i = \frac{c_{P_{x_i}} - c_{N_{x_i}}}{2}$. Therefore, the whole system has the following form:

$$\dot{x} = P + A\tanh(Kx) \quad (3)$$

where P is a constant vector, A is a constant matrix, and

$$\tanh(Kx) = [\tanh(k_1x_1),\ \tanh(k_2x_2), \cdots, \tanh(k_nx_n)]^T.$$

We will call (3) a fuzzy hyperbolic model (FHM).

Definition 2: let S_C be the set of all functions $f(\cdot)$: $R \to R$ satisfying [8], [9] :

1) f is continuous;
2) $f(0) = 0$ and for all other $x \in R$, $f(x)x > 0$;
3) $\int_0^x f(y)dy \to \infty$ as $|x| \to \infty$

Remark 1: From Definition 1, if we set $c_{P_{x_i}}$ and $c_{N_{x_i}}$ negative to each other, we can obtain a homogeneous FHM:

$$\dot{x} = A\tanh(Kx). \quad (4)$$

Since the difference between (3) and (4) is only the constant vector term in (3), there is essentially no difference between the (3) and (4). In this paper, we will design a fuzzy H_∞ filter based on FHM (4).

III. FUZZY HYPERBOLIC H_∞ FILTER ANALYSIS

Consider a class of nonlinear continuous-time systems:

$$\begin{aligned} \dot{x}(t) &= f(x) + g(x(t-d)) + Bw(t) \\ y(t) &= h(x) + Dw(t) \\ s(t) &= Lx(t) \\ x(t) &= \varphi(t),\ t \in [-d_{\max}, 0) \end{aligned} \quad (5)$$

where $x(t) = [x_1(t), x_2(t), \cdots, x_n(t)]^T \in R^{n\times 1}$ denotes the state vector; $y(t) \in R^{m\times 1}$ denotes the measurements vector; $s(t) \in R^{q\times 1}$ denotes the signal to be estimated; $w(t) \in R^{n\times 1}$ is assumed to be bounded disturbance; d is the bounded time-delay in the state and satisfies $0 \le d \le d_{\max}$and d is a known constant.

The ith rule of the FHM fuzzy model for the nonlinear discrete-time systems (5) is proposed as the following form:

R^j : IF x_1 is F_{x_1} and x_2 is F_{x_2}, ..., and x_n is F_{x_n}, $x_{d1}$is F_{xd_1} and x_{d2} is F_{xd_2}, ..., and x_{dn} is F_{xd_n}, THEN

$$\dot{x}_l = c^\pm_{F_{x_1}} + c^\pm_{F_{x_2}} + \cdots + c^\pm_{F_{xn}} + c^\pm_{F_{xd_1}} + c^\pm_{F_{xd_2}} + \cdots + c^\pm_{F_{xdn}} \quad (6)$$

where j=1,...,2^n, x_{di} denotes $x_i(t-d)$, F_{x_i} and F_{xd_i} $(i = 1, \cdots, n)$ are fuzzy sets of x_i and xd_i, respectively ; $c^\pm_{F_{x_i}}$ and $c^\pm_{F_{xd_i}}$ $(i = 1, \cdots, n)$ are $2n$ real constants corresponding to F_{x_i} and F_{xd_i}, respectively.

The fuzzy system in (6) has singleton fuzzifier, product inference, and centroid defuzzifier. Its state dynamics and the output equation are of the following form, respectively:

$$\begin{aligned} \dot{x}(t) &= A\tanh(k_x x) + A_d\tanh(k_x x(t-d)) + Bw(t) \\ y(t) &= C\tanh(k_x x) + Dw(t) \\ s(t) &= Lx(t) \\ x(t) &= \varphi(t),\ t \in [-d_{\max}, 0) \end{aligned} \quad (7)$$

where $A, A_d, B, C, D$and L are known constant real system matrices to be obtained by neural network [14].

Based on the FHM (4), the following H_∞ filter is addressed

$$\begin{aligned} \dot{\hat{x}}(t) &= A\tanh(k_x\hat{x}(t)) + A_d\tanh(k_x\hat{x}(t-d)) \\ &\quad + K(y(t) - C\tanh(k_x\hat{x}(t))) \\ \hat{s}(t) &= L\hat{x}(t),\ \hat{x}(0) = 0 \end{aligned} \quad (8)$$

we call (8) fuzzy hyperbolic H_∞ filter.

