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Robust Nonlinear Excitation Control Based on A Novel Adaptive Back-stepping Design for Power Systems

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***Abstract*— This paper extends a newly developed adaptive back-stepping algorithm for systems in parametric feedback form to general nonlinear systems. The extended method is applied to robust nonlinear design of excitation control of power systems containing some unknown parameters. The proposed method differs from the "classical" adaptive back-stepping design in the way the parameter estimator is constructed and in the way the nonlinear gains are assigned. Simulation results compared with the "classical" adaptive back-stepping method on a single-machine infinite bus system (SMIB) demonstrate that this proposed controller can obtain better performances. In addition, this extended algorithm is applicable to other control systems.**

I. INTRODUCTION

Excitation control of power systems is a very important, effective and economic method in improving stability. Recently, advanced nonlinear control techniques have been used in the excitation control of power system [1-11], such as nonlinear geometric theory [1,2], direct feedback linearization [3-6], sliding-mode control [7], intelligent control [8], back-stepping [9-11], in order to overcome the disadvantages of controllers by using linear control theory and linearized model around an operating point in the case of large uncertainties. A most successful nonlinear excitation scheme is based on the direct feedback linearization [3-6]. It was shown in the literatures that the dynamics of the power system can be exactly linearized by employing nonlinear pre-compensation so that linear control theory can be used while preserving the nonlinearities, however, this method requires the exact model of system and cancels useful nonlinearities, so it will be not effective in the setting where there are some unknown parameters in the mathematical models of systems.

Back-stepping design is a powerful nonlinear control method to keep synchronism between generators following small disturbance and especially large disturbance [9-11], more suitable for the situation where the excitation system models have unknown parameters [10], which use an adaptive law based on certainty equivalence theory to estimate the unknown parameters. However, the response of the system still can be improved and the speed of adaptation is not directly related to parameters of error dynamics. More recently, a novel back-stepping algorithm presented in [12], which relies on the nonlinear stabilization tools developed in [13], betters the "classical" adaptive back-stepping one with respect to the response of the system and the speed of adaptation. Unfortunately, it is only suitable in situation where the "virtual" control coefficients are ones and limits its range of application.

In this paper, we extend the above-mentioned algorithm in [12] to systems in general form and apply it to design a robust adaptive nonlinear controller for excitation systems. The design obtains better performances and an absent property in comparison with the "classical" adaptive back-stepping, namely, the speed of adaptation may be regulated by a related parameter imposed on error dynamics.

The rest of the paper is organized as follows. The extended algorithm including adaptive law design and stabilization is presented in section II. The proposed method is applied to excitation control of power systems in section III. Simulation results are provided in section IV, and finally section V concludes the paper by brief remarks.

II. THE EXTENDED ALGORITHM

This section extends the adaptive back-stepping method [12] which is for systems in so-called parametric feedback to a general form whose "virtual" control parameters are functions of system feedback states.

A. Problem Formulation

Consider a class of systems described by equations of form

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 + \phi_1(x_1)^T\theta_1 \\ \dot{x}_2 = f_2(x_1,x_2) + g_2(x_1,x_2)x_3 + \phi_2(x_1,x_2)^T\theta_2 \\ \quad \vdots \\ \dot{x}_n = f_n(x_1,\dots,x_n) + g_n(x_1,\dots,x_n)u \\ \qquad + \phi_n(x_1,\dots,x_n)^T\theta_n \end{cases} \tag{1}$$

where $x \in R^n$ is system state, $u \in R$ is the control input and f, g are smooth functions, $g_i(x_1,\dots,x_i) \neq 0$ and $\phi_i(x_1,\dots,x_i)$ are smooth vector fields, θ_i is unknown constant vector. Our control objective is to regulate x_1 to a constant reference x_1^*.

B. Adaptive Law Design

Firstly, we define the error variable

$$z_i = \widehat{\theta}_i - \theta_i + \beta_i(x_1, x_2, \cdots, x_i), \qquad i = 1, 2, \cdots, n \tag{2}$$

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where $\beta_i(\cdot)$ is smooth function yet to be defined.

