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Reliable H_∞ Aircraft Flight Controller Design Against Faults With State/Output Feedback

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***Abstract*— This paper presents a novel H_∞ formulation to design a reliable tracking controller for a discrete LTI aircraft system against actuator outage faults and control surface impairment modelled as polytopic uncertainties. Both the state-feedback and output-feedback cases are considered. The approach is based on multiobjective optimization using several parameter dependent Lyapunov functions, each corresponding to a vertex of the uncertainty polytope. The main advantage of this approach with respect to the other well-known techniques is the reduced conservativeness. In the output-feedback case, a new Linear Matrix Inequality (LMI) formulation is derived and solved by an iterative LMI (ILMI) algorithm. Finally, the application to a nonlinear F-16 aircraft model with actuator outage faults and control surface impairment illustrates the effectiveness of the proposed approach.**

***Keywords*: H_∞ Control; Flight Control; Linear Matrix Inequalities; Lyapunov Function; Outage Fault; Control Surface impairment; State/Output Feedback**

I. Introduction

Reliability, maintainability and survivability are three main issues in the aircraft flight controller design area. Reliability means that the controller should optimize the performance in normal condition and meet certain closed-loop stability and performance demands in the presence of a set of given faults simultaneously. Such system is called a reliable control system which is particularly suitable in safety-critical systems, such as aircraft, nuclear power plants where safety and reliability are sometimes of more importance than good performance solely. In the literature to date, reliable control has attracted considerable amount of attention ([1]-[2]), and several applications have been developed ([3]-[4]).

A various reliable flight control system design conception named self-repairing flight control system (SRFCS) was proposed in ([5], [6]). SRFCS is regarded as an *active* approach which includes real-time fault detection and isolation (FDI) procedure, and control system reconfiguration procedure in the presence of faults. Such *active* approach provides satisfying robustness and reliability for aircraft. However, identifying the fault and reconfiguring the controller may require time delay, which may not be available in many safety-critical applications.

Another reliable flight control system design conception is to design a fixed controller which can tolerate a set of faults by exploiting the inherent redundancy of aircraft control surface actuators. As no FDI and/or controller reconfiguration is needed, this *passive* approach provides promising fault handling ability in practical applications ([7], [4], [8]) and has received considerable attention.

In this paper, the reliable H_∞ flight tracking controller design problem is studied, in the presence of actuator outage faults and/or control surface impairment. For control surface impairment fault, each fault is defined as a vertex to a polytopic uncertain system, and the design goal is to find a controller to stabilize this uncertain system. Comparing with approaches that use only one Lyapunov function on all vertices in controller design, parameter dependent Lyapunov functions, one for each and every vertex of the uncertainty polytope, are employed in this paper in order to reduce the controller's conservativeness. Multiobjective optimization methodology is applied to optimize the tracking performance during normal condition when there is no fault and to maintain acceptable degradations when faults occur.

The paper is organized as follows. Section II presents models of outage faults and control surface impairment, together with the formulation of reliable tracking problem. In Section III, both state feedback and output feedback designs are discussed and a new LMI formulation is derived. In Section IV, a nonlinear F-16 aircraft model with mentioned faults is used to illustrate the effectiveness of the proposed approach. Finally, conclusion is given in Section V.

II. Fault model and reliable tracking problem formulation

Following [4], we present the discrete-time aircraft model as follows:

$$\begin{cases} x(k+1)=Ax(k)+B^u u(k)+B^w w(k) \\ \quad y(k)=Cx(k), \qquad\qquad x(0)=x_0 \end{cases} \tag{1}$$

where $x(k)=[u,w,q,v,p,r]^T \in \Re^n$ is the state, $u(k)\in\Re^m$ is the control input, $w(k)\in\Re^h$ is the bounded input disturbance and $y(k)\in\Re^p$ is the output.

To study the reliable flight control and tracking problem in case of faults, the fault models must be established first.

There are two types of faults to be considered in this paper.

Let $u^F(k)$ represent the control surface input vector after actuator outage faults have occurred. Then, the following actuator fault model is adopted in this paper:

$$u^F(k) = \omega_L u(k), \quad L = 0, 1, \ldots, l_p, \; l_p \leqslant 2^m - 1 \tag{2}$$

where $\omega_L \in \Re^{m\times m}$ is the scaling factor satisfying

$$\begin{aligned}\omega_L \in \Omega \triangleq \{&\mathrm{diag}[\omega_{L1}, \omega_{L2}, \ldots, \omega_{Lm}]\\ &\omega_{Lj} = 0 \text{ or } 1, \; j = 1, 2, \ldots, m\}\end{aligned} \tag{3}$$

From equations (2) and (3), it is clear that $\omega_{Lj} = 0$ $(1 \leqslant j \leqslant m)$ represents the outage fault case of the j^{th} actuator and $\omega_{Lj} = 1$ corresponds to normal condition of the j^{th} control channel. We also define $L = 0$ to correspond to the normal condition, i.e., $\omega_0 = I_{m\times m}$.

