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Retrofit Reconfigurable Flight Control in the Presence of Control Effector Damage

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Abstract— **In this paper the development and implementation of an Integrated Retrofit Reconfigurable Flight Control design for control effector damage compensation is presented. The proposed damage-adaptive control system has the capability of detecting and identifying flight-critical actuator failures and control effector damage, and rejecting the state-dependent disturbances arising due to the asymmetry of the damaged vehicle. The proposed damage-adaptive control system is an extension of the Fast on-Line Actuator Recovery Enhancement (FLARE) system that rapidly and accurately detects and identifies actuator failures and the loss of gain of control effectors. The algorithm for estimating the nonlinearities arising due to the damage is based on Variable Structure Control (VSC) and results in a system in which explicit performance bounds can be calculated.**

I. Introduction

Due to increasing terrorist threats, in the recent years there has been a lot of interest in the development of effective adaptive reconfigurable control systems that can compensate for the damage in both military and commercial aircraft caused by man-portable air defense systems (ManPADS). The design of such control systems is a difficult and challenging problem since control effector damage not only causes the variations in the control derivative matrix, but also results in large aerodynamic disturbances due to the asymmetry of the damaged aircraft. Hence techniques of interest are those that can not only compensate for the damaged surface by reconfiguring the remaining control effectors, but also reject the resulting state-dependent disturbances.

In the past decade there has been substantial progress in the development of on-line FDIR techniques in aerospace applications [1], [3], [2], [13], [14], [18], [10], [5], [7], [12]. A large number of techniques has been proposed, and some of those have actually been flight tested [3]. Extensive simulation studies and flight-tests have demonstrated the potential of on-line FDIR systems to achieve the desired flight performance despite severe flight-critical subsystem or component failures, structural damage and large external disturbances. However, there are only a few results related to the compensation of control effector damage and the resulting state-dependent disturbances. To address this problem, in [11] the authors proposed a scheme that assumes that the control effector damage causes the changes in the gain effectiveness matrix only. However, the fact that such damage also generates state-dependent disturbances was not taken into account.

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In order to address this issue, in this paper an efficient algorithm for damage compensation is proposed that estimates the gain effectiveness matrix on line even while rejecting the damage-generated disturbances. The resulting damage-adaptive control system is integrated with the Fast on-Line Actuator Recovery Enhancement (FLARE) system, and implemented in a retrofit fashion, i.e. the baseline controller is retained, and the damage adaptive subsystem is implemented as an add-on signal. An algorithms for estimation of the unknown nonlinearity arising due to the damage using the Variable Structure Control (VSC) approach [16] is presented. It is shown that the estimation of parameters associated with actuator failures is completely decoupled from the estimation of the effect of control effector damage. However, the coupling between these two subsystems appears when the estimates are used in the reconfigurable control law. It is shown that the compensation for this coupling using the VSC approach is straightforward. In addition, explicit performance bounds can be calculated.

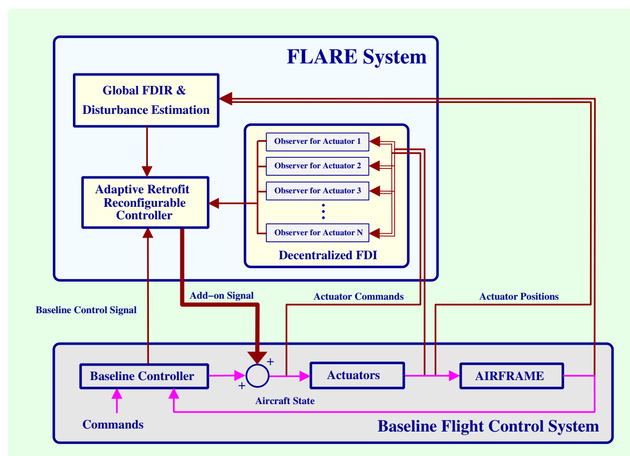

Fig. 1. *Structure of the Fast on-Line Actuator Recovery Enhancement (FLARE) System (*©1999-2004 Scientific Systems Company, Inc.*)*

The proposed damage compensation system is an extension of the Fast on-Line Actuator Recovery Enhancement (FLARE) system that accommodates actuator failures and control effector damage even while compensating for external disturbances [5], [7]. The FLARE system is shown in Figure 1 and is seen to consist of decentralized FDI observers for actuators, Global FDI for damage, and disturbance estimation and rejection. The adaptive reconfigurable controller is implemented in a retrofit fashion, i.e. the baseline flight controller is retained, while the FLARE system is active only if there is a fault whose effect needs to be compensated.