Remark 2: The parameters A, A_d in (8) is the same as in (7), which shows that we utilize all the information we have known. Therefore, only K is needed to design as filter parameter.

Let the state error be $e(t) = x(t) - \hat{x}(t)$, then the augmented filter error system can be written as the following form:

$$\begin{aligned} \dot{e}(t) &= (A-KC)(\tanh(k_x x(t)) - \tanh(k_x\hat{x}(t))) + \\ &\quad A_d(\tanh(k_x x(t-d)) - \tanh(k_x\hat{x}(t-d))) + (B-KD)w(t) \\ s_e(t) &= Le(t) \end{aligned} \quad (9)$$

where $s_e(t) = s(t) - \hat{s}(t)$.

For notational convenience, similar to Wang [12], [13], we give the following definitions

$$l(t) = \tanh(k_x x(t)) - \tanh(k_x\hat{x}(t)) - A_l e(t) \quad (10)$$

$$m(t) = \tanh(k_x x(t-d)) - \tanh(k_x\hat{x}(t-d)) - A_{dl} e(t-d) \quad (11)$$

where A_l, A_{dl}satisfy

$$|\tanh(k_x x(t)) - \tanh(k_x\hat{x}(t)) - A_l(x-\hat{x})| \le a\,|(x-\hat{x})| \quad (12)$$

and

$$|\tanh(k_x x(t-d)) - \tanh(k_x\hat{x}(t-d)) - A_{dl}(x-\hat{x})| \le a_d\,|(x-\hat{x})| \quad (13)$$

where a, a_d are known positive constants, and $|\cdot|$ denotes the Euclidean norm in R^n.

Then, (9) can be written as follows

$$\begin{aligned} \dot{e}(t) &= (A-KC)A_l e(t) + A_d A_{dl} e(t-d) + \\ &\quad (A-KC)l(t) + A_d m(t) + (B-KD)w(t) \\ s_e(t) &= Le(t) \end{aligned} \quad (14)$$

Remark 3: It is clear that $\phi(e, x(t)) = \tanh(k_x x(t)) - \tanh(k_x\hat{x}(t)) = 0$ if $e = x(t) - \hat{x}(t) = 0$. Moreover, $f(e+x) = A\tanh(k_x(e+x))$ is monotonically increasing (or decreasing) for each component e_i of e. Since $e_i > 0$ (or $e_i < 0$) implies that $e_i + x_i > x_i$ (or $e_i + x_i < x_i$) for all x_i, $f_i(e_i + x_i) > f_i(x_i)$ (or $f_i(e_i + x_i) < f_i(x_i)$). This means that $\phi^T(e,x)e = (f(e+x) - f(x))^T e > 0$. Therefore, there exist positive constants γ_1, γ_2, and L_ϕ such that

$$\gamma_1 \|e\|_2^2 \le \phi^T(e,x)e \le \gamma_2 \|e\|_2^2 \quad (15)$$

and

$$\|\phi(e,x)\|_2 < L_\phi \|e\|_2 \quad (16)$$

Therefore, $\phi(e,x)$ is Lipschitz with respect to e.

The aim of this paper is to design an fuzzy hyperbolic H_∞ filter for the nonlinear system (7). H_∞ performance index is defined as follows

$$\int_0^\infty s_e^T(t)\Lambda s_e^T dt \le \int_0^\infty \gamma^2 w^T w dt \quad (17)$$

where Λ is the weight matrix and satisfies positive definite symmetric matrix.

We should design the filter parameter K such that the (17) holds.

Theorem 1: For nonlinear system (7) and a prescribed real number $\gamma > 0$, if there exist positive scalars ε_1, ε_2 , ε_3, L_ϕ and positive definite matrices $P > 0$, $Q > 0$ such that the matrix inequality

$$\begin{bmatrix} \Psi + L^T\Lambda L & PA_dA_{dl} & P(B-KD) \\ * & -R^TQR & 0 \\ * & * & -\gamma^2 I \end{bmatrix} < 0 \quad (18)$$

then the filtering error system (14) is asymptotically stable and satisfies a guaranteed γ level of noise attenuation. where

