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Robust Motion Control for Nonholonomic Constrained Mechanical Systems: Sliding Mode Approach

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Abstract— **This paper addresses the robust trajectory tracking problem for a general class of nonholonomic systems with velocity constraints in the presence of uncertainties. The development of the proposed algorithm is based on sliding mode control technique. First, for the purpose of designing controllers, the conservative upper-bounded function of dynamics model of nonholonomic systems is derived based on the dynamics structure properties. Then, a sliding mode control scheme is presented to guarantee trajectory tracking of closed-loop system. The asymptotical or exponential stability is obtained in the Lyanupov sense. Finally, simulation examples are given to demonstrate the proposed approach.**

I. Introduction

The control of mechanical systems with nonholonomic kinematics constraints has attracted significant attention and becomes a topic of great interest during the last few years. Because such systems can be found frequently in mechanical systems such as wheeled mobile robot, car-like vehicle and so on[1],[2],[19],[24]. It is well known that nonholonomic systems cannot be stabilized using any time-invariant continuous pure state feedback controller as Brockett[3] pointed out. Thus considerable efforts have been made in designing stabilizing control laws and research results can be classified into two categories: kinematics controllers providing kinematics control inputs such as driven speeds[2],[13] and dynamics controllers providing real physical control inputs such as driven torques[5],[7],[14]. Various stabilizing control methods can be found in [2] and the references cited therein.

The problems of trajectory tracking or motion control of constrained nonholonomic systems have received relatively less attention in the literature. Similar to the stabilization case, the trajectory tracking problem can also be classified as kinematics tracking problems and dynamics tracking problems. Up to now, most works reported on the trajectory tracking problems of nonholonomic systems are at kinematics level (See[15] and the references therein), yet, dynamic-based trajectory tracking control is more necessary in practice. Several results based on dynamics model of constrained nonholonomic systems have been published in recent years. The simplest case is that the dynamics models were assumed to be perfect, exactly known and free of external disturbances. Model-based controller proposed by Su[4]guarantees the asymptotic convergence of motion errors. But computation of the scheme was time-consuming due to depending on the regressor matrix. In [5], the uncertainties and external disturbances in dynamics of nonholonomic were considered and an adaptive fuzzy approach was presented. But the method relied on a strong assumption that the approximation errors of fuzzy logical systems plus external disturbances belonged to $L_2 \cap L_\infty$. Ge[6] applied robust adaptive technique to nonholonomic systems with uncertainties and got satisfied performances. In addition, some researches developed tracking control laws for a class of special nonholonomic systems, so-called chained forms, which were brought to the literature in [7]. However, not all the nonholonomic systems can be transformed into chained systems. Necessary and sufficient conditions for the conversion from nonholonomic systems to chained systems are given in [21].

Sliding mode control (SMC) is a special discontinuous control technique applicable to various practical systems [8]. By designing switch functions of state variables or output variables to form sliding surfaces, SMC can guarantee that when trajectories reach the surfaces, the switch functions keep the trajectories on the surfaces, thus yielding the desired system dynamics. The main advantages of using SMC include fast response, good transient and the robustness with respects to system uncertainties and external disturbances. Therefore, it is attractive for many highly nonlinear uncertain systems [8-9]. SMC has been applied to deal with the stabilization problems of nonholonomic

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systems[22-23]. However, for the trajectory tracking problem of nonholonomic systems with kinematics constraints, SMC is merely used to control some special nonholonomic systems including wheeled mobile robots[10-11] and chained form systems[16-17]. As far as wheeled mobile robot is concerned, parameterized kinemics model of wheeled mobile robot makes the design of sliding-surface special and simple. Whereas chained systems cannot stand for general nonholonomic systems as stated above. These methods based on SMC cannot deal with the trajectory tracking problems of general constrained nonholonomic systems. Therefore, the problem of designing tracking controller based on SMC for a general class of nonholonomic systems with uncertainties is challengeable.

In this paper, we are focusing on nonholonomic systems with kinematics constraints. The challenge addressed here is how to ensure desired trajectory tracking at the dynamic control level when the modeling uncertainties and external disturbances are taken into account. The dynamic properties of mechanical systems are used to form a conservative upper boundedness of system dynamics. With this boundedness, a sliding mode control algorithm is derived, guaranteeing that any given trajectories can be tracked in the presence of structured and /or unstructured uncertainties. Stability analysis shows the trajectories can reach the sliding surfaces in finite time and are restricted on the sliding surfaces in the subsequent time. Moreover, the whole closed-loop system exhibits good tracking performance. The application of the developed design procedure is illustrated by an example.