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II. Problem Statement

In this paper the focus is on the class of models of aircraft dynamics of the form:

$$\dot{x} = f(x) + g(x)u, \tag{1}$$

where $x : \mathbb{R}^+ \to \mathbb{R}^n$ denotes the state vector, $f : \mathbb{R}^n \to \mathbb{R}^n$, $g : \mathbb{R}^n \to \mathbb{R}^{n\times m}$, and $u : \mathbb{R}^+ \to \mathbb{R}^m$ denotes the control input vector. It is assumed that the above model describes dominant dynamics of the aircraft. In such a case, the state variables include total velocity V, angle-of-attack α, side-slip angle β, angular velocities p, q and r, and attitude angles ϕ, θ and ψ.

The above model is subject to the following assumption:

Assumption 1:
(a) State of the system is measurable;
(b) For a closed bounded set of states $\mathscr{S}_x$, $g(x)g^T(x)$ is invertible for all $x \in \mathscr{S}_x$;
(c) $f(x)$ and $g(x)$ are sufficiently smooth functions (functionals) of their argument; and
(d) $m > n$.

Since the nonlinear model (1) describes the dynamics of the dominant state variables, these are commonly measurable in the case of advanced fighter aircraft. Hence Assumption 1(a) is justified in most practical situations. In the case of gain-scheduled models of aircraft dynamics, the aerodynamics effects are modeled as $A(x)$ and $B(x)$ where the matrices A and B depend on several variables including the Mach number, altitude, angle of attack and side-slip angle. Each element of A and B can then be obtained from the loop-up tables, and represented as a multi-dimensional polynomial of these state variables. In most of the flight regimes, the resulting matrix $B(x)$ is such that $B(x)B^T(x)$ is invertible over a domain. Assumption 1(b) is justified in such cases. Assumption (c) is commonly justified in the case of nominal nonlinear aircraft dynamics. Assumption 1(d) is justified in the case of advanced fighter aircraft characterized by a high level of control effector redundancy.

In order to design a retrofit controller in the case of control effector damage, the nominal controller is discussed first.

Nominal Controller: In this paper the nominal controller is assumed to have the following form:

$$\dot{\omega}_N = h(\omega_N, x, r),\ \omega(0) = \omega_o \tag{2}$$
$$u_N = q(\omega_N, r), \tag{3}$$

where u_N is the output of the nominal controller, ω_N denotes the internal controller state, and r denotes a vector of bounded piece-wise continuous reference inputs (commands).

The ideal nominal controller is of the form:

$$\dot{\omega}_N^* = h(\omega_N^*, x_N, r),\ \omega^*(0) = \omega_o \tag{4}$$
$$u_N^* = q(\omega_N^*, r). \tag{5}$$

It is seen that the ideal nominal controller has x_N as the feedback signal, where x_N is the nominal system state as defined below.

Assumption 2: *The plant equation, with the initial condition at $x(0) = x_N(0)$, together with the ideal nominal controller, defines the desired dynamics of the plant:*

$$\dot{x}_N = f(x_N) + g(x_N)u_N^*,\ x_N(0) = x_{No}, \tag{6}$$

where $x_N(t)$ is well defined and bounded.

