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Adaptation-Based Reconfiguration in the Presence of Actuator Failures and Saturation

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Abstract— **In this paper, a direct-adaptive flight controller is proposed to enable reconfiguration in the presence of actuator failures, environmental disturbances, and modeling errors. The controller combines reconfiguration and control allocation. In addition, a multivariable saturation compensation technique is introduced to preserve stability in the presence of control saturation phenomena. An adaptation law is derived for which it is conjectured that the closed-loop system states and controller gains remain bounded, provided that sufficient control authority exists. Benefits of the proposed design are demonstrated using a linearized six degree of freedom simulation model of a large, four engine transport aircraft.**

I. Introduction

A RECONFIGURABLE controller is one that automatically redesigns control laws so as to restore nominal control of a plant in the event of actuator failure or other unforeseen changes in the plant dynamics. Reconfigurable controllers have the potential to provide a significant benefit in safety in several applications; most notably in aircraft. It is estimated that 14% of all fatal aircraft failures could have been prevented, in principle, by an effective reconfigurable flight control system [1]. Reconfigurable control is crucial to the development of Unmanned Air Vehicles (UAV's) as well, since UAV's are required to follow navigational commands in the presence of uncertain, possibly failure-related, disturbances. Several approaches to reconfigurable control have been proposed for various applications [2], [3], [4], however, this work will focus primarily on the use of adaptive control for reconfiguration.

Most modern aircraft have redundant control effectors, and thus employ so-called control allocation techniques [5]. For such aircraft, it is logical to use the actuator redundancy for the problem of reconfiguration. In other words, if an actuator fails then it might be possible to compensate for its adverse effects by utilizing the remaining healthy controls. Control allocation algorithms such as Pseudo-Inverse [5], Quadratic Programming [6], Linear Programming [7], Direct Allocation [8], and others have been applied to the problem of reconfiguration with varying degrees of success. Specifically, Durham's Direct Allocation has received much attention recently, and has been shown to be among the most promising allocation algorithms [8], [9].

To be useful for reconfiguration, all of the above allocation methods require an online estimate of the aircraft dynamics after failure. The speed of convergence of such online estimators depends on excitation of the external inputs, which can be somewhat alleviated by using multiple models as demonstrated by Boskovic and Mehra [10]. In light of these potential difficulties, another direction of research has focused on reconfiguration with fixed allocation [11]-[15], where direct-adaptation can be applied straightforwardly in the realm of virtual-inputs. In [11] a reconfiguration retrofit module is developed for transport aircraft, with a direct-adaptive input error approach. In [12] several adaptation methods are examined, and it is concluded that a direct-adaptive input error formulation is most appropriate for the problem of reconfiguration. Similar techniques are used in [13] with a cascading adaptive formulation, and in [14] with an adaptive neural network. In [15], Tao et. al. consider direct-adaptation for reduced order, single input systems representative of aircraft dynamics. In all of these cases, the controller was assumed to operate with fixed control allocation. This is an inherent shortcoming because it fails to utilize the versatility of an over-actuated aircraft. Specifically, it requires that independent control surfaces be restricted to move in constant proportion to one another, thereby limiting the moments attainable by the control system.

In this paper, we develop a controller for a realistic multiple-input aircraft model that simultaneously performs reconfiguration and control allocation by using a direct-adaptive approach. This strategy combines the benefits of both approaches discussed above, while avoiding the limitations of either. In addition, the proposed controller is able to suppress the adverse effects of actuator saturation by using a proposed multivariable extension of the technique developed by Karason and Annaswamy [16], often referred to as "Training Signal Hedging" (TSH). The form of a stability theorem is conjectured based on the

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stability result of [16], though a proof is not currently available.