$$\Psi = A_l^T(A-KC)^TP + P(A-KC)A_l + R^TQR + \varepsilon_1 P^2 + \varepsilon_2 P^2 + \varepsilon_1^{-1} a \left|\lambda((A-KC)^T(A-KC))\right|_{\max} + \varepsilon_2^{-1} a_d \left|\lambda(A_d^TA_d)\right|_{\max}$$

and $*$ denotes the entries induced by symmetry. Matrices A_l, A_{dl} satisfy, respectively

$$|A_l| \le a - L_\phi |k_x| \quad (19)$$

and

$$|A_{dl}| \le a_d - L_\phi |k_x| \quad (20)$$

Proof: Choose the following Lyapunov function candidate for the system (14)

$$V(t) = e^T(t)Pe(t) + \int_{t-d}^t e^T(\alpha)R^TQRe(\alpha)d\alpha \quad (21)$$

where $P = P^T > 0$, $Q = Q^T > 0$.

Along the trajectories of system (14) with $w(t) = 0$, the corresponding time derivative of $V(t)$ is given by

$$\begin{aligned} &\tfrac{d}{dt}V(e(t),t) \\ &= \dot e^T(t)Pe(t) + e^T(t)P\dot e(t) + e^T(t)R^TQRe(t) \\ &\quad - e^T(t-d)R^TQRe(t-d) \\ &= e^T(t)A_l^T(A-KC)^TPe(t) + e^T(t)P(A-KC)A_le(t) \\ &\quad + e^T(t)P(A-KC)l(t) + l^T(t)(A-KC)^TPe(t) + \\ &\quad e^T(t)PA_dm(t) + m^T(t)A_d^TPe(t) + \\ &\quad e^T(t-d)A_{dl}^TA_d^TPe(t) + e^T(t)PA_dA_{dl}e(t-d) + \\ &\quad e^T(t)R^TQRe(t) - e^T(t-d)R^TQRe(t-d) \end{aligned} \quad (22)$$

let ε_1, ε_2, and ε_3 are positive scalars , and $\Phi_1 = \varepsilon_1^{1/2}e^T(t)P - \varepsilon_1^{-1/2}l^T(t)A^T$, $\Phi_2 = \varepsilon_2^{1/2}e^T(t)PK + \varepsilon_2^{-1/2}l^T(t)C^T$, $\Phi_3 = \varepsilon_3^{1/2}e^T(t)P - \varepsilon_3^{-1/2}m^T(t-d)A_d^T$, then we have

$$\Phi_1\Phi_1^T \ge 0, \quad \Phi_2\Phi_2^T \ge 0, \Phi_3\Phi_3^T \ge 0. \quad (23)$$

Thus, we have

$$\begin{aligned} &\varepsilon_1 e^T(t)P^2e(t) + \varepsilon_1^{-1}l^T(t)A^TAl(t) \\ &\le \varepsilon_1 e^T(t)P^2e(t) + \varepsilon_1^{-1}l^T(t)A^TAl(t) \end{aligned} \quad (24)$$

and

$$\begin{aligned} &-e^T(t)PKCl(t) - l^T(t)(KC)^TPe(t) \\ &\le \varepsilon_2 e^T(t)PKK^TPe(t) + \varepsilon_2^{-1}l(t)^TC^TCl(t) \end{aligned} \quad (25)$$

and

$$\begin{aligned} &e^T(t)PA_dm(t-d) + m^T(t-d)A_d^TPe(t) \\ &\le \varepsilon_3 e^T(t)P^2e(t) + \varepsilon_3^{-1}m^T(t-d)A_d^TA_dm(t-d) \end{aligned} \quad (26)$$