Secondly, differentiating (2), we have the dynamics of z_i given by the equation

$$\dot{z}_i = \dot{\widehat{\theta}}_i + \sum_{k=1}^{i} \frac{\partial \beta_i}{\partial x_k}[f_k(x_1,\ldots,x_k) + g_k(x_1,\ldots,x_k)x_{k+1} + \phi_k(x_1,\ldots,x_k)^T(\widehat{\theta}_k + \beta_k(x_1,\ldots,x_k) - z_k)], \quad (3)$$

where $x_{k+1} = u$.

The update law can be selected as follows

$$\dot{\widehat{\theta}}_i = -\sum_{k=1}^{i} \frac{\partial \beta_i}{\partial x_k}\big[f_k(x_1,\ldots,x_k) + g_k(x_1,\ldots,x_k)x_{k+1} + \phi_k(x_1,\ldots,x_k)^T\big(\widehat{\theta}_k + \beta_k(x_1,\ldots,x_k)\big)\big], \quad (4)$$

We have the error dynamics

$$\dot{z}_i = -\sum_{k=1}^{i} \frac{\partial \beta_i}{\partial x_k}\phi_k(x_1,\ldots,x_k)^T z_k, \quad (5)$$

Note that (5) is in lower triangular form.

Thirdly, we adopt the selection of the functions $\beta_i(\cdot)$ and its assumption in [12]. In (5), its diagonal terms can be rendered negative semi-definite by selecting β_i as

$$\beta_i(\ldots,x_i) = \int_0^{x_i} \kappa_i(\ldots,\chi_i)\phi_i(\ldots,\chi_i)d\chi_i, \quad (6)$$

where $\kappa_i(\cdot)$ is some positive function. Consider the following assumption.

Assumption 1 There exist a function $\kappa_i(\cdot)$ and a constant k_i with $\kappa_i(\cdot) \geq k_i > 0$ such that, for $j = 1,\ldots,i-1$,

$$\frac{\partial \beta_i}{\partial x_j} = \delta_{ij}(x_1,\ldots,x_i)\phi(x_1,\ldots,x_i), \quad (7)$$

for some bounded functions $\delta_{ij}(\cdot)$, where $\beta_i(\cdot)$ is given by (6).

In order to establish the stability properties of the estimator, we propose the following lemma.

Lemma 1 Consider the system (5), where the functions $\beta_i(\cdot)$ are given by (6), and suppose that assumption 1 holds for all $i = 1,\ldots,n$, then there exist constants $\varepsilon_i > 0$, such that

$$\frac{d}{dt}(\sum_{i=1}^{n} \varepsilon_i z_i^T z_i) \leq -\sum_{i=1}^{n}(\phi_i(x_1,\ldots,x_i)^T z_i)^2, \quad (8)$$

Proof: Similar to [12].

Remark 1. Since $\sum_{i=1}^{n} \varepsilon_i z_i^T z_i$ is a decreasing function of time, the states z_i are bounded, integrating both sides of (8), we can know that $\phi_i^T z_i$ are square-integrable.

Remark 2. Because $f_i(x_1,\ldots,x_i), g_i(x_1,\ldots,x_i)$ only have impact on (4) directly, remarks 1-3 in [12] still hold for system (1).

If we can make the terms $\phi_i^T z_i$ converge to zero, then the parameter estimation is asymptotic. We will design a controller step by step to realize our goal in next subsection.

C. Control Law Design

This section proposes a control law to render $\phi_i^T z_i$ converge to zero, which implies from (2) that an asymptotic estimate of each term $\phi_i^T \theta_i$ in (1) is given by $\phi_i^T(\widehat{\theta}_i + \beta_i)$, and ensures global asymptotic stability of the desired equilibrium.

Now a controller can be designed by the following procedure.

