2005 American Control Conference
June 8-10, 2005. Portland, OR, USA



# New complex frame to model and control an active filter

Claude Dewez and Laurent Rambault and Jean Paul Gaubert

*Abstract*— **In this paper, a linear model of an active filter and its controller are proposed. The usual models are non linear due to the Park transform. The main interest of our work is the linearity of the model described as a state space representation. The originality of the model obtained is the complex valued state space representation. An experimental approach is proposed to perform controllers with complex valued parameters.**

## I. INTRODUCTION

Power quality has received increased attention in recent year with the widespread application of nonlinear load employing advanced power switching devices in a multitude of industrial and commercial applications. Some devices like power systems draw non sinusoidal load current consisting primarily of lower-order 5th, 7th, 11th and 13th harmonics that distort the system power quality. In order to eliminate the harmonics currents, the structures proposed are usually made up either a simple active system using inverter with ”HF” filter, or a hybrid system (Fig. 1) [1], [2]. The main objective of the active (or hybrid) harmonic filters proposed is to monitor current absorbed by the load, and generate an adaptive current waveform that matches the shape of the nonlinear portion of the load current.

The work developed in this paper concerns the modelling of the system in order to obtain a linear multivariable model [3]. The main difficulty to control different harmonics is to measure the disturbance. Classically the Park transform and a low pass filter allow to measure the disturbance. This approach provides a nonlinear sensor that complicates the design of a controller. Our approach proposes a linear sensor in order to use the control theory in a linear context. The main problem is to control the inverter to eliminate the 11th and the 13th harmonics without taking into account the 5th and the 7th harmonics. So to perform this separation of the inverter and the ”LC” filters it is important to model all the structure proposed.

In this paper, a linear model of an active filter and its controller are proposed in the following sections. The main interest is the linearity of the model. The usual model is a non linear one due to the Park transform. The originality of the model obtained is the complex valued state space representation. The section 3. deals with the controller design with an experimental approach.

Claude Dewez is with LAII, University of Poitiers, France Claude.Dewez@univ-poitiers.fr
Laurent Rambault is with LAII, University of Poitiers, France Laurent.Rambault@univ-poitiers.fr
Jean Paul Gaubert is with LAII, University of Poitiers, France Jean.Paul.Gaubert@univ-poitiers.fr

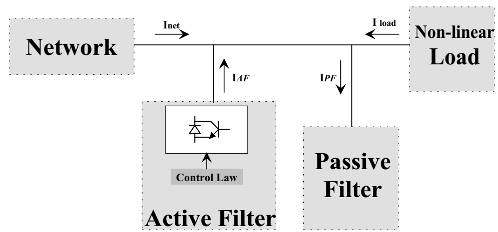


Fig. 1. Principle of hybrid filter structure

## II. LINEAR MODEL OF AN HYBRID FILTER

The three-phase network structure proposed in this paper is shown on Fig. 2. The equations of this system describe the electrical behaviour of each phase $i$ :

$$
\begin{aligned}
[i_{net}]_{i=1,2or3} &= [i_{Caf}]_{i=1,2or3} - [i_{load}]_{i=1,2or3} - [i_{inv}]_{i=1,2or3} \\
[U_{inv}]_{i=1,2or3} &= L_{afi}\left[\frac{di_{inv}}{dt}\right]_{i=1,2or3} + [v_{Caf}]_{i=1,2or3} + [V_{NCaf}] \\
[i_{Caf}]_{i=1,2or3} &= C_{afi}\left[\frac{dv_{Caf}}{dt}\right]_{i=1,2or3} \\
[V_{net}]_{i=1,2or3} &= L_{neti}\left[\frac{di_{net}}{dt}\right]_{i=1,2or3} + r_{neti}[i_{net}]_{i=1,2or3} \\
&\quad + [v_{Caf}]_{i=1,2or3} + [V_{NCaf}]
\end{aligned}
$$

By applying Concordia transform, the three phases system in the particular case where the network is a balancing realization, the $\alpha$ and $\beta$ coordinates are independent. From

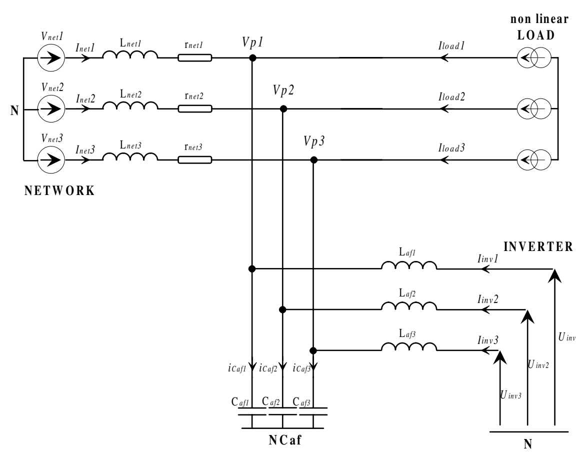


Fig. 2. three phase network with parallel active filter

the structure proposed on Fig.(2) , we obtain the following model for $C = \alpha$ or $\beta$ :

$$
\begin{aligned}
[i_{net}]_C &= [i_{Caf}]_C - [i_{load}]_C - [i_{inv}]_C && (1)\\
[U_{inv}]_C &= L_{af}\left[\frac{di_{inv}}{dt}\right]_C + \left[v_{Caf}\right]_C && (2)\\
\left[i_{Caf}\right]_C &= C_{af}\left[\frac{dv_{Caf}}{dt}\right]_C && (3)\\
[V_{net}]_C &= L_{net}\left[\frac{di_{net}}{dt}\right]_C + r_{net}\,[i_{net}]_C + \left[v_{Caf}\right]_C && (4)
\end{aligned}
$$