Another fault we are dealing with here is the control surface impairment, which is characterized by the percentage loss of the total control surface area. Unlike actuator outage faults of the form (2), control surface impairment will change the aerodynamic characteristics of the aircraft. For different percentage control surface losses, corresponding aircraft models can be obtained and defined as vertices to a polytopic uncertain system. Then, for arbitrary percentage loss on such control surface, aircraft model can be obtained from these models by linear interpolation. To describe this, the following polytopic uncertainties are adopted:

$$\begin{array}{ll} A(\theta) = \sum_{i=0}^{l} A_i\theta_i, & B^u(\theta) = \sum_{i=0}^{l} B_i^u\theta_i, \\ C(\theta) = \sum_{i=0}^{l} C_i\theta_i, & B^w(\theta) = \sum_{i=0}^{l} B_i^w\theta_i \end{array} \tag{4}$$

where the parameter $\theta = [\theta_0, \theta_1, \ldots, \theta_l]^T$ satisfying

$$\theta \in \Theta \triangleq \left\{ \theta \in \Re^{l+1} : \theta_i \geqslant 0, \; \sum_{i=0}^{l} \theta_i = 1 \right\} \tag{5}$$

Vertex matrices A_i, B_i^u, B_i^w, C_i $(i = 0, \ldots, l)$ are known matrices with $i = 0$ corresponding to the normal condition and $i = 1, \ldots, l$, the various vertex fault conditions.

Hence, integrating the actuator outage faults (2) and control surface impairment (4), the aircraft dynamics can be characterized as

$$\begin{cases} x(k+1) = A(\theta)x(k) + B^u(\theta)u^F(k) + B^w(\theta)w(k) \\ \quad y(k) = C(\theta)x(k), \qquad\qquad x(0) = x_0 \end{cases} \tag{6}$$

Consider discrete-time aircraft model (6) with both actuator outage fault (2) and control surface impairment (4), the reliable controller design problem in this paper is to find a controller such that:

1) The closed-loop system is robustly stable for all $\omega_L \in \Omega$ and $\theta \in \Theta$.
2) In the event of no control surface actuator outage faults, the output signal $Sy(k)$ tracks the reference signal $r(k)$ without steady-state error, that is:

$$\lim_{k\to\infty} e(k) = 0, \quad e(k) = r(k) - Sy(k) \tag{7}$$

and with optimized closed-loop performance. Here S is a known matrix of appropriate dimension to determine which outputs are required to track the reference signal.

3) In the case of actuator outage faults and/or control surface impairment, the output signal $Sy(k)$ tracks the reference signal $r(k)$ without steady-state error and with an acceptable degradation in tracking performance.

In order to obtain a reliable flight controller with zero steady state tracking error for step input, we introduce integral action on the continuous-time tracking error $\dot{e}(t) = -SCx(t) + r(t)$. Using a sampling time of $0.01s$, the following discrete-time vertex system model can be obtained.

$$\begin{cases} x_a(k+1) = A_{ai}x_a(k) + B_{ai}^u\omega_L u(k) + B_{ai}^w w_a(k) \\ \quad y_a(k) = C_{ai}x_a(k) \\ \quad z(k) = C_1x_a(k) + D_1u(k) \end{cases} \tag{8}$$

where the augmented state is $x_a(k) = [e^T(k)\; x^T(k)]^T$ and the disturbance vector is $w_a(t) = [r^T(k)\; w^T(k)]^T$, the matrix C_1, D_1 are constant matrices of appropriate dimension, while $z(k)$ is the regulated output. The matrices A_{ai}, B_{ai}^u, B_{ai}^w and C_{ai} are the discretization of the corresponding continuous-time vertex system models.