Step 1. Define $\widetilde{x}_1 = x_1 - x_1^*$ (here x_1^* is a constant reference signal), its dynamics are derived by

$$\dot{\widetilde{x}}_1 = f_1(x_1) + g_1(x_1)x_2 + \phi_1(x_1)^T\theta_1, \quad (9)$$

Take x_2 as a “virtual” control and define

$$\widetilde{x}_2 = x_2 - \xi_2(x_1,\widehat{\theta}_1), \quad (10)$$

Select

$$\xi_2 = \frac{1}{g_1(x_1)}[-\alpha_1(\widetilde{x}_1,\widehat{\theta}_1) - f_1(x_1) - \phi_1(x_1)^T(\widehat{\theta}_1 + \beta_1(x_1))], \quad (11)$$

where $\alpha_1(\cdot)$ yet to be designed. Inserting (11) into (10) and noticing (9) and (2), yield

$$\dot{\widetilde{x}}_1 = -\alpha(\widetilde{x}_1,\widehat{\theta}_1) - \phi_1(x_1)^T z_1, + g_1(x_1)\widetilde{x}_2, \quad (12)$$

Step 2. Differentiating (10) gives dynamics

$$\dot{\widetilde{x}}_2 = f_2(x_1,x_2) + g_2(x_1,x_2)x_3 + \phi_2(x_1,x_2)^T\theta_2 - \frac{\partial \xi_2}{\partial x_1}\dot{x}_1 - \frac{\partial \xi_2}{\partial \widehat{\theta}_1}\dot{\widehat{\theta}}_1, \quad (13)$$

Take x_3 as a ”virtual” control and define

$$\widetilde{x}_3 = x_3 - \xi_3(x_1,x_2,\widehat{\theta}_1,\widehat{\theta}_2), \quad (14)$$

By selecting

$$\xi_3 = \frac{1}{g_2(x_1,x_2)}\Big[-\alpha_2(\widetilde{x}_1,\widetilde{x}_2,\widehat{\theta}_1,\widehat{\theta}_2) - f_2(x_1,x_2) - \phi_2(x_1,x_2)^T(\widehat{\theta}_2 + \beta_2(x_1,x_2)) + \frac{\partial \xi_2}{\partial x_1}(f_1(x_1)$$

$$+g_1(x_1)x_2+\phi_1(x_1)^T(\widehat{\theta}_1+\beta_1(x_1)))+\frac{\partial\xi_2}{\partial\widehat{\theta}_1}\dot{\widehat{\theta}}_1\Big], \tag{15}$$

Following this procedure step by step, we can derive the dynamics of the rest of states until the real control appears.

Step n. The n-th dynamics are given by

$$\begin{aligned}\dot{\widetilde{x}}_n &= f_n(x_1,\ldots,x_n)+g_n(x_1,\ldots,x_n)u+\phi_n(x_1,\ldots,x_n)^T\\ &\cdot\theta_n-\sum_{k=1}^{n-1}\frac{\partial\xi_n}{\partial x_k}[f_k(x_1,\ldots,x_k)+g_k(x_1,\ldots,x_k)x_{k+1}\\ &+\phi_k(x_1,\ldots,x_k)^T\theta_k]-\sum_{k=1}^{n-1}\frac{\partial\xi_n}{\partial\widehat{\theta}_k}\dot{\widehat{\theta}}_k, \end{aligned} \tag{16}$$

The real control input is:

$$\begin{aligned}u &= \frac{1}{g_n(x_1,\ldots,x_n)}\big[-\alpha_n(\widetilde{x}_1,,\ldots,\widetilde{x}_n,\widehat{\theta}_1,\ldots,\widehat{\theta}_n)\\ &-f_n(x_1,\ldots,x_n)-\phi_n(x_1,\ldots,x_n)^T(\widehat{\theta}_n\\ &+\beta_n(x_1,\ldots,x_n))+\sum_{k=1}^{n-1}\frac{\partial\xi_n}{\partial x_k}[f_k(x_1,\ldots,x_k)\\ &+g_k(x_1,\ldots,x_k)x_{k+1}+\phi_k(x_1,\ldots,x_k)^T\\ &\cdot(\widehat{\theta}_k+\beta_k(x_1,\ldots,x_k))]+\sum_{k=1}^{n-1}\frac{\partial\xi_n}{\partial\widehat{\theta}_k}\dot{\widehat{\theta}}_k\big], \end{aligned} \tag{17}$$