So it is possible to write the model as state space representation.

$$
\begin{aligned}
\begin{bmatrix}\dot{X}_\alpha\\ \dot{X}_\beta\end{bmatrix} &= \begin{pmatrix}A & \mathbf{0}\\ \mathbf{0} & A\end{pmatrix}\begin{bmatrix}X_\alpha\\ X_\beta\end{bmatrix} + \begin{pmatrix}B_u & \mathbf{0}\\ \mathbf{0} & B_u\end{pmatrix}\begin{bmatrix}U_{inv_\alpha}\\ U_{inv_\beta}\end{bmatrix}\\
&\quad + \begin{pmatrix}B_w & \mathbf{0}\\ \mathbf{0} & B_w\end{pmatrix}\begin{bmatrix}w_\alpha\\ w_\beta\end{bmatrix}\\
\begin{bmatrix}s_\alpha\\ s_\beta\end{bmatrix} &= \begin{pmatrix}C & \mathbf{0}\\ \mathbf{0} & C\end{pmatrix}\begin{bmatrix}X_\alpha\\ X_\beta\end{bmatrix} + \begin{pmatrix}D_u & \mathbf{0}\\ \mathbf{0} & D_u\end{pmatrix}\begin{bmatrix}U_{inv_\alpha}\\ U_{inv_\beta}\end{bmatrix}\\
&\quad + \begin{pmatrix}D_w & \mathbf{0}\\ \mathbf{0} & D_w\end{pmatrix}\begin{bmatrix}w_\alpha\\ w_\beta\end{bmatrix} \qquad (5)
\end{aligned}
$$

Where $\mathbf{0}$ is a zero matrix with an appropriate size and for $C = \alpha$ or $\beta$ component :

$$
X_C : \begin{bmatrix}i_{net_C}\\ i_{inv_C}\\ v_{Caf_C}\end{bmatrix},\ s_C : \begin{bmatrix}i_{net_C}\\ i_{inv_C}\end{bmatrix} \quad \text{and} \quad w_C : \begin{bmatrix}i_{load_C}\\ V_{net_C}\end{bmatrix}
$$

$$
A : \begin{pmatrix} -\frac{r_{net}}{L_{net}} & 0 & -\frac{1}{L_{net}}\\ 0 & 0 & -\frac{1}{L_{af}}\\ \frac{1}{C_{af}} & \frac{1}{C_{af}} & 0\end{pmatrix} \qquad B_u : \begin{pmatrix}0\\ \frac{1}{L_{af}}\\ 0\end{pmatrix}
$$

$$
B_w : \begin{pmatrix} 0 & \frac{1}{L_{net}}\\ 0 & 0\\ \frac{1}{C_{af}} & 0\end{pmatrix} \qquad C : \begin{pmatrix}1 & 0 & 0\\ 0 & 1 & 0\end{pmatrix}
$$

$$
D_u : \begin{pmatrix}0\\ 0\end{pmatrix} \qquad D_w : \begin{pmatrix}0 & 0\\ 0 & 0\end{pmatrix}
$$

From the model proposed in $(\alpha,\beta)$ frame work, it is difficult to perform a controller in order to cancel the harmonic effect generated by the non linear load. To avoid this difficulty, the Park transform is proposed. The main idea is to translate the frequency component we try to eliminate, to frequency equal 0. So from the equations (1) to (4), the Park transform is applied. The details of the method used is developed in order to justify the main interest of this model. In fact with the help of classical approach, the Park transform is used to isolate and eliminate the frequency component. Generally a low pass filter is added to improve the result given by the Park transform. From this approach the model obtained is non linear due to the transformation. In this paper, we propose to use the Park transform and a decoupling transformation in order to perform a linear model.

Let a vector $\begin{bmatrix}y_\alpha\\ y_\beta\end{bmatrix}$, such as: $\begin{bmatrix}y_d\\ y_q\end{bmatrix} = \rho^{-1}(\theta)\begin{bmatrix}y_\alpha\\ y_\beta\end{bmatrix}$,

with $\rho^{-1}(\theta) = \begin{pmatrix} cos(\theta) & sin(\theta)\\ -sin(\theta) & cos(\theta)\end{pmatrix}$.