Then the following discrete-time model with polytopic uncertainty will be used to model with actuator fault (2)-(3) and control surface impairment fault (4)-(5).

$$\begin{cases} x_a(k+1) = A_a(\theta)x_a(k) + B_a^u(\theta)\omega_L u(k) + B_a^w(\theta)w_a(k) \\ \quad y_a(k) = C_a(\theta)x_a(k) \\ \quad z(k) = C_1x_a(k) + D_1u(k) \end{cases} \tag{9}$$

where

$$\begin{array}{ll} A_a(\theta) = \sum_{i=0}^{l} A_{ai}\theta_i, & B_a^u(\theta) = \sum_{i=0}^{l} B_{ai}^u\theta_i, \\ C_a(\theta) = \sum_{i=0}^{l} C_{ai}\theta_i, & B_a^w(\theta) = \sum_{i=0}^{l} B_{ai}^w\theta_i \end{array} \tag{10}$$

If we obtain a controller to stabilize the augmented system (9), then it also stabilizes the original system (6) and guarantees the steady state error to be zero.

III. H_∞ Flight controller design using parameter-dependent Lyapunov functions

For convenience, we first give the customary H_∞ performance indices over an infinite time horizon. Consider the system (9) and a real number $\gamma_{00} > 0$, the exogenous signal w_a is attenuated by γ_{00} if, assuming $x(0) = 0$,

$$\sum_{k=0}^{\infty} \|z(k)\|^2 < \gamma_{00} \sum_{k=0}^{\infty} \|w_a(k)\|^2 \tag{11}$$

Hereafter, we will design state/output feedback reliable flight controllers in an H_∞ framework by using parameter-dependent Lyapunov functions.

A. State Feedback Controller Design

For the augmented polytopic uncertain system (9), fault model (2)-(4), consider the static state-feedback controller

$$u(k) = K_s x_a(k) \tag{12}$$

The closed-loop system of system (9) is given by

$$\begin{cases} x_a(k+1) = [A_a(\theta) + B_a^u(\theta)\omega_L K_s]x_a(k) + B_a^w(\theta)w_a(k) \\ \quad z(k) = [C_1 + D_1K_s]x_a(k) \end{cases} \tag{13}$$

Theorem 3.1: Consider the discrete linear system (9). There exists a static state-feedback control law of type (12), such that the closed-loop system (13) is stable and the H_∞ constraint (11) is fulfilled for all faults in (2)-(4), if there exist matrices $Q_{Li} = Q_{Li}^T > 0$, non-singular matrix G and V for $i = 0, \ldots, l$, and $L = 0, 1, \ldots, l_p$, $l_p \leqslant 2^m - 1$,

$$\begin{bmatrix} Q_{Li} - G - G^T & * & * & * \\ A_{ai}G + B_{ai}^u \omega_L V & -Q_{Li} & * & * \\ C_1 G + D_1 V & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_{Li} I \end{bmatrix} < 0 \quad (14)$$

The state feedback gain K_s is given by

$$K_s = V G^{-1} \quad (15)$$

Proof: Given $Q_{Li} = Q_{Li}^T > 0$ and the additional non-singular matrix G of suitable dimension, $(G^T - Q_{Li})Q_{Li}^{-1}(G - Q_{Li})$ is nonnegative definite, and consequently,

$$0 < G + G^T - Q_{Li} \leqslant G^T Q_{Li}^{-1} G \quad (16)$$

holds true. Hence, the following LMI holds,

$$\begin{bmatrix} -G^T Q_{Li}^{-1} G & * & * & * \\ A_{ai}G + B_{ai}^u \omega_L V & -Q_{Li} & * & * \\ C_1 G_i + D_1 V & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_{Li} I \end{bmatrix} < 0 \quad (17)$$

where $V = K_s G$. Then, as G is nonsingular and G^{-1} always exists, we can multiply (17) from the left by $diag\{Q_{Li}G^{-T}, I, I, I\}$ and from the right by $diag\{G^{-1}Q_{Li}, I, I, I\}$, the following LMI is obtained

$$\begin{bmatrix} -Q_{Li} & * & * & * \\ A_{ai}Q_{Li} + B_{ai}^u \omega_L W_{Li} & -Q_{Li} & * & * \\ C_1 Q_{Li} + D_1 W_{Li} & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_{Li} I \end{bmatrix} < 0 \quad (18)$$

where $W_{Li} = K_s Q_{Li}$. This is nothing but the H_∞ formulation based on one Lyapunov function in ([9]). ■

B. Output Feedback Controller Design

Consider the static output-feedback controller

$$u(k) = K y_a(k) = K C_a(\theta) x_a(k) \quad (19)$$

Assume that $C_a(\theta)$ is of full row rank. The closed-loop system of system (9) becomes

$$\begin{cases} x_a(k+1) = [A_a(\theta) + B_a^u(\theta)\omega_L K C_a(\theta)]x_a(k) + B_a^w(\theta) w_a(k) \\ z(k) = [C_1 + D_1 K C_a(\theta)] x_a(k) \end{cases} \quad (20)$$

We have the following approach for output feedback case.