So, the closed-loop system is given by

$$\begin{cases}\dot{\widetilde{x}}_1=-\alpha_1(\widetilde{x}_1,\widehat{\theta}_1)-\phi_1(x_1)^Tz_1+g_1(x_1)\widetilde{x}_2\\ \dot{\widetilde{x}}_2=-\alpha_2(\widetilde{x}_1,\widetilde{x}_2,\widehat{\theta}_1,\widehat{\theta}_2)-\phi_2(x_1,x_2)^Tz_2\\ \qquad+\frac{\partial\xi_2}{\partial x_1}\phi_1(x_1)^Tz_1+g_2(x_1,x_2)\widetilde{x}_3\\ \quad\vdots\\ \dot{\widetilde{x}}_n=-\alpha_n(\widetilde{x}_1,\ldots,\widetilde{x}_n,\widehat{\theta}_1,\ldots,\widehat{\theta}_n)-\phi_n(x_1,\ldots,x_n)^T\\ \qquad\cdot z_n+\sum_{k=1}^{n-1}\frac{\partial\xi_n}{\partial x_k}\phi_k(x_1,\ldots,x_k)^Tz_k\end{cases} \tag{18}$$

Our main result is given in the following theorem.

Theorem 1. There exist functions $\alpha_i(\cdot)$ such that the system (18) together with (5) is globally asymptotically stable at the origin and $\lim_{t\to 0}\phi_i\big(x_1(t),\ldots,x_i(t)\big)^Tz_i(t)=0$

Proof: Consider the function

$$V_1=\frac{1}{2}\sum_{k=1}^{n}\widetilde{x}_k^2, \tag{19}$$

Differentiate (19) along the trajectories of (18), we have

$$\begin{aligned}\dot{V}_1 &= \widetilde{x}_1[-\alpha_1(\widetilde{x}_1,\widehat{\theta}_1)-\phi_1(x_1)^Tz_1+g_1(x_1)\widetilde{x}_2]\\ &+\widetilde{x}_2[-\alpha_2(\widetilde{x}_1,\widetilde{x}_2,\widehat{\theta}_1,\widehat{\theta}_2)-\phi_2(x_1,x_2)^Tz_2\\ &+\frac{\partial\xi_2}{\partial x_1}\phi_1(x_1)^Tz_1+g_2(x_1,x_2)\widetilde{x}_3]+\ldots\\ &+\widetilde{x}_n[-\alpha_n(\widetilde{x}_1,\ldots,\widetilde{x}_n,\widehat{\theta}_1,\ldots,\widehat{\theta}_n)-\phi_n(x_1,\ldots,x_n)^T\\ &\cdot z_n+\sum_{k=1}^{n-1}\frac{\partial\xi_n}{\partial x_k}\phi_k(x_1,\ldots,x_k)^Tz_k], \end{aligned} \tag{20}$$

To (20), firstly applying Young's Inequality, then selecting the functions $\alpha_i(\cdot)$ as follows

$$\begin{aligned}\alpha_1 &= (c_1+n/4\gamma)\widetilde{x}_1,\\ \alpha_i &= g_{i-1}(x_1,\ldots,x_{i-1})x_{i-1}+(c_i+\frac{n-i+1}{4\gamma})\widetilde{x}_i\\ &+\sum_{k=1}^{i-1}\frac{n-k+1}{4\gamma}(\frac{\partial\xi_i}{\partial x_k})^2\widetilde{x}_i, \end{aligned} \tag{21}$$

for $i=2,\ldots,n$, where $c_i,\gamma>0$ are arbitrary positive tuning constants, we have

$$\dot{V}_1\le-\sum_{k=1}^{n}c_k\widetilde{x}_k^2+\gamma\sum_{k=1}^{n}(\phi_k(x_1,\ldots,x_k)^Tz_k)^2, \tag{22}$$

By Lemma 1, $V=V_1+\gamma\sum_{i=1}^n\varepsilon_iz_i^Tz_i$ is the Lyapunov function for the closed-loop system (18), the proof is completed.

Remark 3. When $f_i(x_1,\ldots,x_i)\equiv 0,\ g_i(x_1,\ldots,x_i)\equiv 1$ in (1), our result coincides with that of [12].