So we have: $\frac{d}{dt}\begin{bmatrix}y_\alpha\\ y_\beta\end{bmatrix} = \rho(\theta)\frac{d}{dt}\begin{bmatrix}y_d\\ y_q\end{bmatrix} + \dot{\rho}(\theta)\begin{bmatrix}y_d\\ y_q\end{bmatrix}$

with the property

$$
\rho^{-1}(\theta)\dot{\rho}(\theta) = \begin{pmatrix}0 & -\dot{\theta}\\ \dot{\theta} & 0\end{pmatrix}.
$$

Then it is possible to write :

$$
\rho^{-1}(\theta)\begin{bmatrix}\dot{y}_\alpha\\ \dot{y}_\beta\end{bmatrix} = \begin{bmatrix}\dot{y}_d\\ \dot{y}_q\end{bmatrix} + \begin{pmatrix}0 & -\dot{\theta}\\ \dot{\theta} & 0\end{pmatrix}\begin{bmatrix}y_d\\ y_q\end{bmatrix}.
$$

It allows to perform the following state space representation:

$$
\begin{aligned}
\begin{bmatrix}\dot{X}_d\\ \dot{X}_q\end{bmatrix} &= \begin{pmatrix}A & \dot{\theta}\mathbb{I}_n\\ -\dot{\theta}\mathbb{I}_n & A\end{pmatrix}\begin{bmatrix}X_d\\ X_q\end{bmatrix} + \begin{pmatrix}B_u & \mathbf{0}\\ \mathbf{0} & B_u\end{pmatrix}\begin{bmatrix}U_{inv_d}\\ U_{inv_q}\end{bmatrix}\\
&\quad + \begin{pmatrix}B_w & \mathbf{0}\\ \mathbf{0} & B_w\end{pmatrix}\begin{bmatrix}w_d\\ w_q\end{bmatrix}\\
\begin{bmatrix}s_d\\ s_q\end{bmatrix} &= \begin{pmatrix}C & \mathbf{0}\\ \mathbf{0} & C\end{pmatrix}\begin{bmatrix}X_d\\ X_q\end{bmatrix} + \begin{pmatrix}D_u & \mathbf{0}\\ \mathbf{0} & D_u\end{pmatrix}\begin{bmatrix}U_{inv_d}\\ U_{inv_q}\end{bmatrix}\\
&\quad + \begin{pmatrix}D_w & \mathbf{0}\\ \mathbf{0} & D_w\end{pmatrix}\begin{bmatrix}w_d\\ w_q\end{bmatrix} \qquad (6)
\end{aligned}
$$

We observe that the rotation matrix from Park transform expresses itself through the parameter $\dot{\theta}$ in the state matrix. However, if we consider $\dot{\theta}$ equals to a constant, we obtain a state space representation with a linear time invariant model. It is important to notice that the Park transform induces coupling between $d$ and $q$ axes. In order to simplify the design of controller, we propose the following transformer. In this case, all the linear control theory is available to perform a controller.

$$
\Delta = T^{-1}A_{dq}T \quad \text{with} \quad T = \frac{1}{\sqrt{2}}\begin{pmatrix}\mathbb{I}_n & j\mathbb{I}_n\\ j\mathbb{I}_n & \mathbb{I}_n\end{pmatrix}, \qquad (7)
$$

$$
\Delta = \begin{pmatrix}A + j\dot{\theta}\mathbb{I}_n & \mathbf{0}_n\\ \mathbf{0}_n & A - j\dot{\theta}\mathbb{I}_n\end{pmatrix} \qquad (8)
$$

$n$ represents the order of equivalent system for one phase and $j$ is defined as $j = \sqrt{-1}$.

The state space representation in the new framework can be written as:

$$
\begin{aligned}
\begin{bmatrix} \dot{X}_{cd} \\ \dot{X}_{qi} \end{bmatrix} &= \begin{pmatrix} A + j\dot{\theta}I\!I_n & \mathcal{O}_n \\ \mathcal{O}_n & A - j\dot{\theta}I\!I_n \end{pmatrix} \begin{bmatrix} X_{cd} \\ X_{qi} \end{bmatrix} \\
&+ \begin{pmatrix} B_u & \mathcal{O} \\ \mathcal{O} & B_u \end{pmatrix} \begin{bmatrix} U_{inv_{cd}} \\ U_{inv_{qi}} \end{bmatrix} + \begin{pmatrix} B_w & \mathcal{O} \\ \mathcal{O} & B_w \end{pmatrix} \begin{bmatrix} w_{cd} \\ w_{qi} \end{bmatrix} \\
\begin{bmatrix} s_{cd} \\ s_{qi} \end{bmatrix} &= \begin{pmatrix} C & \mathcal{O} \\ \mathcal{O} & C \end{pmatrix} \begin{bmatrix} X_{cd} \\ X_{qi} \end{bmatrix} + \begin{pmatrix} D_u & \mathcal{O} \\ \mathcal{O} & D_u \end{pmatrix} \begin{bmatrix} U_{inv_{cd}} \\ U_{inv_{qi}} \end{bmatrix} \\
&+ \begin{pmatrix} D_w & \mathcal{O} \\ \mathcal{O} & D_w \end{pmatrix} \begin{bmatrix} w_{cd} \\ w_{qi} \end{bmatrix}, \qquad (9) \\
\text{with: } \begin{bmatrix} X_{cd} \\ X_{qi} \end{bmatrix} &= T^{-1} \begin{bmatrix} X_d \\ X_q \end{bmatrix}.
\end{aligned}
$$

For a frequency analysis, we can notice that the transfer matrix from the state representation (9) is for the "cd" component :

$$C(sI\!I - (A + j\dot{\theta}I\!I))^{-1}B + D = C((sI\!I - j\dot{\theta}I\!I) - A)^{-1}B + D.$$

This is equivalent, for $s' = s - j\dot{\theta}$, to the $\alpha - \beta$ (or a - b - c) reference frame-based transfer matrix from the state representation (5). That is to say, it is the same frequency response with a right translation (if $\dot{\theta} > 0$) from the frequency. Alike, the frequency response for the "qi" component correspond to a left translation (if $\dot{\theta} > 0$) from the frequency.