Theorem 3.2: Consider the discrete linear system (9). There exists a static output-feedback control law of type (19) such that the closed-loop system (20) is stable and the H_∞ constraint (11) is fulfilled for all faults in (2)-(4), if there exist matrices $Q_{Li} = Q_{Li}^T > 0$, non-singular matrix G, G_0 and K, K_0 such that for $i = 0, \ldots, l$, and $L = 0, 1, \ldots, l_p$, $l_p \leqslant 2^m - 1$,

$$\begin{bmatrix} Z_{11} & * & * & * & * & * \\ A_{ai}G & Z_{22} & * & * & * & * \\ C_1 G & 0 & Z_{33} & * & * & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_{Li} I & * & * \\ G & (B_{ai}^u \omega_L K C_{ai})^T & 0 & 0 & -I & * \\ G & 0 & (D_1 K C_{ai})^T & 0 & 0 & -I \end{bmatrix} < 0 \quad (21)$$

where

$$Z_{11} = Q_{Li} - G - G^T - 2G^T G_0 - 2G_0^T G + 2G_0^T G_0$$
$$Z_{22} = -Q_{Li} - B_{ai}^u \omega_L K C_{ai}(B_{ai}^u \omega_L K_0 C_{ai})^T - B_{ai}^u \omega_L K_0 C_{ai}(B_{ai}^u \omega_L K C_{ai})^T + B_{ai}^u \omega_L K_0 C_{ai}(B_{ai}^u \omega_L K_0 C_{ai})^T$$
$$Z_{33} = -I - D_1 K C_{ai}(D_1 K_0 C_{ai})^T - D_1 K_0 C_{ai}(D_1 K C_{ai})^T + D_1 K_0 C_{ai}(D_1 K_0 C_{ai})^T$$

Proof: By using the Schur lemma, (21) can be rewritten as follows:

$$\begin{bmatrix} Z_{11} & * & * & * \\ A_{ai}G & Z_{22} & * & * \\ C_1 G & 0 & Z_{33} & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_{Li} I \end{bmatrix} + \begin{bmatrix} G^T G & * & * & * \\ B_{ai}^u \omega_L K C_{ai} G & B_{ai}^u \omega_L K C_{ai} \times (B_{ai}^u \omega_L K C_{ai})^T & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} G^T G & * & * & * \\ 0 & 0 & * & * \\ D_1 K C_{ai} G & 0 & D_1 K C_{ai}(D_1 K C_{ai})^T & * \\ 0 & 0 & 0 & 0 \end{bmatrix} < 0 \quad (22)$$

which is, in turn, equivalent to,

$$\begin{bmatrix} Q_{Li} - G - G^T & * & * & * \\ (A_{ai} + B_{ai}^u \omega_L K C_{ai})G & -Q_{Li} & * & * \\ (C_1 + D_1 K C_{ai})G & 0 & -I & * \\ 0 & (B_{ai}^w)^T & 0 & -\gamma_{Li} I \end{bmatrix} + \begin{bmatrix} (G - G_0)^T \times (G - G_0) & * & * & * \\ 0 & (B_{ai}^u \omega_L K C_{ai} - B_{ai}^u \omega_L K_0 C_{ai}) \times (B_{ai}^u \omega_L K C_{ai} - B_{ai}^u \omega_L K_0 C_{ai})^T & * & * \\ 0 & 0 & 0 & * \\ 0 & 0 & 0 & 0 \end{bmatrix} + \begin{bmatrix} (G - G_0)^T \times (G - G_0) & * & * & * \\ 0 & 0 & * & * \\ 0 & 0 & (D_1 K C_{ai} - D_1 K_0 C_{ai}) \times (D_1 K C_{ai} - D_1 K_0 C_{ai})^T & * \\ 0 & 0 & 0 & 0 \end{bmatrix} < 0 \quad (23)$$

Because the second and third terms in (23) is nonnegative definite, then (23) (and hence, (21)) is a sufficient condition to the first term of (23). By Theorem 3.1, this implies the closed-loop stability and H_∞ performance (11) of the output feedback system (20). ■

Remark 3.3: Theorem 3.2 gives a sufficient condition to achieve closed-loop stability and H_∞ performance via a static output feedback control law. In fact, (21) is not an LMI. However, if G_0 and K_0 are given, then (21) is an LMI in G, K and Q_{Li}, $i = 0, \ldots, l$, and $L = 0, 1, \ldots, l_p$, $l_p \leqslant 2^m - 1$. The conservativeness of this sufficient condition lies in the differences between $G - G_0$ and $K - K_0$. By using the following iterative algorithm, this conservativeness can be minimized.