The rest of paper is organized as follows: The dynamics models of nonholonomic systems with kinematics constraints and some preliminaries are included in section II. Section III presents sliding mode control law, its stability analysis and some remarks about the proposed scheme. The illustrative example to valid the proposed algorithm is given in section IV. Finally, some conclusions are given in section V.

II. SYSTEM DESCRIPTIONS

According to Lagrange theory[12], the dynamic equations incorporating constraint effects of mechanical systems with external disturbances have been derived as:

$$M(q)\ddot{q} + C(q,q)\dot{q} + G(q) + d(t) = E(q)\tau + A^T(q)\lambda \quad (1)$$

and m kinematics constraints are considered independent of time and can be expressed as:

$$A(q)\dot{q} = 0 \quad (2)$$

where $q \in R^n$ denotes the vector of generalized coordinates; $\tau \in R^r$ is the vector of generalized input torque; $M(q) \in R^{n\times n}$ is symmetric bounded positive definite inertia matrix; $C(q,\dot{q})\dot{q} \in R^n$ represents the vector of centripetal and Coriolis torques; $G(q) \in R^n$ is the gravitational torque vector; $d(t) \in R^n$ is external disturbances; $E(q) \in R^{n\times r}$ is a full rank input transformation matrix and is assumed to be known because it is a function of fixed geometry of system; $A(q) \in R^{m\times n}$ is the kinematics constraints matrix, $\lambda \in R^m$ is constraints force vector. Here, the effect of nonholonomic constraints can be viewed as restricting the dynamics on a constraint manifold as:

$$M = \{(q,\dot{q}) \in R^n \times R^n : A(q)\dot{q} = 0\} \quad (3)$$

The constraint manifold M in equation (3) is assumed to be exactly known in this study. Following popular handling way, so-called embedding approach [4-6], the nonholonomic dynamic equations (1) and (2) can be reduced to the so-called reduced-form dynamic equations.

Let $r_1(q),\cdots,r_{n-m}(q)$ be a set of smooth and linear independent vector fields in the null space of $A(q)$, then the following relation holds:

$$A(q)R(q) = 0 \quad (4)$$

where $R(q) = [r_1(q),\cdots,r_{n-m}(q)]$, the constraint equation (2) and equality (4) imply that there exists a $n-m$ dimensional vector $\dot{z}$ such that:

$$\dot{q} = R(q)\dot{z} \quad (5)$$

Then dynamic equation (1) satisfying nonholonomic constraint (2) can be written in terms of the internal state variable $\dot{z}$ as:

$$M(q)R(q)\ddot{z} + C_1(q,\dot{q})\dot{z} + G(q) + d(t) = E(q)\tau + A^T(q)\lambda \quad (6)$$

where $C_1(q,\dot{q}) = M(q)\dot{R}(q) + C(q,\dot{q})R(q)$, some fundamental properties are listed about this dynamic structure (6).

Property 1: The matrix $\overline{M}(q) = R^T(q)M(q)R(q)$ is symmetric and positive definite.

Property 2: A suitable definition of $C_1(q,\dot{q})$ makes the matrix $\dot{\overline{M}}(q) - 2R^T(q)C_1(q,\dot{q})$ skew-symmetric.

Property 3: $R^T(q)A^T(q) = 0$

Property 4: There exist positive scalars $\beta_i (i = 1,\cdots,5)$ such that $\forall q \in R^n, \forall \dot{q} \in R^n$: $\|M(q)\| \le \beta_1 < \infty, \|C(q,\dot{q})\| \le \beta_2 + \beta_3\|\dot{q}\|$ $\|G(q)\| \le \beta_4$ and $\sup_{t\ge 0}\|d(t)\| \le \beta_5$

It should be noted that the whole system consists of a new dynamic model (6) together with a pure kinematics relationship (5). This structure makes it possible to design the sliding mode control law.

III. SLIDING MODE CONTROL DESIGN

We now consider the trajectory tracking problem of the uncertain nonholonomic mechanical systems discussed above. In order to derive the controller, the following assumptions are required throughout this section.

*Assumption*1[5]: The matrix $R^T(q)E(q)$ is of full rank.