Furthermore, let consider $(i_1, i_2, i_3)$ a system of balancing three-phase variables with harmonics (as $i_1 = \sum_k I_k \cos(k\omega t + \alpha_k)$).

After the Concordia and Park transforms, we obtain :

$$
\left\{
\begin{aligned}
i_d &= \sqrt{3/2} \left( \sum_{k=3n+1} I_k \cos((k-h)\omega t + \alpha_k) \right. \\
&\quad \left. + \sum_{k=3n+2} I_k \cos((k+h)\omega t + \alpha_k) \right) \\
i_q &= \sqrt{3/2} \left( \sum_{k=3n+1} I_k \sin((k-h)\omega t + \alpha_k) \right. \\
&\quad \left. + \sum_{k=3n+2} I_k \sin((k+h)\omega t + \alpha_k) \right)
\end{aligned}
\right. \qquad (10)
$$

where $\dot{\theta}$ is constant as $\theta = \dot{\theta}t = \text{h}.\omega.t$,

and $\left\{ \begin{aligned} i_{cd} &= \sqrt{1/2}\ (i_d - \text{j}\ i_q) \\ i_{qi} &= \sqrt{1/2}\ (i_q - \text{j}\ i_d) \end{aligned} \right.$

The choice of $\dot{\theta}$ allows to extract a particular frequency harmonic such $k - h = 0$ for the harmonics of rank $3k+1$ ($\dot{\theta} > 0$ : direct frame), or $k + h = 0$ for the harmonics of rank $3k+2$ ($\dot{\theta} < 0$ : indirect frame) ; the others harmonics are moving away to a frequency $\dot{\theta} = (k-h)\omega$ or $\dot{\theta} = (k+h)\omega$.

The main interest of the model proposed in this paper is in control design. The model is a linear one and it includes the harmonic estimation without non linear sensor to estimate frequency harmonics. This technique allows to perform a linear controller by using tools developed in control theory [5], [6]. One of the main interest is the possibility to take into account the robustness. In fact, in linear control, the modification of physical parameters can be studied in two ways. The first one is to integrate the robustness after the design step by using analysis tools to verify the stability or some particular performances with an uncertain model. The second way is to include the uncertainties in the design step for example with the help of polytopic representation. This approach provides good results for linear systems. Furthermore, we try to design a structured controller like PID controller that represents an interest in industrial context.

## III. CONTROLLER DESIGN

From the suggested modelization, we have seen that it is possible to distinguish a specific frequency when we choose an appropriate frame with the parameter $\dot{\theta}$. In this frame, the signal to be controlled corresponds to the pulsation $\omega$ = 0 rd/s.

For the control law synthesis, we use the standard representation Fig. 3, where the network voltage $V_{net}$ and the load current $i_{load}$ represent two perturbations (exogenous inputs : $w$) and the inverter voltage $U_{inv}$ is the control input ($u$). The measurement ($y$) of the network current and the inverter current act on the controller.

The main objective of the synthesis is to minimize the transfer $z/w$ to the meaning of norm (e.g. $H_2$ or $H_\infty$), where $z$ is the signal which represents harmonics to eliminate (e.g. $I_{inv}$ or $I_{net}$).

For example, the main specifications to obtain are :

- to eliminate the fundamental component of the inverter current (no compensation of the reactive power)
- to eliminate harmonics (e.g. 5th, 7th, 11th or 13th)
- to control the resonance (limitation of the overvoltage to connection point with the inverter).

In the framework considered, the controller is equivalent to a single low-pass filter about which we can easily specify the performances to the meaning of static gain (compensation of the harmonic considered as a perturbation) and band width. Defining a band width allows to obtain the robustness towards the variations of the network frequency, these variations are similar to an uncertainty on $\dot{\theta}$.

In first step, we design a controller in order to cancel the fundamental component of the inverter current (no

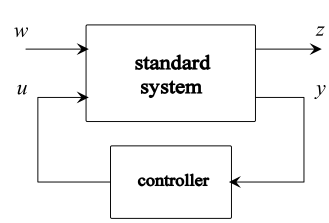


Fig. 3. Standard representation

compensation of the reactive power). Since the "$cd-qi$" frame is a complex frame, we can choose a controller whose transfer has complex parameters. However, this induces a constraint on the structure of controller in order to be able to implement it in the $\alpha-\beta$ frame or $a-b-c$ frame with real parameters.

The structure of the controller is defined as :

$$\begin{pmatrix} K & 0 \\ 0 & conj(K) \end{pmatrix},$$

where $K$ is the transfer matrix with complex parameters for the "$cd$" component and $conj(K)$ is the congugate of $K$.

For the synthesis of the controller, the measurement vector $y$ is limited to the inverter current. The transfer $u/y$, let $U_{inv}/I_{inv}$, is :

$$K(s) = \frac{0,2(1+i)s+120}{0,25(1+i)s+1},$$

for the cd component.

The frequency response is shown Fig. 4 in the "$cd-qi$" frame for $\dot{\theta} = 100\pi$. For a variation of the network frequency about one percent, we choose a band width equal to $\pi$ rd/s that allows to achieve the robustness properties required. The other frequencies are beyond $6\dot{\theta}$.