Algorithm:

1) Calculate a stabilizing static output feedback controller K_0^0. If there is no such controller can be found, then stop and the algorithm fails to get a solution. The detail can be found from *Algorithm 5.2* of [10] and is omitted here for brevity.

2) Choose performance upper bounds γ_{Li} for $i = 1, \ldots, l$ and $L = 1, \ldots, l_p$, $l_p \leqslant 2^m - 1$, with K_0^0 from Step 1, minimize γ_{00} (the performance upper bound in normal condition) subject to the inequality (14). This gives G_0^0.
3) At the j^{th} ($j = 1, 2, \ldots$) iteration, set $G_0 = G_0^{j-1}$, $K_0 = K_0^{j-1}$ and minimize γ_{00}^j subject to linear matrix inequality (21). This gives G^j, K^j and γ_{00}^j.
4) If $|\gamma_{00}^j - \gamma_{00}^{j-1}| < \epsilon$ where $\epsilon > 0$ is a given error tolerance, the obtained K^j is the optimal output feedback controller K_{opt}. Otherwise, set $j = j + 1$ and return to Step 3.

Remark 3.4: As the LMI (21) is solvable, the given algorithm is convergent. In the case of state feedback control, the method proposed here is based on LMIs instead of ILMIs. Compared with [11], where only one common Lyapunov function is adopted, the approach developed in this paper employs several parameter dependent Lyapunov functions, each one corresponding to a different vertex of the uncertainty polytope. By this method, the conservativeness can be reduced theoretically. The design and simulation in the Section IV also illuminates the advantage.

IV. Design and Simulation

In modern aircraft flight controller design, more and more challenging maneuvers such as high angle of attack flight or high roll rate flight under a low air speed condition should be considered. Under these conditions, significant kinematic and inertial couplings are prevalent and must be handled properly. Therefore, three variables: stability axis roll rate $\dot{\mu}_{rat}$, angle of attack α and sideslip angle β, must be stabilized and tracked closely.
In this section, two examples of reliable tracking control are given to demonstrate the proposed method. The nonlinear dynamic model for this F-16 aircraft can be found in [4]. The following parameters are used for the two examples, where S determines the outputs required to track.

$$S=\begin{bmatrix}0&1&0&0&0\\0&0&0&1&0\\0&0&0&0&1\end{bmatrix},\ C_1=\begin{bmatrix}I_{3\times3}&0_{3\times6}\\0_{3\times3}&0_{3\times6}\end{bmatrix},\ D_1=\begin{bmatrix}0_{3\times3}\\0.1I_{3\times3}\end{bmatrix}$$
$$\gamma_{Li}=1,\quad i=1,\ldots,l,\quad L=1,2,\ldots,l_p,\ l_p\leqslant 2^m-1$$

A. Example 1 (Actuator Outage Faults)

First, only actuator outage faults are considered. The trim condition is as follows,

$$\begin{array}{lll} T = 0.1464, & X_{cg} = 0.35, & H = 500m, \\ V_t = 152.4m/s, & \alpha = 2.3703°, & \beta = 0.0°, \\ \delta_{el} = -0.5831°, & \delta_{er} = -0.5831°, & \delta_{al} = 0.0°, \\ \delta_{ar} = 0.0°, & \delta_r = 0.0° & \end{array} \tag{24}$$

where T is the engine throttle, H is the altitude, V_t is the total airspeed, X_{cg} is the center of gravity location, δ_{el}, δ_{er}, δ_{al}, δ_{ar} and δ_r are inputs to the model (i.e., the left and right elevators, the left and right ailerons and the rudder). Then, the F-16 aircraft with independent control surfaces can be approximately modelled as follows,

$$\begin{cases} x(k+1) = Ax(k) + B^u\omega_L u(k) + B^w w(k) \\ \quad y(k) = Cx(k) \qquad L = 0, \ldots, 8 \end{cases} \tag{25}$$

where $x(k) = [u, w, q, v, p, r]^T$, $y(k) = [q, \dot{\mu}_{rat}, r, \alpha, \beta]^T$, $w(k)$ represents the vertical gust disturbance and A, B^u, B^w, C are system matrices trimmed from nonlinear aircraft model and discretized with a sampling period of $0.01s$.