Assumption2[6]: The variable z and $\dot{z}$ are bounded, moreover the matrices $R(q)$ and $\dot{R}(q)$ are also bounded as

$\|R(q)\| \le \beta_6, \|\dot{R}(q)\| \le \beta_7\|\dot{q}\|$,where $\beta_i (i = 6,7)$ are positive constants.

Remarks1: Assumption1 can guarantee all $n-m$ degrees of freedom are independently actuated, which always holds for a large class of nonholonomic mechanical systems such as wheeled mobile robot and so on. Moreover, in these systems, as long as we appropriately select a set of $n-m$ dimensional vector $z(q)$ and $\dot{z}(q)$, internal variable $z(q)$ and $\dot{z}(q)$ possess the practical physical meaning[5-6].

Remark2: According to Ge's claim[6], if z is bounded, then $R(q)$ is bounded. If $\dot{z}$ is bounded, then $\dot{R}(q)$ is bounded. Therefore, assumption 2 is reasonable. The purpose of assumption 2 is to try to make full use of the available knowledge so as to reduce control gains.

A trajectory tracking control problem for perturbed nonholonomic mechanical systems is formulated as: Appropriately selecting $n-m$ dimensional vector $z(q)$ and $\dot{z}(q)$, given a desired z_d and $\dot{z}_d$, develop a sliding mode controller such that for any $(q(0),\dot{q}(0)) \in M$ as in (3), then $z(q)$ and $\dot{q}$ can converge to a manifold M_d specified as:

$$M_d = \{\{q,\dot{q}) \in R^n \times R^n | z(q) = z_d, \dot{q} = R(q)\dot{z}_d\} \quad (7)$$

Here, the vector $z(q)$ can be considered as $n-m$ "output equations" of the nonholonomic system.

Assumption3: The desired reference trajectory z_d is assumed to be bounded and uniformly continuous, and has bounded and uniformly continuous derivative up to the second order.

In the following, some variables are defined:

$$e = z - z_d \quad (8)$$

$$\dot{z}_r = \dot{z}_d - \Lambda e \quad (9)$$

where e and z_r denote the tracking error and a set of auxiliary signals, respectively. Λ is a positive definite matrix whose eigenvalues are strictly in the right-hand of complex plane. Then a sliding variable is defined as:

$$s = \dot{z} - \dot{z}_r = \dot{e} + \Lambda e \quad (10)$$

If the sliding surface exists on $s = 0$, according to the theory of SMC, the sliding mode is governed by the following differential equation:

$$\dot{e} = -\Lambda e$$

Obviously, the behavior of the system on the sliding surface is determined by the structure of matrix Λ . In other words, if the sliding mode exists on $s = 0$, the tracking error transient response is completely governed by the above equation.

The closed-loop system in terms of sliding variable s can be obtained:

$$\begin{aligned} M(q)R(q)\dot{s} &= E(q)\tau - (M(q)R(q)\ddot{z}_r + C_1(q,\dot{q})\dot{z}_r \\ &+ G(q) + d(t)) - C_1(q,\dot{q})s + A^T(q)\lambda \end{aligned} \quad (11)$$

Multiplying left $R^T(q)$ and using property 3 lead to:

$$\overline{M}(q)\dot{s} = \overline{E}(q)\tau - H(q,\dot{q},t) - \overline{C}(q,\dot{q})s \quad (12)$$

where

$\overline{M}(q) = R^T(q)M(q)R(q),\ \overline{C}(q,\dot{q}) = R^T(q)C_1(q,\dot{q})$,

$H(q,\dot{q},t) = R^T(q)M(q)R(q)\ddot{z}_r + R^T(q)C_1(q,\dot{q})\dot{z}_r + R^T(q)G(q) + R^T(q)d(t)$.

$\overline{E}(q) = R^T(q)E(q)$,

Before we design control input, following Vicente's work[18], we have the result as:

Lemma 1: The expression $H(q,\dot{q},t)$ in equation (12) is upper bounded, moreover, the boundedness is a state-dependent function.