This approach allows to synthesize similar structures of controller, however, it requires to recalculate the transfer of the closed-loop system with the controller previously synthesized.

Let consider the system specified by the state space

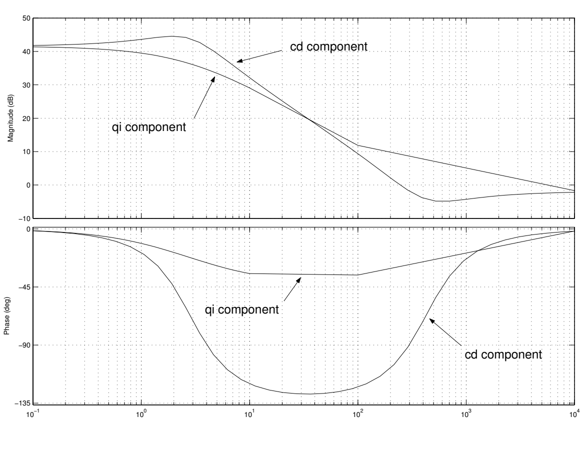


Fig. 4. Controller frequency response performed for the fundamental of inverter current

representation (9) and a complex controller $K$ such as :

$$\begin{aligned}\begin{bmatrix}\dot{X}_{Kcd}\\ \dot{X}_{Kqi}\end{bmatrix} &= \begin{pmatrix} A_K & \mathbf{0} \\ \mathbf{0} & conj(A_K)\end{pmatrix}\begin{bmatrix}X_{Kcd}\\ X_{Kqi}\end{bmatrix} + \begin{pmatrix} B_K & \mathbf{0}\\ \mathbf{0} & B_K\end{pmatrix}\begin{bmatrix}y_{cd}\\ y_{qi}\end{bmatrix}\\ \begin{bmatrix}U_{Kinv_{cd}}\\ U_{Kinv_{qi}}\end{bmatrix} &= \begin{pmatrix} C_K & \mathbf{0} \\ \mathbf{0} & conj(C_K)\end{pmatrix}\begin{bmatrix}X_{Kcd}\\ X_{Kqi}\end{bmatrix} + \begin{pmatrix} D_K & \mathbf{0}\\ \mathbf{0} & D_K\end{pmatrix}\begin{bmatrix}y_{cd}\\ y_{qi}\end{bmatrix}.\end{aligned} \tag{11}$$

The current frame, where the controller has been synthesized, is defined for an angular velocity $\dot{\theta}$ equal to $\dot{\theta}_1$. In this case, when $D_u$ and $D_w$ are null matrices (9), the state space representation of the closed-loop system is:

- for the "cd" component :

$$\begin{aligned}\begin{bmatrix}\dot{X}_{cd}\\ \dot{X}_{Kcd}\end{bmatrix} &= \begin{pmatrix} A+j\dot{\theta}_1 I\!I_n + B_u D_K C & B_u(RC_K+jIC_K)\\ B_K C & RA_K + jIA_K\end{pmatrix}\begin{bmatrix}X_{cd}\\ X_{Kcd}\end{bmatrix} + \begin{pmatrix}B_u\\ \mathbf{0}\end{pmatrix}u_{cd} + \begin{pmatrix}B_w\\ \mathbf{0}\end{pmatrix}w_{cd}\\ y_{cd} &= \begin{pmatrix} C & \mathbf{0}\end{pmatrix}\begin{bmatrix}X_{cd}\\ X_{Kcd}\end{bmatrix}\end{aligned} \tag{12}$$

- for the "qi" component :

$$\begin{aligned}\begin{bmatrix}\dot{X}_{qi}\\ \dot{X}_{Kqi}\end{bmatrix} &= \begin{pmatrix} A-j\dot{\theta}_1 I\!I_n + B_u D_K C & B_u(RC_K-jIC_K)\\ B_K C & RA_K - jIA_K\end{pmatrix}\begin{bmatrix}X_{qi}\\ X_{Kqi}\end{bmatrix} + \begin{pmatrix}B_u\\ \mathbf{0}\end{pmatrix}u_{qi} + \begin{pmatrix}B_w\\ \mathbf{0}\end{pmatrix}w_{qi}\\ y_{qi} &= \begin{pmatrix} C & \mathbf{0}\end{pmatrix}\begin{bmatrix}X_{qi}\\ X_{Kqi}\end{bmatrix}\end{aligned} \tag{13}$$

with $A_K = RA_K + jIA_K$ and $C_K = RC_K + jIC_K$.
Then (12) end (13) can be written as follows :

$$\begin{aligned}\begin{bmatrix}\dot{X}_{1cd}\\ \dot{X}_{1qi}\end{bmatrix} &= \begin{pmatrix} AA+j\Phi & \mathbf{0}\\ \mathbf{0} & AA-j\Phi\end{pmatrix}\begin{bmatrix}X_{1cd}\\ X_{1qi}\end{bmatrix} + \begin{pmatrix}Bf_u & \mathbf{0}\\ \mathbf{0} & Bf_u\end{pmatrix}\begin{bmatrix}U_{inv_{cd}}\\ U_{inv_{qi}}\end{bmatrix} + \begin{pmatrix}Bf_w & \mathbf{0}\\ \mathbf{0} & Bf_w\end{pmatrix}\begin{bmatrix}w_{cd}\\ w_{qi}\end{bmatrix}\\ \begin{bmatrix}y_{cd}\\ y_{qi}\end{bmatrix} &= \begin{pmatrix}Cf & \mathbf{0}\\ \mathbf{0} & Cf\end{pmatrix}\begin{bmatrix}X_{1cd}\\ X_{1qi}\end{bmatrix}\end{aligned} \tag{14}$$