III. EXCITATION CONTROL OF POWER SYSTEMS

In this section, we apply the proposed method to the stabilization of excitation systems where the damping coefficient can not be measured accurately [10]. Note that the mechanical input power P_m is treated as a constant in our design of excitation control, i. e. , it is assumed that the governor action is slow enough not to have any significant impact on the machine dynamics.

A. Dynamic Model of SMIB

Consider a dynamic model of single machine infinite bus system which is described by a third-order dynamic model [14].

$$\begin{cases}\dot{\delta}=\omega-\omega_0\\ \dot{\omega}=-\frac{D}{H}(\omega-\omega_0)+\frac{\omega_0}{H}(P_m-\frac{E_q'V_s sin\delta}{X_{d\Sigma}'})\\ \dot{E}_q'=-\frac{1}{T_d'}E_q'+\frac{1}{T_{d0}}\frac{X_d-X_d'}{X_{d\Sigma}'}V_s cos\delta+\frac{1}{T_{d0}}V_f\end{cases} \tag{23}$$

where

- $\delta(t)$ the power angle of the generator (in radian);
- $\omega(t)$ the relative speed (in rad/s);
- E_q' the transient EMF in the quadrature axis (in p.u.);
- P_m the mechanical input power (in p.u.);
- V_f voltage of excitation coil (in p.u.);
- V_s infinite bus voltage (in p.u.);
- D damping constant (in p.u.);

H inertia constant (in second);

The notation for other variables, electrical equations and parameters are standard and readers are referred to [14,15,5].

Generally speaking, the damping constant can not be measured accurately [10], hence D is an unknown constant parameter, so is the term $-\frac{D}{H}$ in (23). By comparing (24) with (23), θ should be defined by $[\theta_1, \theta_2, \theta_3]^T = [0, -\frac{D}{H}, 0]^T$.

Let $x_1 = \delta - \delta_0$, $x_2 = \omega - \omega_0$, $x_3 = E'_q - E'_{q0}$ ($\delta_0, \omega_0, E'_{q0}$ are the initial values of the corresponding variables).

So we can put the model (23) below in form of (1).

$$\begin{cases} \dot{x}_1 = f_1(x_1) + g_1(x_1)x_2 + \phi_1(x_1)^T\theta_1 \\ \dot{x}_2 = f_2(x_1,x_2) + g_2(x_1,x_2)x_3 + \phi_2(x_1,x_2)^T\theta_2 \\ \dot{x}_3 = f_3(x_1,x_2,x_3) + g_3(x_1,x_2,x_3)u \\ \qquad +\phi_3(x_1,x_2,x_3)^T\theta_3 \end{cases} \tag{24}$$

where

$$f_1(x_1) = 0,$$
$$g_1(x_1) = 1,$$
$$f_2(x_1,x_2) = \tfrac{\omega_0}{H}(P_{m0} - \tfrac{V_s}{X'_{d\Sigma}} sin(x_1+\delta_0)),$$
$$g_2(x_1,x_2) = \tfrac{\omega_0 V_s}{H X'_{d\Sigma}} sin(x_1+\delta_0)),$$
$$f_3(x_1,x_2,x_3) = -\frac{1}{T'_d}(E'_q + X_3) + \frac{1}{T_{d0}}\frac{X_d - X'_d}{X'_{d\Sigma}}V_s \cdot cos(X_1+\delta_0), \tag{25}$$
$$g_3(x_1,x_2,x_3) = \tfrac{1}{T_{d0}},$$
$$\phi_1(x_1) = 0,$$
$$\phi_2(x_1,x_2) = -x_2,$$
$$\phi_3(x_1,x_2,x_3) = 0,$$
$$u = V_f,$$

B. Design of a Robust Nonlinear Excitation Controller

The control objective is to regulate x_1 to zero, i. e. , $x_1^* = 0$. Furthermore, since (24) together with (25) is a special form of (1), x_2 also will converge to zero and x_3 will be bounded. In other words, synchronism between generators in power systems is kept.

In the following procedure, we will design the control law to realize the goal .