according to (24) and (25), yields

$$\begin{aligned} &e^T(t)P(A-KC)l(t) + l^T(t)(A-KC)^TPe(t) \\ &= e^T(t)PAl(t) - e^T(t)PKCl(t) + l^T(t)A^TPe(t) - \\ &\quad l^T(t)(KC)^TPe(t) \\ &= e^T(t)PAl(t) + l^T(t)A^TPe(t) - e^T(t)PKCl(t) - \\ &\quad l^T(t)(KC)^TPe(t) \\ &\le \varepsilon_1 e^T(t)P^2e(t) + \varepsilon_1^{-1}l^T(t)A^TAl(t) + \\ &\quad [\varepsilon_2 e^T(t)PKK^TPe(t) + \varepsilon_2^{-1}l^T(t)C^TCl(t)] \\ &= \varepsilon_1 e^T(t)P^2e(t) + \varepsilon_2 e^T(t)PKK^TPe(t) + \\ &\quad \varepsilon_1^{-1}l^T(t)A^TAl(t) + \varepsilon_2^{-1}l^T(t)C^TCl(t) \\ &\le \varepsilon_1 e^T(t)P^2e(t) + \varepsilon_2 e^T(t)PKK^TPe(t) + \\ &\quad \varepsilon_1^{-1}\left|\lambda(A^TA)\right|_{\max} ae^T(t)e(t) + \\ &\quad \varepsilon_2^{-1}\left|\lambda(C^TC)\right|_{\max} ae^T(t)e(t) \end{aligned} \quad (27)$$

from (26) and (27), (22) becomes

$$\begin{aligned} &\tfrac{d}{dt}V(e(t),t) \\ &\le e^T(t)A_l^T(A-KC)^TPe(t) + e^T(t)P(A-KC)A_le(t) + \\ &\varepsilon_1 e^T(t)P^2e(t) + \varepsilon_2 e^T(t)PKK^TPe(t) + \\ &\varepsilon_1^{-1}\left|\lambda(A^TA)\right|_{\max} ae^T(t)e(t) + \\ &\varepsilon_2^{-1}\left|\lambda(C^TC)\right|_{\max} ae^T(t)e(t) + \varepsilon_3 e^T(t)P^2e(t) + \\ &\varepsilon_3^{-1}a_d\left|\lambda(A_d^TA_d)\right|_{\max} e^T(t-d)e(t-d) + \\ &e^T(t)PA_dA_{dl}e(t-d) + e^T(t-d)A_{dl}^TA_d^TPe(t) + \\ &e^T(t)R^TQRe(t) - e^T(t-d)R^TQRe(t-d) \\ &= \begin{bmatrix} e(t) \\ e(t-d) \end{bmatrix}^T \Theta \begin{bmatrix} e(t) \\ e(t-d) \end{bmatrix} \end{aligned} \quad (28)$$

where $\Theta = \begin{bmatrix} \Psi & PA_dA_{dl} \\ A_{dl}^TA_d^TP & -R^TQR + \varepsilon_3^{-1}a_d\left|\lambda(A_d^TA_d)\right|_{\max} I \end{bmatrix}$,

$$\begin{aligned} \Psi = &A_l^T(A-KC)^TP + P(A-KC)A_l + \varepsilon_1 P^2 + \\ &\varepsilon_2 PKK^TP + \varepsilon_1^{-1}\left|\lambda(A^TA)\right|_{\max} aI + \\ &\varepsilon_2^{-1}\left|\lambda(C^TC)\right|_{\max} aI + \varepsilon_3 P^2 + R^TQR \end{aligned} \quad (29)$$

if let $\Theta < 0$, then $\frac{d}{dt}V(e(t),t) < 0$. Since $\lim_{t\to\infty} V(e(t),t) = \mu$, $V(e(0),0) = V((x(0) - \hat x(0)),0) = V(0,0) = 0$. it can be concluded that the filtering error system (14) is globally asymptotically stable.

$$\begin{aligned} J &= \int_0^\infty (s_e^T(t)\Lambda s_e^T - \gamma^2 w^T w)dt \\ &= \int_0^\infty (s_e^T(t)\Lambda s_e^T - \gamma^2 w^T w + \dot{V})dt + V|_{t=0} - V|_{t\to\infty} \\ &\le \int_0^\infty (s_e^T(t)\Lambda s_e^T - \gamma^2 w^T w + \dot{V})dt \\ &= \int_0^\infty \xi^T \Xi \xi dt \end{aligned} \tag{30}$$

where $\xi = \begin{bmatrix} e^T(t) & e^T(t-d) & w^T(t) \end{bmatrix}^T$, and $\Xi = \begin{bmatrix} \Psi + L^T\Lambda L & PA_dA_{dl} & P(B-KD) \\ * & \Xi_{22} & 0 \\ * & * & -\gamma^2 I \end{bmatrix}$, $\Xi_{22} = -R^TQR + \varepsilon_3^{-1} a_d \left|\lambda(A_d^TA_d)\right|_{\max} I$.