In this example, the following possible control surface actuator faults are considered:

1. $\omega_0 = \text{diag}\{1,1,1,1,1\} \rightarrow$ Normal condition;
2. $\omega_1 = \text{diag}\{0,1,1,1,1\} \rightarrow$ Left elevator outage fault
3. $\omega_2 = \text{diag}\{1,0,1,1,1\} \rightarrow$ Right elevator outage fault
4. $\omega_3 = \text{diag}\{1,1,0,1,1\} \rightarrow$ Left aileron outage fault
5. $\omega_4 = \text{diag}\{1,1,1,0,1\} \rightarrow$ Right aileron outage fault
6. $\omega_5 = \text{diag}\{0,1,0,1,1\} \rightarrow$ Left elevator and left aileron outage faults;
7. $\omega_6 = \text{diag}\{0,1,1,0,1\} \rightarrow$ Left elevator and right aileron outage faults;
8. $\omega_7 = \text{diag}\{1,0,0,1,1\} \rightarrow$ Right elevator and left aileron outage faults;
9. $\omega_8 = \text{diag}\{1,0,1,0,1\} \rightarrow$ Right elevator and right aileron outage faults;

By using the proposed approach in Section III-A and Section III-B, a reliable state feedback tracking controller and a reliable output feedback tracking controller can be designed to tolerate the above outage faults.
Figure 1 and Figure 2 shows the linear simulation results of the reliable static state/output feedback controller, for all eight actuator outage faults, $\omega_1 - \omega_8$. The tracking commands are unit steps.

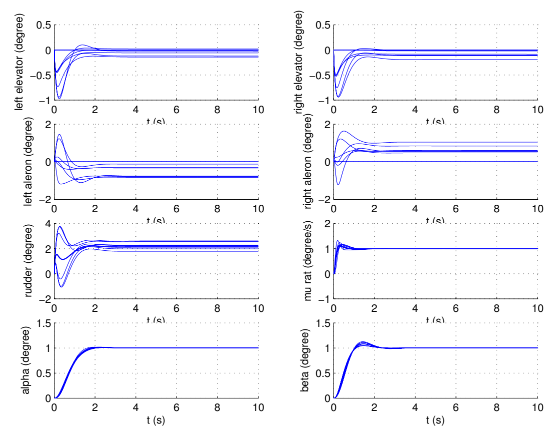

Fig. 1. Response curves of the normal and actuator outage fault cases with reliable state feedback controller

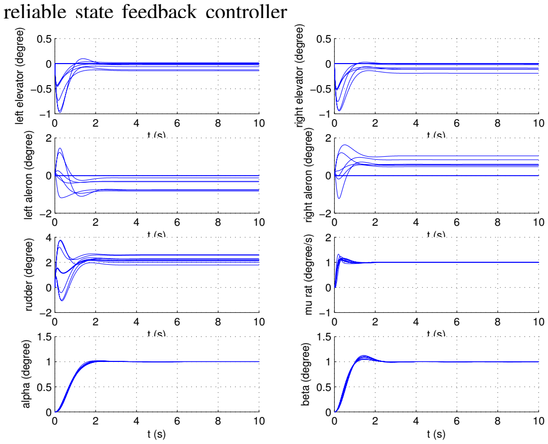

Fig. 2. Response curves of the normal and actuator outage fault cases with reliable output feedback controller

For comparison, we designed two standard (without considering the actuator outage faults, state feedback and output feedback) controllers, by using the same LMI formulation. The simulation results are shown by Figure 3 and 4.

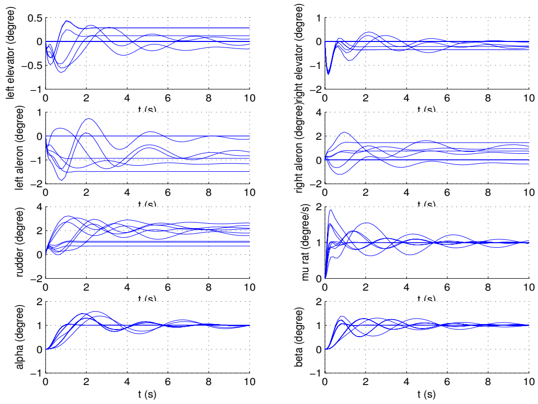

Fig. 3. Response curves of the normal and actuator outage fault cases with standard state feedback controller