We begin with the construction of the estimation error

$$z_2 = \widehat{\theta}_2 - \theta_2 + \beta_2(x_1,x_2), \tag{26}$$

Selecting the function $\beta_2(\cdot)$ according to equation (6),

$$\beta_2(x_1,x_2) = \frac{1}{2}kx_2^2, \tag{27}$$

Yields the error dynamics together with the update law (4),

$$\dot{z}_2 = -kx_2^2 z_2, \tag{28}$$

Clearly, Assumption 1 holds for any constant $k > 0$, Lemma 1 also does. Following the design methodology of the previous section, the error equations can be written as

$$\begin{cases} \dot{\widetilde{x}}_1 = -\alpha_1(\widetilde{x}_1) + g_1(x_1)\widetilde{x}_2 \\ \dot{\widetilde{x}}_2 = -\alpha_2(\widetilde{x}_1,\widetilde{x}_2,\widehat{\theta}_2) - \phi_2(x_1,x_2)^T z_2 + g_2(x_1,x_2)\widetilde{x}_3 \\ \dot{\widetilde{x}}_3 = -\alpha_3(\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3,\widehat{\theta}_2) + \frac{\partial\xi_3}{\partial x_2}\phi_2(x_1,x_2)^T z_2 \end{cases} \tag{29}$$

Following (19), define the function

$$V_1 = \frac{1}{2}\sum_{k=1}^{3}\widetilde{x}_k^2, \tag{30}$$

whose time derivative along the trajectories of (29) is given by

$$\begin{aligned} \dot{V}_1 = {} & \widetilde{x}_1[-\alpha_1(\widetilde{x}_1) + \widetilde{x}_2] + \widetilde{x}_2[-\alpha_2(\widetilde{x}_1,\widetilde{x}_2,\widehat{\theta}_2) \\ & -\phi_2(x_1,x_2)^T z_2 + g_2(x_1,x_2)\widetilde{x}_3] + \widetilde{x}_3 \\ & \cdot [-\alpha_3(\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3,\widehat{\theta}_2) + \frac{\partial\xi_3}{\partial x_2}\phi_2(x_1,x_2)^T z_2], \end{aligned} \tag{31}$$

Hence

$$\begin{aligned} \dot{V}_1 \le {} & -\alpha_1(\widetilde{x}_1,\widehat{\theta}_1)\widetilde{x}_1 + \widetilde{x}_1\widetilde{x}_2 - \alpha_2(\widetilde{x}_1,\widetilde{x}_2,\widehat{\theta}_2)\widetilde{x}_2 \\ & +g_2(x_1,x_2)\widetilde{x}_2\widetilde{x}_3 - \alpha_3(\widetilde{x}_1,\widetilde{x}_2,\widetilde{x}_3,\widehat{\theta}_2)\widetilde{x}_3 \\ & +(\gamma/2)[\phi_2(x_1,x_2)^T z_2]^2 + (1/2\gamma)\widetilde{x}_2^2 + (\gamma/2) \\ & \cdot [\phi_2(x_1,x_2)^T z_2]^2 + (1/2\gamma)[\frac{\partial\xi_3}{\partial x_2}]^2\widetilde{x}_3^2, \end{aligned} \tag{32}$$

The functions $\alpha_i(\cdot)$ can be selected as follows

$$\begin{aligned} & \alpha_1 = c_1\widetilde{x}_1, \\ & \alpha_2 = \widetilde{x}_1 + (c_2 + \frac{1}{2\gamma})\widetilde{x}_2, \\ & \alpha_3 = g_2(x_1,x_2)\widetilde{x}_2 + [c_3 + (\frac{1}{2\gamma})(\frac{\partial\xi_3}{\partial x_2})^2]\widetilde{x}_3, \end{aligned} \tag{33}$$

where c_1, c_2, c_3 , γ are arbitrary positive tuning constants.

Therefore,

$$\dot{V}_1 \le -c_1\widetilde{x}_1^2 - c_2\widetilde{x}_2^2 - c_3\widetilde{x}_3^2 + \gamma[\phi_2(x_1,x_2)^T z_2]^2, \tag{34}$$

Hence, from (28), the closed-loop system is asymptotically stable with the Lyapunov function $V = V_1 + (\frac{\gamma}{2k})z^2$.