If let $\Xi < 0$, then $J < 0$, i. e. $\int_0^\infty s_e^T(t)\Lambda s_e^T dt \le \int_0^\infty \gamma^2 w^T w dt$.

Hence, the fuzzy hyperbolicH_∞ filter index is achieved with a prescribed γ^2.

This completes the proof.

IV. Fuzzy Hyperbolic H_∞ Filter design

Theorem 1 provides a sufficient condition for the existence of fuzzy H_∞filter assuring a robust H_∞ performance for system in (7). In order to obtain fuzzy H_∞ filter, the most important work is to solve $P = P^T$, $Q = Q^T$, and ε_1, ε_2, $\varepsilon_2, \varepsilon_3, L_\phi$ from (18). The (18) can be solved in a computationally efficient manner using a convex optimization technique such as the interior point method. Then, the matrix inequalities in (18) are converted into LMI using Schur complements as follows

$$\begin{bmatrix} \Omega & PA_dA_{dl} & PB-MD & L^T & P & P & M \\ * & -R^TQR + \varepsilon_3^{-1} a_d \left|\lambda(A_d^TA_d)\right|_{\max} I & 0 & 0 & 0 & 0 & 0 \\ * & * & -\gamma^2 I & 0 & 0 & 0 & 0 \\ * & * & * & -\Lambda^{-1} & 0 & 0 & 0 \\ * & * & * & * & -\varepsilon_1^{-1} & 0 & 0 \\ * & * & * & * & * & -\varepsilon_3^{-1} I & 0 \\ * & * & * & * & * & * & -\varepsilon_2^{-1} I \end{bmatrix} < 0 \tag{31}$$

where $\Omega = A_l^TA^TP + PAA_l - A_l^TC^TM - MCA_l + \varepsilon_1^{-1}\left|\lambda(A^TA)\right|_{\max} aI + \varepsilon_2^{-1}\left|\lambda(C^TC)\right|_{\max} aI + R^TQR$, $M = PK$.

Remark 4: In view of Theorem 1, the H_∞ filtering problem (7) can be solved in terms of the feasibility of the LMIs of (31). In fact, any feasible solution to (31) yields a suitable robust filter. To obtain a better robust filtering performance against disturbances, the attenuation level γ^2 can be reduced to the minimum possible value such that (31) is satisfied. Then, the H_∞ filter with the smallest γ attenuation level obtained from Theorem 1 can easily be determined by solving the following convex optimization problem:

$$\min_{P,P_j,K_l} \delta, \text{subject to (31) with } \gamma^2 = \delta. \tag{32}$$

The design procedure for fuzzy H_∞ filter is summarized as follows:

Step 1: Construct the FHM fuzzy model in (7) for the nonlinear system by the BP network [14];

Step 2: Solve the LMI problem in (32) to obtain P, Q and M ($K = P^{-1}M$) ;

Step 5: Obtain the fuzzy H_∞ filter constructed as in (8).

Remark 5: A_l and A_{dl} in (10) and (11) can be obtain as follows According to Lipschitz condition (16), we have

$$\begin{aligned} &|\tanh(k_x x(t)) - \tanh(k_x\hat{x}(t)) - A_l(x(t)-\hat{x}(t))| \\ &\le |\tanh(k_x x(t)) - \tanh(k_x\hat{x}(t))| + |-A_l(x(t)-\hat{x}(t))| \\ &\le L_\phi |k_x(x(t)-\hat{x}(t))| + |-A_l(x(t)-\hat{x}(t))| \\ &\le L_\phi |k_x| \cdot |(x(t)-\hat{x}(t))| + |A_l| \cdot |(x(t)-\hat{x}(t))| \\ &\le a|(x(t)-\hat{x}(t))| \end{aligned} \tag{33}$$

and

$$\begin{aligned} &|\tanh(k_x x(t-d)) - \tanh(k_x\hat{x}(t-d)) - A_{dl}(x(t-d)-\hat{x}(t-d))| \\ &\le |\tanh(k_x x(t-d)) - \tanh(k_x\hat{x}(t-d))| + \\ &\quad |-A_{dl}(x(t-d)-\hat{x}(t-d))| \\ &\le L_\phi |k_x(x(t-d)-\hat{x}(t-d))| + |-A_{dl}(x(t-d)-\hat{x}(t-d))| \\ &\le L_\phi |k_x| \cdot |(x(t-d)-\hat{x}(t-d))| + |A_{dl}| \cdot |(x(t-d)-\hat{x}(t-d))| \\ &\le a_d |(x(t-d)-\hat{x}(t-d))| \end{aligned} \tag{34}$$

we can obtain from (33)

$$|A_l| \le a - L_\phi |k_x| \tag{35}$$

and we can obtain from (34)

$$|A_{dl}| \le a_d - L_\phi |k_x| \tag{36}$$

we can construct A_l, A_{dl} according to $|A_l|$, $|A_{dl}|$.