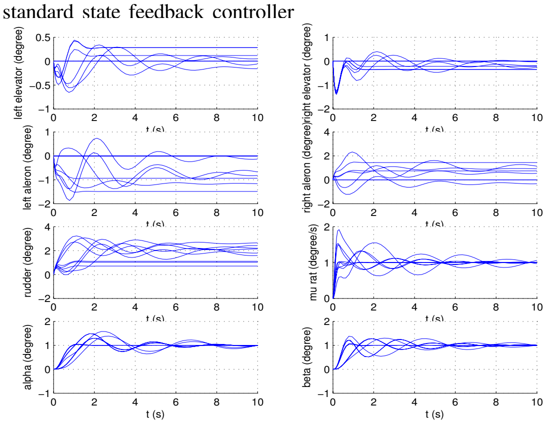

Fig. 4. Response curves of the normal and actuator outage fault cases with standard output feedback controller

From these figures above, it can be seen that, although the standard controllers perform almost the same as the reliable controllers in the normal condition, the reliable controllers give much better tracking performance in the presence of any of the eight actuator outage faults. Meanwhile, while using the ILMI method, the output feedback controller can perform as good as the state feedback controller.

B. Example 2 (Control Surface Impairment)

In this example, control surface impairment (up to 75% efficiency loss to the elevator, symmetrically) is considered. Since the aerodynamic characteristics of the aircraft will change nonlinearly from normal condition to 75% loss of elevator, in order to achieve better modelling accuracy in interpolating polytope vertices, we consider two additional vertices (25% and 50% efficiency loss to the elevator, symmetrically) besides the existing two. Then, the aircraft model with control surface impairment can be written as

$$\begin{cases} x(k+1) = A(\theta)x(k) + B^u(\theta)u(k) + B^w(\theta)w(k) \\ \quad\quad y(k) = C(\theta)x(k) \end{cases} \tag{26}$$

where $x(k) = [u, w, q, v, p, r]^T$, $y(k) = [q, \dot{\mu}_{rat}, r, \alpha, \beta]^T$, $u(k) = [\delta_e, \delta_a, \delta_r]^T$, $w(k)$ represents the vertical gust disturbance and

$$\begin{array}{ll} A(\theta) = \sum_{i=0}^{3} A_i\theta_i, & B^u(\theta) = \sum_{i=0}^{3} B_i^u\theta_i, \\ C(\theta) = \sum_{i=0}^{3} C_i\theta_i, & B^w(\theta) = \sum_{i=0}^{3} B_i^w\theta_i \end{array} \tag{27}$$

By using the proposed approach in Section III-A and III-B, a reliable state feedback tracking controller and a reliable output feedback tracking controller can be designed to tolerate the above control surface impairment. Figure 5 and 6 shows the linear simulation results of the reliable static state/output feedback controller, for normal case and 3 fault cases. The tracking commands are unit steps.

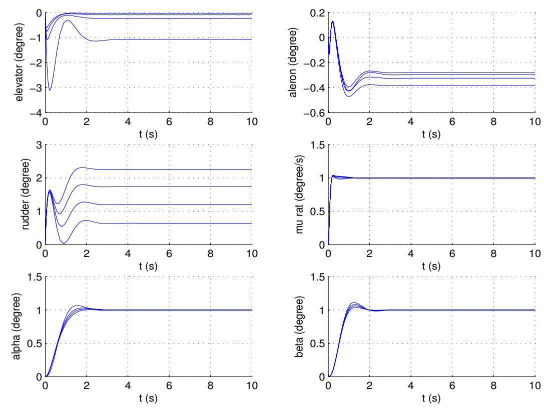

Fig. 5. Response curves of the normal condition and control surface impairment with reliable state feedback controller

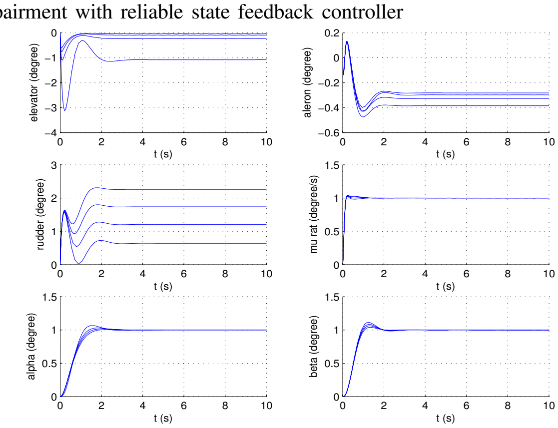

Fig. 6. Response curves of the normal condition and control surface impairment with reliable output feedback controller

For comparison, we designed two standard (without considering the control surface impairment, state/output feedback) controllers, by using the same LMI formulation. The simulation results are shown by Figure 7 and 8.