In order to complete our task presented in the beginning of this subsection, the controller can be obtained by (17), (33) and definitions of $\widetilde{x}_i(\cdot)$; the update law can be given from (4) and (27), respectively.

$$V_f = T_{d0}\Big[(x_3 + E'_{q0})/T'_d + x_2\dot{\widehat{\theta}}_2/[\frac{\omega_0}{H}\frac{V_s}{X'_{d\Sigma}} sin(x_1+\delta_0)]$$

$$+\frac{\partial\xi_3}{\partial x_1}x_2+\frac{\partial\xi_3}{\partial x_2}[\frac{\omega_0}{H}(P_{m0}-\frac{V_s}{X'_{d\Sigma}}sin(x_1+\delta_0))$$

$$+\frac{\omega_0}{H}\frac{V_s x_3}{X'_{d\Sigma}}sin(x_1+\delta_0)-x_2(\widehat{\theta}_2+\frac{1}{2}x_2^2)]$$

$$-(c_1x_1+x_2)-(x_3-\xi_3)(c_3+(1/2\gamma)[\frac{\partial\xi_3}{\partial x_2}]^2)\Big], \quad (35)$$

where $\xi_3(\cdot)$ is available from (15).

$$\dot{\widehat{\theta}}_2=-kx_2[\frac{\omega_0}{H}(P_{m0}-\frac{V_s}{X'_{d\Sigma}}sin(x_1+\delta_0))$$

$$+\frac{\omega_0}{H}\frac{V_s x_3}{X'_{d\Sigma}}sin(x_1+\delta_0)-x_2(\widehat{\theta}_2+\frac{1}{2}x_2^2)], \quad (36)$$

IV. SIMULATION RESULTS

The system under study is shown in Fig. 1 (there is an equivalence relationship between E'_q and P_e), and the data of the system are given in Appendix A. Suppose the related variables in feedback law are available by estimation and measurement [14]. In order to investigate the effectiveness of the proposed controller, we will make comparisons with the classical adaptive back-stepping controller and full-information controller, which is obtained by assuming the parameters are known and applying standard back-stepping from [16]. The classical adaptive back-stepping controller derived by the adaptive back-stepping method from [17] or [18] is given in Appendix B.

Fig. 2 shows the response of the system with initial conditions $x_1(0) = 0.1744,\ x_2(0) = x_3(0) = 0$. It reveals that the proposed adaptive scheme recovers the performance of the full-information controller and much better than the classical adaptive back-stepping controller. The speed of response can be further increased (or reduced) by tuning the parameter k.

Fig. 3 gives the performance of the estimator. Finally, we should note that an absent property in classical adaptive back-stepping: the speed of adaptation is directly related to the gain k, by the reason that z dynamics (28) is imposed by the selection of the function $\beta_i(\cdot)$.

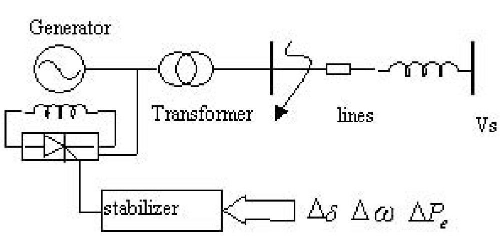

Fig. 1. Single machine infinite bus power system configuration.

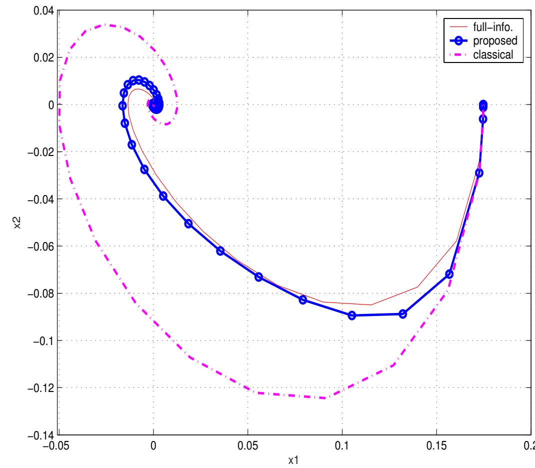

Fig. 2. Trajectories on the $x_1 - x_2$ coordinates.