V. Simulation Example

Suppose that we have the following HFRBs:

If $x_1(t)$ is P_{x1} and $x_2(t)$ is P_{x2} and $x_1(t-d_1)$ is P_{xd1} , then $\dot{x}_2 = C_{x1} + C_{x2} + C_{xd1}$;

If x_1 is N_{x1} and x_2 is P_{x2} and $x_1(t-d_1)$ is P_{xd1}, then $\dot{x}_2 = -C_{x1} + C_{x2} + C_{xd1}$;

If x_1 is P_{x1} and x_2 is N_{x2} and $x_1(t-d_1)$ is P_{xd1}, then $\dot{x}_2 = C_{x1} - C_{x2} + C_{xd1}$;

If x_1 is N_{x1} andx_2 is N_{x2} and $x_1(t-d_1)$ is P_{xd1}, then $\dot{x}_2 = -C_{x1} - C_{x2} + C_{xd1}$;

If x_1 is P_{x1} and x_2 is N_{x2} and $x_1(t-d_1)$ is N_{xd1}, then $\dot{x}_2 = C_{x1} - C_{x2} - C_{xd1}$;

If x_1 is N_{x1} and x_2 is N_{x2} and $x_1(t-d_1)$ is N_{xd1}, then $\dot{x}_2 = -C_{x1} - C_{x2} - C_{xd1}$;

If $x_2(t)$ is P_{x2}, then $\dot{x}_1 = C_{x2}$;

If x_2 is N_{x2}, then $\dot{x}_1 = -C_{x2}$;

Here, we choose membership functions of P_{xi} and N_{xi} as follows:

$$\begin{aligned} &\mu_{P_{x_i}}(x) = e^{-\frac{1}{2}(x_i - k_i)^2}, \quad \mu_{N_{x_i}}(x) = e^{-\frac{1}{2}(x_i + k_i)^2}, \\ &\mu_{P_{xd_i}}(x_d) = e^{-\frac{1}{2}(x_{di} - k_i)^2}, \mu_{N_{xd_i}}(x) = e^{-\frac{1}{2}(x_{di} + k_i)^2}. \end{aligned} \tag{37}$$

Then, we have the following model:

$$\dot{x} = A\tanh(k_x x) + A_d \tanh(k_x x(t-d)) \tag{38}$$

where
$x(t) = [x_1(t) \quad x_2(t)]^T$, $x(t-d) = [x_1(t-d) \quad x_2(t-d)]^T$,

$$A = \begin{bmatrix} 0 & C_{x2} \\ C_{x1} & C_{x2} \end{bmatrix}, \quad A_d = \begin{bmatrix} 0 & 0 \\ C_{xd1} & 0 \end{bmatrix},$$

$$\tanh(k_x x) = \begin{bmatrix} \tanh(k_1 x_1) & \tanh(k_2 x_2) \end{bmatrix}^T,$$

$\tanh(k_x x(t-d)) = [\tanh(k_1 x_1(t-d)) \quad \tanh(k_2 x_2(t-d))]^T$.

For measurement $y(t)$, we choose the following FRBs:
If $x_1(t)$ is P_{x1} and $x_2(t)$ is P_{x2}, then $y(t) = D_{x1} + D_{x2}$;
If $x_1(t)$ is P_{x1} and $x_2(t)$ is N_{x2}, then $y(t) = D_{x1} - D_{x2}$;
If x_1 is N_{x1} and $x_2(t)$ is P_{x2}, then $y(t) = -D_{x1} + D_{x2}$;
If x_1 is N_{x1} and $x_2(t)$ is N_{x2}, then $y(t) = -D_{x1} - D_{x2}$;
Then, we have the following model:

$$y(t) = C\tanh(k_x x) \tag{39}$$

where $C = \begin{bmatrix} D_{x1} & D_{x2} \end{bmatrix}$.