Assumption 3 : *For a set of initial conditions $x(0) = x_o$ and $x_N(0) = x_{No}$, the following holds: $\lim_{t\to\infty}[x(t,t_o;x_o,u_N) - x_N(t,t_o;x_{No},u_N^*)] = 0$.*

The implications of this assumption are discussed next. With the nominal controller, the plant equation is of the form:

$$\dot{x} = f(x) + g(x)u_N,\ x(0) = x_o, \tag{7}$$

while that of the desired dynamics is of the form (6). Let $e = x - x_N$ denote the tracking error. Then the system:

$$\dot{e} = f(x) - f(x_N) + g(x)u_N - g(x_N)u_N^*, \tag{8}$$

is locally asymptotically stable and $\lim_{t\to\infty} e(t) = 0$. Hence, to achieve asymptotic tracking of the desired dynamics (6), it is sufficient to set $u = u_N$ in (1). This is an important conclusion that will be used in the following sections. To further simplify the analysis, the following assumption is introduced:

Assumption 4 : *The right-hand side of (8) can be expressed as $A_m e$, i.e. expression (8) implies that*

$$\dot{e} = A_m e, \tag{9}$$

where A_m is a constant asymptotically stable matrix such that the system (9) is Bounded-Input Bounded-Output (BIBO) stable.

Remark: Expressing the right-hand side of (8) in the form (9) is possible in the case when the expression (8) is linearized around $e = 0$, i.e. around $x = x_N$. In such a case the linearization error is much smaller than in the case when the original plant equation is linearized around a trim. It is noted that A_m can be time-varying, which is commonly the case when the system is linearized around a time-varying trajectory. However, if the nominal controller keeps the states at a constant trim, and if the expression (8) is linearized around such a trim, the resulting A_m matrix is constant.

Example: Let the nominal controller be static and linear:

$$u_N = K_1 x + K_2 r, \tag{10}$$

and let the plant equation be of the form:

$$\dot{x} = Ax + Bu. \tag{11}$$

Then, with the nominal controller, the closed-loop system is of the form:

$$\dot{x} = (A + BK_1)x + BK_2 r. \tag{12}$$

Let a matrix K_1 exist so that $A + BK_1$ is asymptotically stable, and let

$$A + BK_1 = A_m, \quad BK_2 = B_m, \tag{13}$$

define the matrices of a reference model. Then the closed-loop system is of the form:

$$\dot{x} = A_m x + B_m r, \; x(0) = x_o. \tag{14}$$

Now, let the ideal nominal controller be of the form:

$$u_N^* = K_1 x_N + K_2 r. \tag{15}$$

The nominal plant equation is

$$\begin{aligned} \dot{x}_N &= Ax_N + Bu_N^* = (A + BK_1)x_N + BK_2 r && (16)\\ &= A_m x_N + B_m r, \; x_N(0) = x_{No}. && (17) \end{aligned}$$

The last equation specifies a reference model that defines the desired dynamics of the plant. Since A_m is asymptotically stable, the error $e = x - x_N$ tends to zero asymptotically for arbitrary initial conditions in x and x_N.

Control Objective: Design an add-on signal v such that

$$u = u_N + v, \tag{18}$$

that assures that the tracking error $e(t)$ is bounded for all time in the presence of control effector and actuator failures and state-dependent disturbances.

The control design that achieves this control objective is discussed in the following sections.

III. Actuator Failures and Control Effector Damage Modeling

Actuator failure modeling: Typical actuator and control effector failures include: (i) Lock-In-Place (LIP); (ii) Hard-Over Failure (HOF); (iii) Float; and (iv) Loss of Effectiveness (LOE). In the case of LIP failures the effector ”freezes” at a certain condition and does not respond to subsequent commands. HOF is characterized by the effector moving to the upper or lower position limit regardless of the command. The speed of response is limited by the effector rate limit. Float failure occurs when the effector ”floats” with zero moment and does not contribute to the control authority. Loss of effectiveness is characterized by lowering the effector gain with respect to its nominal value. Different types of actuator and control effector failures can be parameterized as follows:

$$u_i(t) = \begin{cases} u_c(t), & k_i(t) = 1, \; \forall t \geq t_o \\ & \textbf{(No-Failure Case)} \\ k_i(t)u_c(t), & 0 < \varepsilon_i \leq k_i(t) < 1, \forall t \geq t_{Fi} \\ & \textbf{(Loss of Effectiveness)} \\ 0, & k_i(t) = 1, \; \forall t \geq t_{Fi} \\ & \textbf{(Float Type of Failure)} \\ u_{ci}(t_{Fi}), & k_i(t) = 1, \; \forall t \geq t_{Fi} \\ & \textbf{(Lock-in-Place Failure)} \\ u_{im} \text{ or } u_{iM}, & k_i(t) = 1, \; \forall \; t \geq t_{Fi} \\ & \textbf{(Hard-Over Failure)} \end{cases}$$