with $AA = \begin{pmatrix} A+B_u D_K C & B_u RC_K\\ B_K C & RA_K\end{pmatrix}$,
$\Phi = \begin{pmatrix}\dot{\theta}_1 I\!I_n & B_u IC_K\\ \mathbf{0} & IA_K\end{pmatrix}$, $Bf_u = \begin{pmatrix}B_u\\ \mathbf{0}\end{pmatrix}$, $Bf_w = \begin{pmatrix}B_w\\ \mathbf{0}\end{pmatrix}$,
$Cf = \begin{pmatrix} C & \mathbf{0}\end{pmatrix}$ and $X_{1cd} = \begin{bmatrix}X_{cd}\\ X_{Kcd}\end{bmatrix}$, $X_{qi} = \begin{bmatrix}X_{qi}\\ X_{Kqi}\end{bmatrix}$.

The representation (14) is similar to (9), the closed-loop system to the new "cd-qi" synthesis frame for $\dot{\theta} = \dot{\theta}_h$ is explained in the following steps, at first, with the transformation to "dq" frame from (7).

Let consider $\begin{bmatrix}X_{1cd}\\ X_{1qi}\end{bmatrix} = T^{-1}\begin{bmatrix}X_{1d}\\ X_{1q}\end{bmatrix}$, then

$$\begin{bmatrix}\dot{X}_{1d}\\ \dot{X}_{1q}\end{bmatrix} = \begin{pmatrix}AA & \Phi\\ -\Phi & AA\end{pmatrix}\begin{bmatrix}X_{1d}\\ X_{1q}\end{bmatrix} + \begin{pmatrix}Bf_u & \mathbb{O}\\ \mathbb{O} & Bf_u\end{pmatrix}\begin{bmatrix}U_{inv_{1d}}\\ U_{inv_{1q}}\end{bmatrix} + \begin{pmatrix}Bf_w & \mathbb{O}\\ \mathbb{O} & Bf_w\end{pmatrix}\begin{bmatrix}w_{1d}\\ w_{1q}\end{bmatrix}$$

$$\begin{bmatrix}y_{1d}\\ y_{1q}\end{bmatrix} = \begin{pmatrix}Cf & \mathbb{O}\\ \mathbb{O} & Cf\end{pmatrix}\begin{bmatrix}X_{1d}\\ X_{1q}\end{bmatrix} . \tag{15}$$

From the Park Transform to Concordia frame ($\alpha\beta$), it allows to perform the following state space representation:

$$\begin{bmatrix}\dot{X}_{1\alpha}\\ \dot{X}_{1\beta}\end{bmatrix} = \begin{pmatrix}AA & \Phi-\dot{\theta}_1 \mathbb{I}_k\\ -(\Phi-\dot{\theta}_1 \mathbb{I}_k) & AA\end{pmatrix}\begin{bmatrix}X_{1\alpha}\\ X_{1\beta}\end{bmatrix} + \begin{pmatrix}Bf_u & \mathbb{O}\\ \mathbb{O} & Bf_u\end{pmatrix}\begin{bmatrix}U_{inv_\alpha}\\ U_{inv_\beta}\end{bmatrix} + \begin{pmatrix}Bf_w & \mathbb{O}\\ \mathbb{O} & Bf_w\end{pmatrix}\begin{bmatrix}w_\alpha\\ w_\beta\end{bmatrix}$$

$$\begin{bmatrix}y_\alpha\\ y_\beta\end{bmatrix} = \begin{pmatrix}Cf & \mathbb{O}\\ \mathbb{O} & Cf\end{pmatrix}\begin{bmatrix}X_{1\alpha}\\ X_{1\beta}\end{bmatrix} \tag{16}$$

where k is the rank of $AA$.

The closed-loop system is written in a new "cd-qi" frame with a new $\dot{\theta}$ equal to $\dot{\theta}_h$. Set $\Lambda = \Phi - \dot{\theta}_1 \mathbb{I}_k$, for each component $i$ of the state vector $X_{1\alpha}$ and $X_{1\beta}$ (**??**), we apply again the Park Transform :

$$\begin{aligned}\begin{bmatrix}\dot{x}_{1\alpha_i}\\ \dot{x}_{1\beta_i}\end{bmatrix} &= \rho(\theta_h)\begin{bmatrix}\dot{x}_{hd_i}\\ \dot{x}_{hq_i}\end{bmatrix} + \dot{\rho}(\theta_h)\begin{bmatrix}x_{hd_i}\\ x_{hq_i}\end{bmatrix}\\ &= \sum_{j=1}^{k}\begin{pmatrix}aa_{ij} & \lambda_{ij}\\ -\lambda_{ij} & aa_{ij}\end{pmatrix}\rho(\theta_h)\begin{bmatrix}x_{hd_j}\\ x_{hq_j}\end{bmatrix}\\ &+ \begin{pmatrix}bf_{u_i} & 0\\ 0 & bf_{u_i}\end{pmatrix}\rho(\theta_h)\begin{bmatrix}U_{inv_{hd}}\\ U_{inv_{hq}}\end{bmatrix}\\ &+ \sum_{j=1}^{2}\begin{pmatrix}bf_{w_{ij}} & 0\\ 0 & bf_{w_{ij}}\end{pmatrix}\rho(\theta_h)\begin{bmatrix}w_{hd_j}\\ w_{hq_j}\end{bmatrix}\end{aligned} \tag{17}$$