We construct the FHM as system (7). So (38) and (39) becomes

$$\begin{aligned} \dot{x} &= A\tanh(k_x x) + A_d\tanh(k_x x(t-d)) + Bw(t) \\ y(t) &= C\tanh(k_x x) + Dw(t) \end{aligned} \tag{40}$$

we choose $k_x = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $A = \begin{bmatrix} 0 & -7 \\ 10 & -7 \end{bmatrix}$, $A_d = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}$, $L = \begin{bmatrix} -1 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 1 & 1 \end{bmatrix}$, $B = \begin{bmatrix} -0.02 & 0.01 \end{bmatrix}^T$, $D = 0.01$, $d = 1$.
We assume that the noise is normally distributed with zero mean and variance 0. 01. It is assumed that the initial condition is: $(x_1(0), x_2(0), \hat{x}_1(0), \hat{x}_2(0))^T = (0,0,0,0)^T$. we solve (32) to obtain parameters given by

$$P = \begin{bmatrix} 0.3866 & 7.6305 \\ 7.6305 & 150.6413 \end{bmatrix}, M = \begin{bmatrix} 1.3629\times 10^3 \\ 1.3627\times 10^3 \end{bmatrix},$$

$$Q = \begin{bmatrix} 145.2621 & 0.0017 \\ 0.0017 & 0.0024 \end{bmatrix}, K = \begin{bmatrix} 2.2504 \\ -0.1140 \end{bmatrix},$$

$$\gamma^2 = 0.1363, \quad \varepsilon_1 = 11.0293, \quad \varepsilon_2 = 7.2087\times 10^8,$$

$$\varepsilon_2 = 5.4552\times 10^8.$$

Fig. 1 shows the estimation for $s(k)$. Fig. 2 shows the estimation errors for $s(k)$. From the results of this simulation, it shows that the proposed fuzzy hyperbolic H_∞ filter can obtain better estimation.

VI. CONCLUSIONS

In this paper, based on the FHM fuzzy model, fuzzy H_∞ filter is designed for a class of nonlinear continuous-time systems.

The advantages of fuzzy hyperbolic H_∞ filter are: (i) No statistical assumption on the external disturbances and measurement noise is needed. Fuzzy hyperbolic H_∞ filter can tolerate the approximation error based on the model error bounds. Therefore, the proposed filter has strong robustness. (ii) we utilize all the information we have known about the system when we design filter, and only fuzzy estimator gain parameter is needed to design as filter parameter. Therefore, the proposed design procedure is very simple. (iii) The design of fuzzy hyperbolic H_∞ filter is converted into a linear matrix inequality problem which can efficiently be solved using the interior point algorithm.

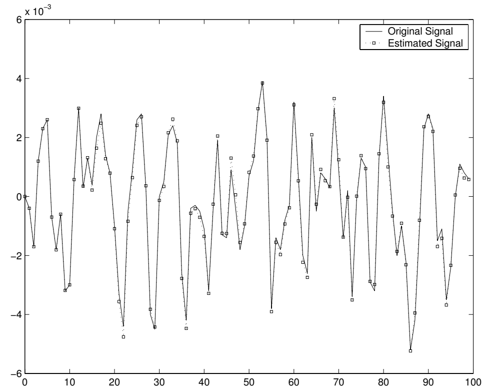

Fig. 1. Original signal $s(t)$ (solid line) and restored signal $\hat{s}(t)$ by fuzzy hyperbolic H_∞ filter (dashdot line with squares)

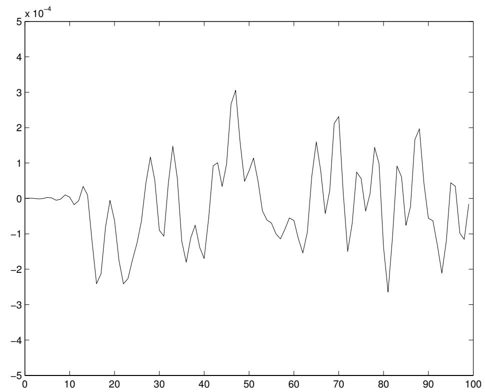

Fig. 2. Estimation error signal for $s(t)$ by fuzzy hyperbolic H_∞ filter

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