where t_{Fi} denotes the time instant of failure of the ith effector, k_i denotes its effectiveness coefficient such that $k_i \in [\varepsilon_i, 1]$, and $\varepsilon_i > 0$ denotes its minimum effectiveness.

The following model:

$$u_i = \sigma_i k_i u_{ci} + (1 - \sigma_i)\bar{u}_i, \tag{19}$$

includes all above cases, where u_i is the actuator output, u_{ci} is the output of the controller (which is, at the same time, an input to the actuator), $\sigma_i = 1$ in the no-failure case, and $\sigma = 0$ in the case of failure, while $\bar{u}_i$ is a position at which the control effector locks in the case of float, lock-in-place and hard-over failures.

Uncertainty: It is seen that the uncertainty in the above model is due to the unknown actuator gain k_i in the case of loss of effectiveness, and the unknown position $\bar{u}_i$ at which the actuator locks in the case of float, lock-in-place and hard-over failures.

Control Effector Damage: Control effector damage can be modeled using the diagonal *control effector damage matrix* D whose elements d_i are equal to one in the no-failure case (i.e. $D_N = I$), while in the case of control effector damage assume values over an interval $[\varepsilon, 1]$, where $\varepsilon << 1$, and each value of d is proportional to the percentage of the loss of surface. The resulting model is of the form:

$$\dot{x} = f(x) + g(x)Du + \xi(x), \tag{20}$$

where $D = \text{diag}[d_1 \; d_2 \; ... \; d_m]$, and $\xi(x)$ is a nonlinearity arising due to the asymmetry of the damaged aircraft.

IV. Actuator FDIR

Actuator FDIR algorithms have been developed for the case of first-order [8] and second-order [4] actuator dynamics. In this paper it will be assumed that the actuator dynamics are much faster that those of the system. Hence the basic failure model is (19). It will also be assumed that the outputs of the actuators are measurable.

Observer: The FDI observers in this case are chosen in the form:

$$\hat{u}_i = \hat{\sigma}_i \hat{k}_i u_{ci} + (1 - \hat{\sigma}_i)u_i, \tag{21}$$

where $\hat{\sigma}_i$ and $\hat{k}_i$ denote respectively the estimates of σ_i and k_i. Subtracting (19) from (21) yields:

$$\hat{e}_{ui} = \phi_{\sigma i}(\hat{k}_i u_{ci} - u_i) + \sigma \phi_{ki} u_{ci} + (1-\sigma)(u_i - \bar{u}_i), \quad (22)$$

where $\hat{e}_{ui} = \hat{u}_i - u_i$, $\phi_{\sigma i} = \hat{\sigma}_i - \sigma_i$ and $\phi_{ki} = \hat{k}_i - k_i$.

Now the following assertion is considered:

Assertion 1: $(1-\sigma)(u_i - \bar{u}_i) \equiv 0$.

The proof follows trivially since, when $\sigma = 1$ (no-failure case), the assertion is true, while for $\sigma = 0$ (failure case), $u(t_{Fi}) = \bar{u}_i$.

The resulting error model is of the form:

$$\hat{e}_{ui} = \phi_{\sigma i}(\hat{k}_i u_{ci} - u_i) + \sigma \phi_{ki} u_{ci}. \quad (23)$$

Theorem IV.1: *Adjustment laws:*

$$\dot{\hat{\sigma}}_i = \dot{\phi}_{\sigma i} = Proj_{[0,1]}\{-\gamma_{\sigma i}\hat{e}_{ui}(\hat{k}_i u_{ci} - u_i)\} \quad (24)$$

$$\dot{\hat{k}}_i = \dot{\phi}_{ki} = Proj_{[\varepsilon_i,1]}\{-\gamma_{ki}\hat{e}_{ui}u_{ci}\}, \quad (25)$$

where $\gamma_{\sigma i} > 0$ *and* $\gamma_{ki} > 0$*, assure that* $\hat{e}_{ui} \in \mathscr{L}^2$.