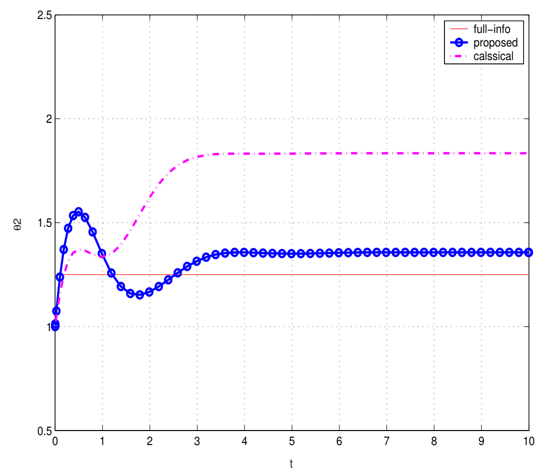

Fig. 3. Comparison of parameter estimates. (The solid line also denotes the true vale of θ_2)

V. CONCLUSION

The main contributions of this paper are extension of a novel adaptive back-stepping algorithm to a general case and its application to excitation control of single machine infinite bus systems to improve performances of system responses and parameter estimation. Simulation results compared with the classical back-stepping and full-information schemes show that the validation of the extension and superiority of the proposed controller for excitation systems. This method can be easily applied to multi-machine systems to realize the decentralized control. Further researches will be devoted to extend this algorithm to systems with more general unknown parameters and disturbance.

VI. ACKNOWLEDGMENTS

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VII. APPENDIX

A. Parameters for the proposed control law, classical adaptive back-stepping control law and full information control law.

$$H = 4,\ \delta_0 = \pi/3,\ \omega_0 = 314.159,$$

$$E'_q = 1.08,\ V_s = 1, X_d = 1.863,$$

$$X'_d = 0.32,\ X'_{d\Sigma} = 0.32,\ T'_d = 0.02,$$

$$T_{d0} = 0.02,\ P_{m0} = 1.0,\ k = 50,$$

$$c_1 = c_2 = c_3 = 0.5,$$

$$\bar{c}_1 = \bar{c}_2 = \bar{c}_3 = 2,$$

$$\gamma = 10,\ \bar{\gamma} = 2,$$

B. The classical adaptive back-stepping control law and update law:

$$V_f = T_{d0}\Bigg[(x_3 + E'_{q0})/T'_d - a_2 cos(x_1 + \delta_0) - \bar{c}_3 e_3 - \frac{(1 + \bar{c}_1\bar{c}_2)x_2 + \dot{\widehat{\theta}}_2 x_2}{a_1 sin(x_1 + \delta_0)} - \frac{(\bar{c}_1 + \bar{c}_2 + \widehat{\theta}_2)(\widehat{\theta}_2 x_2 + a_0 + a_1 sin(x_1 + \delta_0)(x_3 + E'_{q0}))}{a_1 sin(x_1 + \delta_0)} + \frac{(\bar{c}_2 e_2 + e_1 + \bar{c}_1 x_2 + \widehat{\theta}_2 x_2 + a_0)(x_2 cos(x_1 + \delta_0))}{(a_1)[sin(x_1 + \delta_0)]^2}\Bigg], \tag{37}$$

$$\dot{\widehat{\theta}}_2 = \bar{\gamma}[e_2 + \frac{(\bar{c}_1 + \bar{c}_2 + \widehat{\theta}_2)e_3}{a_1 sin(x_1 + \delta_0)}]x_2, \tag{38}$$

where

$$e_1 = x_1,$$

$$e_2 = x_2 - x_2^*,$$

$$e_3 = x_3 - x_3^*,$$

$$x_2^* = \bar{c}_1 x_1,$$

$$x_3^* = \frac{\bar{c}_2 e_2 + e_1 + \bar{c}_1 x_2 + \widehat{\theta}_2 x_2 + a_0}{a_1 sin(x_1 + \delta_0)},$$

$$a_0 = \frac{\omega_0}{H} P_{m0},$$

$$a_1 = \frac{\omega_0 V_s}{H X'_{d\Sigma}},$$

$$a_2 = \frac{1}{T_{d0}} \frac{X_d - X'_d}{X'_{d\Sigma}} V_s.$$

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