Proof: Obviously the fact that desired reference trajectory and its derivative up to the second order are bounded implies the existence of two positive constants β_8 and β_9 such that $\|\dot{z}_r\| \le \beta_8 + \Lambda\|e\|$ and $\|\ddot{z}_r\| \le \beta_9 + \Lambda\|\dot{e}\|$, Therefore, according to property 4, the following can be reached:

$$\begin{aligned} &\|H(q,\dot{q},t)\| \\ &\le \|R(q)\| \cdot \|M(q)\| \cdot \|R(q)\| \cdot \|\ddot{z}_r\| + \\ &\|R(q)\| \cdot (\|M(q)\| \cdot \|\dot{R}(q)\| + \|C(q,\dot{q})\| \cdot \|R(q)\|) \cdot \|\dot{z}_r\| \\ &+ \|R(q)\| \cdot \|G(q)\| + \|R(q)\| \cdot \|d(t)\| \\ &\le \beta_6^2 \cdot \beta_1 \cdot \|\ddot{z}_r\| + \beta_6(\beta_1 \cdot \beta_7 \cdot \|\dot{q}\| + (\beta_2 + \beta_3 \cdot \|\dot{q}\|) \cdot \beta_6)\|\dot{z}_r\| \\ &+ \beta_6\beta_4 + \beta_6\beta_5 \\ &= \overline{\beta}_1 \cdot \|\ddot{z}_r\| + \overline{\beta}_2\|\dot{z}_r\| + \overline{\beta}_3\|\dot{z}_r\| \cdot \|\dot{q}\| + \overline{\beta}_4 \end{aligned} \quad (13)$$

where

$\overline{\beta}_1 = \beta_6^2\beta_1, \overline{\beta}_2 = \beta_2\beta_6^2, \overline{\beta}_3 = \beta_1\beta_6\beta_7 + \beta_6\beta_2, \overline{\beta}_4 = \beta_6(\beta_4 + \beta_5)$.

Here, we denote $\eta(t) = \overline{\beta}_1 \cdot \|\ddot{z}_r\| + \overline{\beta}_2\|\dot{z}_r\| + \overline{\beta}_3\|\dot{z}_r\| \cdot \|\dot{q}\| + \overline{\beta}_4$ and call $\eta(t)$ as the upper bounded function, which is obviously a function of state variables.

Remark 3: Lemma 1 formulates the upper boundedness of systems dynamics, which is a positive definite second order polynomial. What we are concerned with is not the concrete values of variables but boundedness of their values, thus the measurement of acceleration signals is not required. This forms the subsequent foundation of the derivation of the proposed control law.

Assume that the parameters $\overline{\beta}_i\ (i = 1,2,3,4)$ of upper boundedess of the system dynamics are known, we consider the following controller:

$$\overline{E}(q)\tau = -Ks - \frac{s \cdot (\eta(t))^2}{\|s\| \cdot \eta(t) + \varepsilon(t)} - K_d \operatorname{sgn}(s) \quad (14)$$

where K and K_d are $(n-m) \times (n-m)$ positive definite gain matrices determined by the designer, $\eta(t)$ is the upper boundedness of the system dynamics as stated in lemma 1, $\varepsilon(t) > 0$ is a time-varying function and satisfies $\int_0^t \varepsilon(\rho)d\rho < \infty$.

Theorem: Consider the uncertain nonholonomic dynamic system (1) and (2), the SMC laws are chosen as (14). Then, for any $(q(0),\dot{q}(0)) \in M$, the tracking error e and its derivative $\dot{e}$ converge to the sliding surface and are restricted to the surface for all future time.

Proof: Consider the Lyapunov candidate function as:

$$V = \frac{1}{2}s^T \overline{M}s \qquad (15)$$

The time derivative of V leads to:

$$\dot{V} = s^T \overline{M}\dot{s} + \frac{1}{2}s^T \dot{\overline{M}}s \qquad (16)$$

Using (12), the above equality becomes:

$$\dot{V} = s^T (E(q)\tau - H(q,\dot{q},t) - \overline{C}(q,\dot{q})s) + \frac{1}{2}s^T \dot{\overline{M}}s \qquad (17)$$