In the above theorem, Proj$\{\cdot\}$ denotes the projection operator whose role is to keep the estimates within the pre-specified bounds. Properties of the projection type operator are described e.g. in [10].

Proof: Let a tentative Lyapunov function be of the form:

$$V(\phi_{\sigma i}, \phi_{ki}) = \frac{1}{2}\left[\frac{\phi_{\sigma i}^2}{\gamma_{\sigma i}} + \sigma_i \frac{\phi_{ki}^2}{\gamma_{ki}}\right]. \quad (26)$$

Its first derivative along the solutions of (24), (25) yields:

$$\dot{V}(\phi_{\sigma i}, \phi_{ki}) = \frac{\phi_{\sigma i}\dot{\phi}_{\sigma i}}{\gamma_{\sigma i}} + \sigma_i \frac{\phi_{ki}\dot{\phi}_{ki}}{\gamma_{ki}} \leq -\hat{e}_{ui}^2.$$

Hence $\phi_{\sigma i}$ and ϕ_{ki} are bounded, which implies that $\hat{\sigma}_i$ and $\hat{k}_i$ are bounded as well.

Upon integrating $\dot{V}$ one obtains:

$$V(0) - V(\infty) \leq \int_0^t \hat{e}_{ui}^2(\tau)d\tau.$$

Since the left-hand side terms are bounded, it follows that $\hat{e}_{ui} \in \mathscr{L}^2$. ■

It is seen that, if the signals u_{ci} are sufficiently persistently exciting [17], the parameter estimation errors will tend to zero asymptotically. However, the algorithm proposed in this paper does not require persistent excitation of the signals.

V. System Decomposition and Disturbance Estimation

As discussed in the previous section, the actuator FDI is carried out using the actuator inputs and outputs. A question that arises in this context is whether the disturbance estimation can be performed without interacting with the actuator FDI. This is discusses below.

Integrated Control Effector Failure & Damage Model: Using expression (19) and assertion 1, the overall model that describes both actuator failures and control effector damage is of the form:

$$\dot{x} = f(x) + g(x)D[\Sigma K u_c + (I - \Sigma)u] + \xi(x), \quad (27)$$

where x is the state, u_c is the output of the controller (i.e. input into the actuators), u is the actuator output, $\Sigma = \text{diag}[\sigma_1\ \sigma_2\ \ldots\ \sigma_m]$, σ_i are actuator failure indicators such that $\sigma_i = 1$ in the nominal (no-failure) case, and $\sigma_i = 0$ in the case of float, lock-in-place or hard-over failures; K denotes the diagonal actuator loss-of-effectiveness matrix, D is the control effector damage matrix, and $\xi(x)$ is a nonlinearity due to the control effector damage such that $\xi(x) = 0$ in the no-damage case.

System Decomposition: Control Effector Damage Model. It is next noted that, if u is measurable, the dynamics between u and x is of the form:

$$\dot{x} = f(x) + g(x)Du + \xi(x). \quad (28)$$

Hence, *if the actuator outputs are measurable*, the system is decomposed into the part whose input is u and output x, and the part whose input is u_c and the output is u. It is this decomposition that will be shown to facilitate the design of damage-adaptive control algorithms and minimize the interaction between the actuator FDIR and control effector damage compensation.

An algorithm for the estimation of D and ξ is discussed next. The design is based on the model (28) and the following assumption:

Assumption 5: *Bound on* $\xi(x)$ *is known, i.e.* $\|\xi(x)\| \leq c_1 + c_2\varphi(x)$ *for all* x*, where* $c_1, c_2 > 0$ *and* $\varphi(x) \geq 0$ *for all* $x \geq 0$.

In this case the following observer is used:

$$\dot{\hat{x}} = f(x) + g(x)U\hat{d} + \hat{\xi} + A_m\hat{e}, \quad (29)$$

where U and $\hat{d}$ re defined previously, and $\hat{\xi}$ is to be designed.