therefore

$$\begin{aligned}\begin{bmatrix}\dot{x}_{hd_i}\\ \dot{x}_{hq_i}\end{bmatrix} &= \sum_{j=1}^{k}\rho^{-1}(\theta_h)\begin{pmatrix}aa_{ij} & \lambda_{ij}\\ -\lambda_{ij} & aa_{ij}\end{pmatrix}\rho(\theta_h)\begin{bmatrix}x_{hd_j}\\ x_{hq_j}\end{bmatrix}\\ &- \rho^{-1}(\theta_h)\dot{\rho}(\theta_h)\begin{bmatrix}x_{hd_i}\\ x_{hq_i}\end{bmatrix}\\ &+ \rho^{-1}(\theta_h)\begin{pmatrix}bf_{u_i} & 0\\ 0 & bf_{u_i}\end{pmatrix}\rho(\theta_h)\begin{bmatrix}U_{inv_{hd}}\\ U_{inv_{hq}}\end{bmatrix}\\ &+ \sum_{j=1}^{2}\rho^{-1}(\theta_h)\begin{pmatrix}bf_{w_{ij}} & 0\\ 0 & bf_{w_{ij}}\end{pmatrix}\rho(\theta_h)\begin{bmatrix}w_{hd_j}\\ w_{hq_j}\end{bmatrix}\end{aligned}$$

then

$$\begin{bmatrix}\dot{X}_{hd}\\ \dot{X}_{hq}\end{bmatrix} = \begin{pmatrix}AA & \Phi+(\dot{\theta}_h-\dot{\theta}_1)\mathbb{I}_k\\ -(\Phi+(\dot{\theta}_h-\dot{\theta}_1)\mathbb{I}_k) & AA\end{pmatrix}\begin{bmatrix}X_{hd}\\ X_{hq}\end{bmatrix} + \begin{pmatrix}Bf_u & \mathbb{O}\\ \mathbb{O} & Bf_u\end{pmatrix}\begin{bmatrix}U_{inv_{hd}}\\ U_{inv_{hq}}\end{bmatrix} + \begin{pmatrix}Bf_w & \mathbb{O}\\ \mathbb{O} & Bf_w\end{pmatrix}\begin{bmatrix}w_{hd}\\ w_{hq}\end{bmatrix}$$

$$\begin{bmatrix}y_{hd}\\ y_{hq}\end{bmatrix} = \begin{pmatrix}Cf & \mathbb{O}\\ \mathbb{O} & Cf\end{pmatrix}\begin{bmatrix}X_{hd}\\ X_{hq}\end{bmatrix} . \tag{18}$$

Therefore, the state space representation of the closed-loop system to the new "cd-qi" frame is:

$$\begin{bmatrix}\dot{X}_{hcd}\\ \dot{X}_{hqi}\end{bmatrix} = \begin{pmatrix}AA+j(\Phi+(\dot{\theta}_h-\dot{\theta}_1)\mathbb{I}_k) & \mathbb{O}\\ \mathbb{O} & AA-j(\Phi+(\dot{\theta}_h-\dot{\theta}_1)\mathbb{I}_k)\end{pmatrix}\begin{bmatrix}X_{hcd}\\ X_{hqi}\end{bmatrix} + \begin{pmatrix}Bf_u & \mathbb{O}\\ \mathbb{O} & Bf_u\end{pmatrix}\begin{bmatrix}U_{inv_{hcd}}\\ U_{inv_{hqi}}\end{bmatrix} + \begin{pmatrix}Bf_w & \mathbb{O}\\ \mathbb{O} & Bf_w\end{pmatrix}\begin{bmatrix}w_{hcd}\\ w_{hqi}\end{bmatrix}$$

$$\begin{bmatrix}y_{hcd}\\ y_{hqi}\end{bmatrix} = \begin{pmatrix}Cf & \mathbb{O}\\ \mathbb{O} & Cf\end{pmatrix}\begin{bmatrix}X_{hcd}\\ X_{hqi}\end{bmatrix} . \tag{19}$$

We can now synthesize another part of controller to eliminate a network current harmonic, for example, we choose $\dot{\theta}_h = 7\omega$. Here, the technic used needs the network current measure only. The frequency response is shown in Fig. (5) in the new "cd-qi" framework for the transfer function $U_{inv}/I_{net}$, such as :

$$K(s) = \frac{0,05(1+i)s+10}{(0,025(1+i)s+1)(0.05(1+i)s+1)},$$

for the cd component.

It is very important to note that all changes of frame preserve the linearity of the different state space representations. We apply therefore this method to synthesize all controllers for any network current harmonics to eliminate. For example, we produce the simulation results for different controllers : to eliminate the 5th harmonic only ($\dot{\theta} =$ -5$\omega$), the 7th harmonic only ($\dot{\theta} = 7\omega$), next 5th 7th 11th and 13th harmonics.