Substituting control law (14) into (17) and paying attention to property 2, the following can be obtained:

$$\begin{aligned}\dot{V} &= s^T(-Ks - \frac{s\cdot(\eta(t))^2}{\|s\|\cdot\eta(t)+\varepsilon(t)} - H(q,\dot{q},t) - K_d\, sgn(s)) \\ &\le -s^T Ks - \frac{\|s\|^2\cdot(\eta(t))^2}{\|s\|\cdot\eta(t)+\varepsilon(t)} + \|s\|\cdot\|H(q,\dot{q},t)\| - s^T K_d\, sgn(s) \\ &\le -s^T Ks - \frac{\|s\|^2\cdot(\eta(t))^2}{\|s\|\cdot\eta(t)+\varepsilon(t)} + \frac{\|s\|\cdot\eta(t)(\|s\|\cdot\eta(t)+\varepsilon(t))}{\|s\|\cdot\eta(t)+\varepsilon(t)} - K_d|s| \\ &\le -s^T Ks + \varepsilon(t) - K_d|s|\end{aligned} \qquad (18)$$

Thus,by selecting the large gain K_d , it is easy to check:

$$\dot{s}^T s \le -\mu|s| \qquad (19)$$

where μ is a positive constant. The above expression shows that any trajectories away from the sliding surface are forced to reach the sliding surface in finite time. Subsequently, we discuss the convergence rate of trajectories. From (18), we can obtain:

$$\dot{V} \le \omega V + \varepsilon(t)$$

where $\omega = \lambda_{\min}(K)/\lambda_{\max}(\overline{M})$, therefore, from Lyapunov theory, we know that if $\varepsilon(t)$ asymptotically or exponentially tends to zero, s asymptotically or exponentially converges to zero, respectively. Since s and $e,\dot{e}$ are related by (10). This implies in turn the position tracking error e and velocity tracking error $\dot{e}$ will also asymptotically or exponentially tend to sliding surfaces $s=0$, respectively, depending on the selection of $\varepsilon(t)$.

From definition (8) of variable e and the relationship (5) between q and z , we can safely say that the control objective stated at the paragraph below remark 2 is realized using above theorem.

Remark 4: Although many researchers obtain the simultaneous tracking of z, q and therefore the tracking of $\dot{z},\dot{q}$, the control plants considered in their works are either concrete nonholonimic systems such as wheeled mobile robot[10-11] and so on, or chained systems[16-17], as a special case of nonholonomic systems. It is well known that wheeled mobile robot and other concrete plants of nonholnomic systems have parameterized kinematics model about q , and also chained systems have precise equation of the state variables q , so Lyapunov candidate function in terns of the error of q is easily founded for the two cases and then the simultaneous tracking of z, q and $\dot{z},\dot{q}$ can be acquired. These methods mentioned above need kinematics controller and dynamics controller at the same time. Namely, they are methods combining kinematics and dynamics. *However, the problems under the consideration in our works is different from the above and similar to* Su[4], Ge[6], Chen[5] et al. *Our work is for a general class of nonholonomic systems and at dynamics level, where kinematics model is not used for controller design but only for simplifying dynamics model.* Namely, The methods of Su, Ge, Chen and us are dynamics approaches. Therefore, a theoretical result for dynamic-based trajectory tracking control of nonholonomic dynamic systems is proposed in this paper.

Remark 5: The upper boundedness of system dynamics in lemma 1 is the linear combination of system states and includes four parameters $\overline{\beta}_i$ $(i=1,2,3,4)$. The control law (14) is designed on condition that $\overline{\beta}_i$ $(i=1,2,3,4)$ are known. If $\overline{\beta}_i$ $(i=1,2,3,4)$ are unknown, we can replace $\overline{\beta}_i$ $(i=1,2,3,4)$ with their estimations $\hat{\overline{\beta}}_i$ $(i=1,2,3,4)$, and then take adaptive technique to learn the parameters $\hat{\overline{\beta}}_i$ $(i=1,2,3,4)$. The design of controller, the selection of adaptive algorithm and the constitution of Lynapunov function are similar to Ge's approach, when $\overline{\beta}_i$ $(i=1,2,3,4)$ are unknown.

Remark 6: $\varepsilon(t)$ is a designed parameter in the proposed scheme. When $\varepsilon(t)$ tends to zero as t go to infinity, on the one hand, the closed-loop systems can be characterized as asymptotical stability or exponential stability, which benefits to theoretical analysis. On the other hand, the second term $\frac{s\cdot(\eta(t))^2}{\|s\|\cdot\eta(t)+\varepsilon(t)}$ becomes discontinuous as t go to infinity. Therefore, the influences of $\varepsilon(t)$ on closed-loop system are two folds. If we use a constant to replace $\varepsilon(t)$, the second term is always continuous, but only global uniformly ultimately boundedness can be guaranteed and asymptotical stability or exponential stability is lost.