The resulting error model is of the form:

$$\dot{\hat{e}} = A_m\hat{e} + g(x)U\phi_D + \hat{\xi} - \xi(x) \quad (30)$$

where $\phi_D = \hat{d} - d$.

Theorem V.1: *If the elements of* $\hat{D}$ *and* $\hat{\xi}$ *are adjusted using the following adjustment laws:*

$$\dot{\hat{d}} = Proj_{[0,1]}\{-\gamma U g^T(x)P\hat{e}\} \quad (31)$$

$$\hat{\xi} = -\frac{P^{-1}\hat{e}\|P\|(c_1 + c_2\varphi(x))}{\|\hat{e}\|\|P\|(c_1 + c_2\varphi(x)) + \delta}, \quad (32)$$

where $\gamma > 0$*, the resulting closed-loop system is stable and the estimation error is Uniformly Ultimately Bounded (UUB) such that*

$$\lim_{t\to\infty}\|\hat{e}(t)\| = \delta_E = \sqrt{\frac{m(1-\varepsilon)\lambda_m(Q) + 2\delta\lambda_M(P)\gamma}{\lambda_m(P)\lambda_m(Q)\gamma}}, \quad (33)$$

where $\lambda_m(P)$ and $\lambda_M(P)$ denote respectively the minimum and maximum eigenvalue of P, and m is the number of control effectors.

Proof: The following Lyapunov function candidate is chosen:

$$V(\hat{e},\phi_D)=\frac{1}{2}[\hat{e}^T P\hat{e}+\frac{1}{\gamma}\phi_D^T\phi_D], \tag{34}$$

where $\gamma>0$, and $P=P^T>0$ is a solution of the Lyapunov Matrix Equation $A_m^TP+PA_m=-Q$, where $Q=Q^T>0$.

First derivative of V along the solution of the system yields:

$$\dot{V}(\hat{e},\phi_D)\leq-\frac{1}{2}\lambda_m(Q)\|\hat{e}\|^2+\hat{e}^TP\hat{\xi}+\|\hat{e}\|\|P\|(c_1+c_2\varphi(x)),$$

where $\lambda_m(Q)$ denotes the minimum eigenvalue of Q.

Using (32) one further has:

$$\dot{V}\leq-\frac{1}{2}\lambda_m(Q)\|\hat{e}\|^2+\delta. \tag{35}$$

It follows that $\hat{e}$ is bounded [17].

From (34) it follows that:

$$\begin{aligned}V &\leq \frac{1}{2}\lambda_M(P)\|\hat{e}\|^2+\frac{1}{2\gamma}\|\phi_D\|^2\\ &\leq \frac{1}{2}\lambda_M(P)\|\hat{e}\|^2+\frac{m(1-\varepsilon)}{2\gamma},\end{aligned}$$

since $\phi_{Dj}=\hat{d}_j-d_j$, each $d_j\in[\varepsilon,1]$, and projection-type adjustment law assures that $\hat{d}(t)\in[\varepsilon,1]$ for all time. Hence $\|\phi_D\|^2\leq m(1-\varepsilon)$, where m denotes the number of control effectors. From the above expression it follows that:

$$\|\hat{e}\|^2\geq\frac{2V}{\lambda_M(P)}-\frac{m(1-\varepsilon)}{2\lambda_M(P)\gamma}.$$

From (35) one now has:

$$\dot{V}\leq-c_1V+c_2, \tag{36}$$

where

$$c_1=\frac{\lambda_m(Q)}{\lambda_M(P)},\ c_2=\frac{m(1-\varepsilon)\lambda_m(Q)}{2\lambda_M(P)\gamma}+\delta.$$