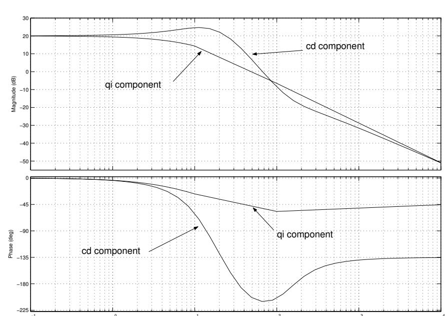


Fig. 5. Controller frequency response performed for 7-th harmonic

## IV. SIMULATION RESULTS

We have verified the validity of the suggested method with a load current showing Fig.6.

We show Fig.7 the results with a controller that eliminate the 5th, 7th, 11th and the 13th harmonics. In these figures, we show the network current, the inverter output current and the active filter node voltage ($V_p$) without the switching due to the frequency of the pulse width modulation (PWM) at 10 kHz. The THD, initially equal to 27.3% , is now equal to 6%.

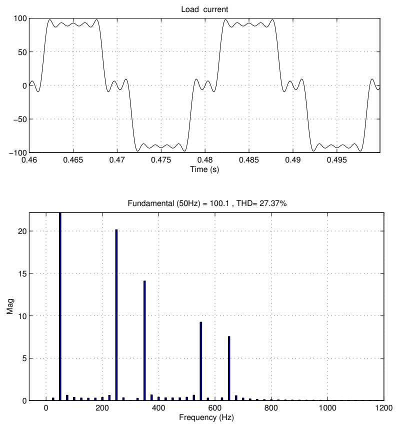


Fig. 6. Load current and this harmonics response

## V. CONCLUSION

This paper developed a model and controller design applied to an active filter in a linear framework. The main problem in to avoid the non linearities introduced with the use of Park transform. Classically, the Park transform is used to measure a particular frequency. Our approach preserves the linearity, but the model obtained is a complex-valued parameters. The advantage is the controller synthesis in a linear context. To implement the controller in a real time board it is important to apply the inverse transform of the transformation developed in section 2. The inverse transform turns the complex-valued coefficient of the controller into real-valued.

## REFERENCES

[1] G.W. Chang and S.K. Chen and M. Chu, "An efficient a-b-c reference frame-based compensation strategy for three-phase active power filter control", *Electric Power Systems Research*, 2002, pp 161-166.

[2] D. Butt and M. Summer and J. Clare, "Harmonic compensation in active shunt filters using controllers employing harmonic rotating frames of reference", *EPE'99 Lausane - Suisse*, 1999.

[3] N. Bruyant, *Etude et commande generalisée de filtres actifs parallèles*, Thesis, GE44 LARGE St Nazaire, France, 1999.

[4] C. Dewez and L. Rambault and J.P. Gaubert, "Linear model of electrical three phase network to design robust controller", *EPE-PMEC'04 Riga - Latvia*, 2004.

[5] P. De Larminat and H.Pouliquen, "$H_2$ control of active filter", *PESC'95*, Atlanta, 1995, pp 1015-1020.

[6] P. Chevrel and F Auger and M. Machmoum, "$H_\infty$ control for a single-phase active power filter : experimental results", *Electrimacs'96*, Saint Nazaire, septembre 1996, Vol3, pp 905-910.

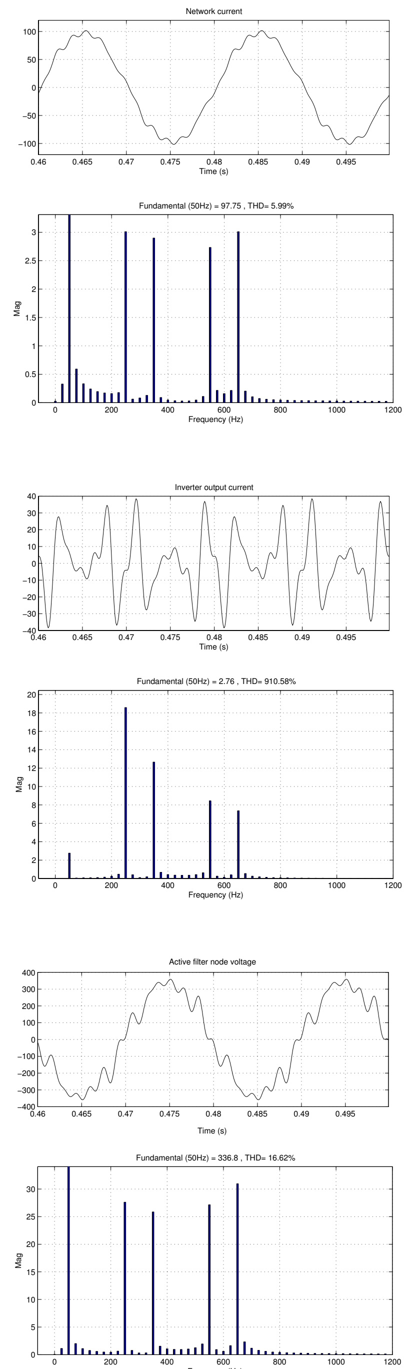


Fig. 7. Network current and inverter current with a controller performed for all harmonics