Remark 7: During the development of motion control of nonholonomic systems with velocity constraints, the approaches based on regressor matrix have become popular [4],[16], where the dynamics models of mechanical systems are assumed to be expressed as product of a regressor matrix and an unknown parameter vector, which is so-called the property of linearity in the system parameters. This property can be employed to simplify the design of control laws and to constitute the adaptive algorithm and so on. However, there are some difficulties associated with this method. For instance, the task of computing regressor matrix is time-consuming, the implementation of this approach requires the perfect knowledge of dynamic models and so on.

To remove the drawbacks, the proposed scheme avoids the use of regressor matrix.

Rmark8: The proposed scheme is similar to Ge's[6] controller in form. But it should be noted that Ge's approaches did not utilize the boundedness of some variables but directly use their values in controller, which results in complex calculation on line, for example, Ge's controller includes the term like $\|(d/dt)[R(q)\dot{z}_r]\|$.

IV. Simulation Results

As an example to verify the proposed controller, here a simplified model of wheeled mobile robot moving on a horizontal plane, as given in detail in [20], is considered. The dynamics model can be expressed as:

$$m\ddot{x} = \lambda\cos\theta - \frac{1}{P}(\tau_1 + \tau_2)\sin\theta$$

$$m\ddot{y} = \lambda\sin\theta + \frac{1}{P}(\tau_1 + \tau_2)\cos\theta$$

$$I\ddot{\theta} = \frac{L}{P}(\tau_1 - \tau_2)$$

where (x, y) denotes the position coordinate in the inertial frame, θ denotes the orientation with respects to the inertial frame. m and I are the mass of the mobile robot and its inertial moment about the vertical axis, respectively. P represents the radius of the wheels and $2L$ the distance between two front wheels. τ_1 and τ_2 are the torques provided by the motors and determined by the designer. We set $L = P = 1$ for simplicity. The nonholonomic constraints is of the form: $\dot{x}\cos\theta + \dot{y}\sin\theta = 0$. Therefore vector q and the matrix $A(q)$ are defined as: $q = [x, y, \theta]^T$ and $A(q) = [\cos\theta, \sin\theta, 0]^T$, respectively. It can be easily obtained $M(q) = diag[m, m, I]$, $C(q, \dot{q}) = 0$ and $G(q) = 0$. Like [4] and [5], the variable $z(q)$ is selected as: $z(q) = [y, \theta]^T$ and therefore $R(q)$ becomes:

$$R(q) = \begin{bmatrix} -\tan\theta & 0 \\ 1 & 0 \\ 0 & 1 \end{bmatrix}$$

So the relation $\dot{q} = R(q)\dot{z}$ naturally holds. The desired manifold is chosen as:

$$M_d = \{\{q, \dot{q}\} | z(q) = [2\sin 2t, 2t]^T, \dot{q} = R(q)\dot{z}_d\}$$

In our simulation, Like in [4] and [5], the real physical parameters take $m = 0.5Kg$ and $I = 0.5Kg \cdot m^2$, respectively. The inertial conditions are taken as: $q(0) = [0,0,0]^T$ and $\dot{q}(0) = [0,0,0]^T$. Suppose the external disturbances $d_i (i = 1,2)$ are defined as $d_1 = \sin(t)$ and $d_2 = \cos(t)$, respectively. The control parameters in equation (14) are selected as: $\Lambda = diag[4,4]$, $K = diag[10,10]$, $K_d = diag[2,2]$ and $\eta = 5$, respectively.

The simulation results that the proposed approach is applied to the constrained nonholonomic systems are show in from Fig.1-6. Position tracking errors of y, θ are shown in Fig.1 and Fig.2, respectively. Velocity tracking errors of y, θ are shown in Fig.3 and Fig.4, respectively. It can be seen that the tracking performance is good. Fig. 5 and Fig.6 show the sliding variables which confirm that the trajectories quickly reach the sliding surfaces. However, the phenomena of chattering occur near the sliding surfaces, which can be seen clearly from the local magnifying graphs inside the corresponding figures.

V. Conclusion

In this paper, the trajectory tracking problem is addressed for a large class of nonholonomic mechanical systems with uncertainties and external disturbances, and a sliding mode controller is presented at the dynamic level. The controller can drive the system motion to the desired manifold. The stability analysis is carried out using Lyapunov approach. The application to a simplified mobile robot is described, and the simulation results show the effectiveness of the development.