Since V is positive definite, the inequality (36) can be integrated from $t_o=0$ to t to obtain:

$$V(t)\leq V(0)\exp(-c_1t)+\frac{c_2}{c_1}[1-\exp(-c_1t)].$$

From (34) one also has:

$$V\geq\frac{1}{2}\lambda_m(P)\|\hat{e}\|.$$

Combining this expression with (37) yields:

$$\begin{aligned}\|\hat{e}\|^2 &\leq \frac{2V(0)\exp(-c_1t)}{\lambda_m(P)}+\frac{2c_2}{\lambda_m(P)c_1}[1-\exp(-c_1t)]\\ &\leq \frac{2[\lambda_M(P)\|\hat{e}(0)\|^2\gamma+m(1-\varepsilon)]\exp(-c_1t)}{\lambda_m(P)\gamma}\\ &\quad+\frac{2c_2}{\lambda_m(P)c_1}[1-\exp(-c_1t)].\end{aligned}$$

Taking the square root of the above expression and letting the time tend to infinity, one obtains that the assertion (33) is true. ■

Remark: For a fixed Q, there are two design parameters in the expression (33), δ and γ. If, for instance, δ is chosen as $\delta=1/\gamma$, the UUB bound can be made arbitrarily small by increasing γ. However, very small values of δ will result in chattering of the signals in the system. Hence these parameters are chosen as a trade-off between the estimation accuracy and the smoothness of the signals.

VI. Retrofit Reconfigurable Control

The retrofit control design is based on the expression (27). It is assumed that the signal generated by the controller is of the form:

$$u_c=u_N+v, \tag{37}$$

where v is to be designed to achieve the control objective. The ideal add-on signal v^* is designed first by solving the following expression:

$$g(x)D[\Sigma K(u_N+v^*)+(I-\Sigma)u]+\xi(x)=g(x)u_N,$$

which yields the following expression for v^*:

$$\begin{aligned}v^* &= (g(x)D\Sigma K)^T[g(x)D^2\Sigma^2K^2g(x)^T]^{-1}\{g(x)(I-D\Sigma K)\\ &\quad -g(x)D(I-\Sigma)u-\xi(x)\}.\end{aligned}$$

The actual add-on signal is implemented as:

$$\begin{aligned}v &= (g(x)\hat{D}\hat{\Sigma}\hat{K})^T[g(x)\hat{D}^2\hat{\Sigma}^2\hat{K}^2g(x)^T]^{-1}\{g(x)(I-\hat{D}\hat{\Sigma}\hat{K})\\ &\quad -g(x)\hat{D}(I-\hat{\Sigma})u-\hat{\xi}(x)+\zeta\},\end{aligned}$$

where ζ is to be designed to minimize the interactions between the actuator FDI and damage estimation.

Theorem VI.1: *The add-on signal of the form (38), where the estimates are adjusted using (31), (32), and ζ is adjusted using:*

$$\zeta=-\frac{P^{-1}\|\hat{e}_m\|\|P\|\|g(x)\|\|\Omega\|}{\|\hat{e}_m\|\|P\|\|g(x)\|\|\Omega\|+\delta}, \tag{38}$$

results in a stable system in which the tracking error is UUB with the UUB bound defined as $\delta_T=\delta_E+\delta_M$, where δ_E is defined by (33), and

$$\delta_M=\sqrt{\frac{2\delta\lambda_M(P)}{\lambda_m(P)\lambda_m(Q)}}.$$

Proof: The expressions (37) and (38) are substituted into (27) to obtain:

$$\begin{aligned}\dot{x} &= f(x)+g(x)\hat{D}[\Sigma K(u_N+v)+(I-\Sigma)u]+\hat{\xi}+A_m\hat{e}\\ &= f(x)+g(x)\hat{D}[\hat{\Sigma}\hat{K}(u_N+v)+(I-\hat{\Sigma})u]\\ &\quad +g(x)\hat{D}\Phi^T\Omega+\hat{\xi}+A_m\hat{e}\\ &= f(x)+g(x)u_N+g(x)\hat{D}\Phi^T\Omega+\zeta+A_m\hat{e},\end{aligned}$$

where $\Phi=[\Phi_\sigma^T\ \Phi_k^T]^T,\Phi_\sigma=\hat{\Sigma}-\Sigma,\Phi_k=\hat{K}-K$, and $\Omega=[(\hat{K}u_c-u)^T\ \Sigma u^T]^T$.