From Figure 7 and 8, the same conclusion as that of section IV-A can be drawn. Furthermore, to verify the effectiveness, the reliable output feedback controller is applied to a nonlinear F-16 aircraft model to perform a Herbst-like maneuver, which consists of a large angle-of-attack pull-up, followed by a stability axis roll. For comparison, the standard output feedback controller is also tested. The simulation result is given by Figure 9 and 10. In these simulations, a 30% loss of the elevator is introduced at 12s, followed by a 60% loss at 15s and a 70% loss at 18s.

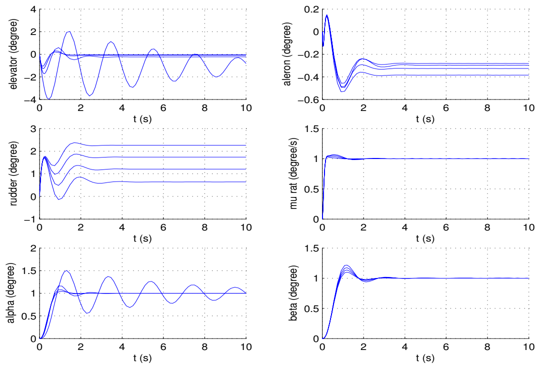

Fig. 7. Response curves of the normal condition and control surface impairment with standard state feedback controller

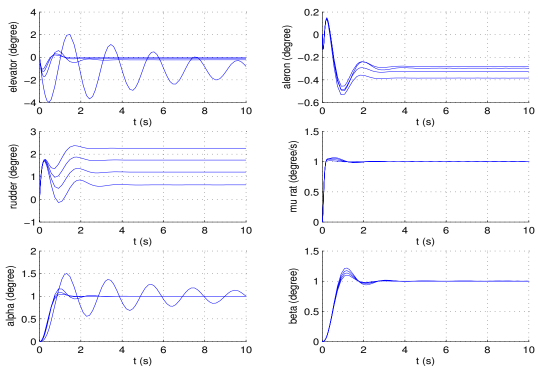

Fig. 8. Response curves of the normal condition and control surface impairment with standard output feedback controller

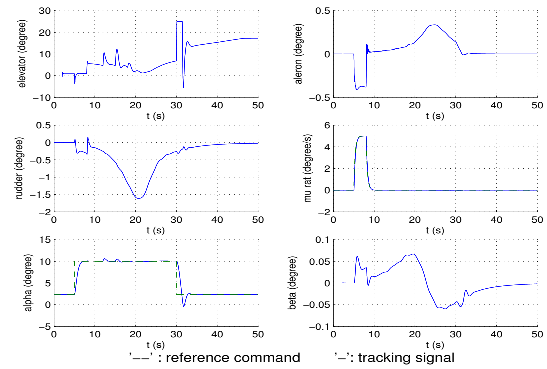

Fig. 9. Nonlinear simulation of Herbst-like maneuver by reliable output feedback controller

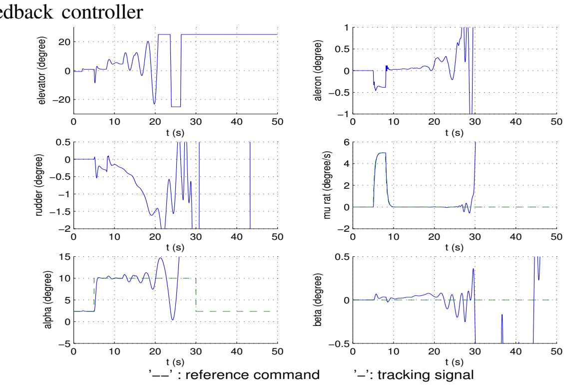

Fig. 10. Nonlinear simulation of Herbst-like maneuver by standard output feedback controller

Clearly, from Figure 9 and 10, it can be seen that the standard controller can not stabilize the aircraft in the presence of more than 60% efficiency loss of elevator, while the reliable controller can still work properly with acceptable degradation in performance.

Summarizing these two examples, it is noted that the proposed reliable controller design method can significantly improve the system performance in the presence of faults, without much sacrifice of performance in normal condition.

V. Conclusion

This paper discusses the reliable flight controller design problems subject to H_∞ constraint. The actuator outage faults and control surface impairment have been considered by using polytopic fault models. To reduce the controller's conservativeness, the proposed approach is based on several parameter dependent Lyapunov functions, each one corresponding to a different vertex of the uncertainty polytope. Furthermore, both state feedback and output feedback cases are discussed in this paper. By using an ILMI based method, the output feedback case can be solved. Finally, the proposed method is applied to a nonlinear F-16 aircraft model and the effectiveness is illuminated.

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