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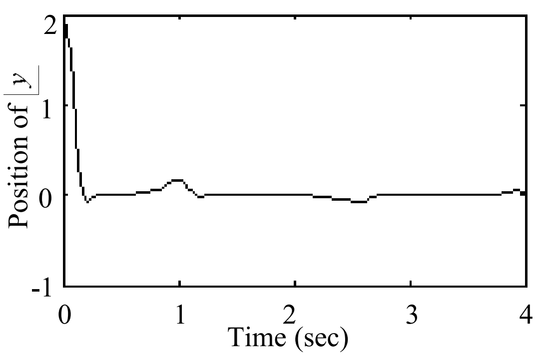

Fig.1 Position error of y

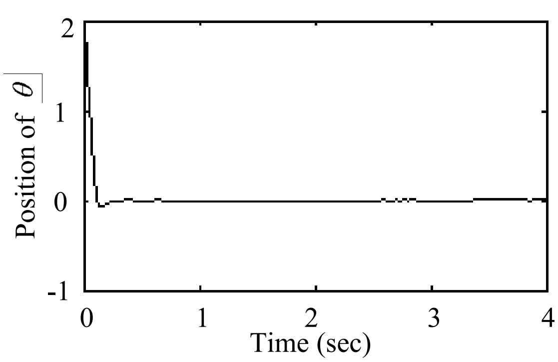

Fig.2 Position error of θ

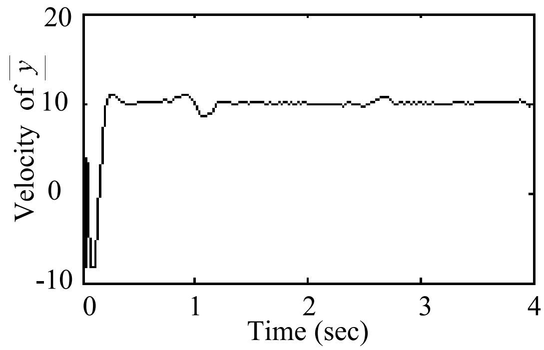

Fig.3 Velocity error of $\dot{y}$

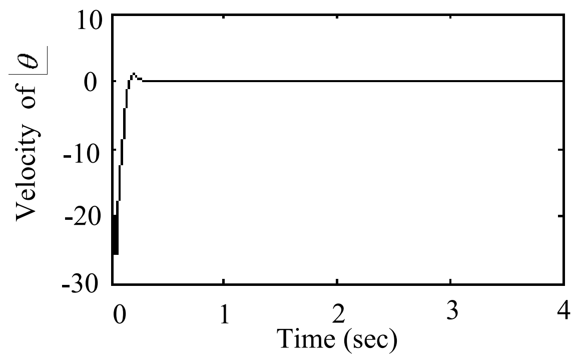

Fig.4 Velocity error of $\dot{\theta}$

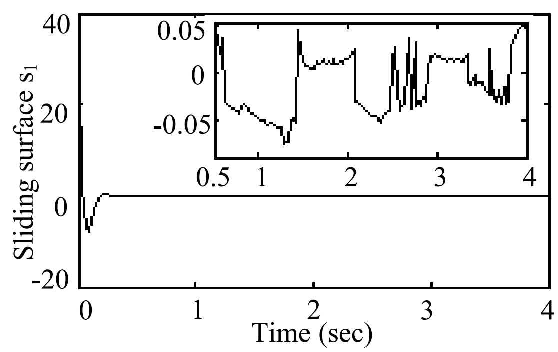

Fig.5 Sliding surface s_1

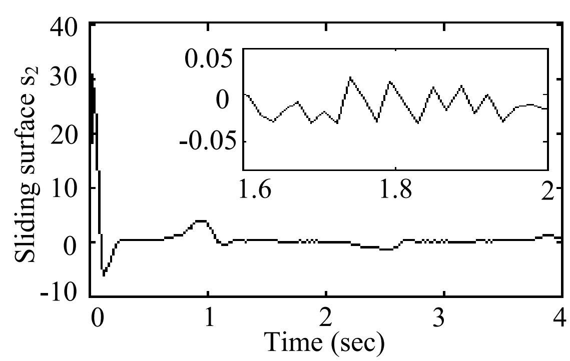

Fig.6 Sliding surface s_2