Let $e_m = \hat{x} - x_N$, i.e. the error between the observer state and that of the desired dynamics. Nominal plant dynamics (6) is subtracted from the above expression next to obtain:

$$\begin{aligned}\hat{e}_m &= f(x) - f(x_N) + g(x)u_N - g(x_N)u_N^* \quad (39)\\ &\quad + g(x)\hat{D}\Phi^T\Omega + \zeta + A_m\hat{e}\\ &= A_m\hat{e}_m + g(x)\hat{D}\Phi^T\Omega + \zeta, \quad (40)\end{aligned}$$

where Assumption 4 is used. It is seen that there is an extra term that depends on unknown Φ. While this term is unknown, its bound can be calculated since $\hat{\sigma}(t) \in [0,1]$, $\sigma(t) \in [0,1]$, $\hat{k}_i(t) \in [\varepsilon_i, 1]$, $k_i(t) \in [\varepsilon_i, 1]$, $\hat{d}_i(t) \in [\varepsilon_i, 1]$ and $d_i(t) \in [\varepsilon_i, 1]$ for all time, which results in $\|\Phi\| \leq 1$.

The following tentative Lyapunov function is chosen next:

$$V(\hat{e}_m) = \frac{1}{2}\hat{e}_m^T P \hat{e}.$$

Its first derivative along the solutions of (40) yields

$$\begin{aligned}\dot{V}(\hat{e}_m) &= -\frac{1}{2}\hat{e}_m^T Q \hat{e}_m + \hat{e}_m P g(x)\hat{D}\Phi^T\Omega + \hat{e}_m^T P\zeta\\ &\leq -\frac{1}{2}\lambda_m(Q)\|\hat{e}_m\|^2 + \|\hat{e}_m\|\|P\|\|g(x)\|\|\Omega\| + \hat{e}_m^T P\zeta\\ &\leq -\frac{1}{2}\lambda_m(Q)\|\hat{e}_m\|^2 + \delta,\end{aligned}$$

where the expression (38) is used.

From (41) one has:

$$\frac{1}{2}\lambda_M(P)\|\hat{e}_m\|^2 \geq V(\hat{e}_m) \geq \frac{1}{2}\lambda_m(P)\|\hat{e}_m\|^2.$$

Hence:

$$\dot{V} \leq -c_1 V + c_2,$$

where $c_1 = \lambda_m(Q)/\lambda_M(P)$ and $c_2 = \delta$. It now follows that:

$$\|\hat{e}_m\| \leq \sqrt{\frac{2\delta\lambda_M(P)}{\lambda_m(P)\lambda_m(Q)}} = \delta_M.$$

Since $\hat{e}_m = \hat{x} - x_N \pm x = \hat{e} - e$ where $e = x - x_N$ is the tracking error, it follows that $\|e\| = \|\hat{e} - \hat{e}_m\| \leq \|\hat{e}\| + \|\hat{e}_m\| = \delta_E + \delta_M$, which completes the proof. ■

Since e and x_N are bounded, this also implies the boundedness of x, which in turn implies that u_{ci} and u_i are bounded. It can now be concluded that each $\hat{e}_{ui}$ defined in (23) is bounded and, as shown previously, belongs to $\mathscr{L}^2$. By taking the derivative of (23), it can be shown that $\dot{\hat{e}}_{ui}$ is also bounded, which now implies that $\lim_{t\to\infty}\hat{e}_{ui}(t) = 0$.

VII. Conclusion

In this paper the development and implementation of an Integrated Retrofit Reconfigurable Flight Control design for control effector damage compensation is presented. The proposed damage-adaptive control system has the capability of detecting and identifying flight-critical actuator failures and control effector damage, and rejecting the state-dependent disturbances arising due to the asymmetry of the damaged vehicle. The proposed damage-adaptive control system is an extension of the Fast on-Line Actuator Recovery Enhancement (FLARE) system that rapidly and accurately detects and identifies actuator failures and the loss of gain of control effectors. The algorithm for estimating the nonlinearities arising due to the damage is based on Variable Structure Control (VSC) and results in a system in which explicit performance bounds can be calculated.

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