II. Problem Statement

We start with a linearized model of a failed plant of the form

$$\dot{x} = A_p x + \Lambda B_p u + B_p f\,, \qquad (1)$$

where $x \in \Re^n$ is the state vector, $u \in \Re^m$ is the input to the control actuators, $A_p \in \Re^{n\times n}$ is unknown, and $B_p \in \Re^{n\times m}$ is known. Actuator failures are represented by the unknown matrix $\Lambda \in \Re^{m\times m}$ and the unknown vector $f \in \Re^m$. Λ is a diagonal positive semi definite matrix with elements $0 \le \lambda_i \le 1$ for $i = 1,\ldots,m$. For each element, $\lambda_i = 0$ represents a complete failure of the corresponding actuator, $0 < \lambda_i < 1$ represents a class of partial actuator failures, and $\lambda_i = 1$ denotes a healthy actuator. The constant vector f accounts for the possibility that a failed actuator may be locked in an out-of-trim position. Additionally, the dynamics matrix A_p is considered to be unknown to account for two separate failure phenomena. First, if a nominal state feedback controller is in place prior to actuator failure, the feedback structure will cause A matrix changes in the event of failed actuators. Secondly, actuator failures may be accompanied by aerodynamic changes, which are expressed as uncertainties in the A matrix. Equation (1) can be considered as describing the dynamics of a perturbation about a known trimmed state. The state is assumed to be fully measurable, which is a common assumption and is reasonable given that modern aircraft are equipped with a large number of sensors [10].

In the case of reconfiguration after a failure, we can expect a vigorous utilization of the remaining operational actuators. It is therefore important to consider the effects of saturation on the system dynamics. We introduce multidimensional saturation into the model as

$$u = sat(u_c)\,,$$

where the $sat()$ function is defined element-wise by

$$sat_i(u_{ci}) = \begin{cases} u_{\min i} & if \quad u_i < u_{\min i} \\ u_i & if \quad u_{\min i} \le u_i \le u_{\max i} \\ u_{\max i} & if \quad u_i > u_{\max i} \end{cases}, \qquad (2)$$

$$\text{for } i = 1,\ldots,m\,,$$

and where the vectors u_{min} and u_{max} define the minimum and maximum position limits respectively of the m actuators. We have assumed that the effects of actuator dynamics are negligible, which is a reasonable and common assumption for large, slowly maneuvering aircraft [10], [15].

The task of control allocation is to determine a suitable control input, u, given a pilot command, $r \in \Re^l$, where $l < m$. We therefore choose a reference model of the form

$$\dot{x}_m = A_m x_m + B_m r\,, \qquad (3)$$

where $x_m \in \Re^n, r \in \Re^l, A_m \in \Re^{n\times n}$ is Hurwitz, and $B_m \in \Re^{n\times l}$, so that x_m represents the desired state plant. The goal is to choose u so that $e(t) = x - x_m$ is as small as possible, and all signals in the closed-loop system remain bounded.

III. Adaptive Controller

The controller we propose has a standard state-feedback/feedforward structure [17], [18]:

$$u_c = K_x x + K_r r + \hat{f}\,. \qquad (4)$$

We assume that there exist ideal gain matrices $K_x^* \in \Re^{m\times n}$ and $K_r^* \in \Re^{m\times l}$, and an ideal disturbance vector $f^* \in \Re^m$ that result in perfect model following, so that

$$A_p + B_p \Lambda K_x^* = A_m \qquad (5)$$

$$B_p \Lambda K_r^* = B_m \qquad (6)$$

$$B_p(\Lambda f^* + f) = 0\,. \qquad (7)$$

Equations (5)-(7) are the so-called matching conditions for the adaptive system. The ideal gains need not be known; their existence is sufficient to show stability. Conditions (5) and (6) are standard in multivariable adaptive control [17], [18] and (7) is required because of the possible constant moment from a locked actuator. Tao et al [15], considered similar matching conditions in detail. In particular, situations in which the conditions are achievable and unachievable were investigated. Such matters will not be pursued here as this work is mostly concerned with the novelty of the control allocating design and the incorporation of a multi-input TSH technique. We conjecture below, based on the results of [16], that the controller in (4) guarantees that the closed-loop system has bounded solutions for a limited set of initial conditions.

Substituting (2) into (1) gives

$$\dot{x} = A_p x + B_p \Lambda u_c + B_p \Lambda \Delta u + Bf\,, \qquad (8)$$

where $\Delta u = u - u_c$ represents the control deficiency signal, as formulated in [16]. Then, after substituting (4) into (8), simplifying using (5)-(7) and taking the difference with (3), the error dynamics of the system can be written

$$\dot{e} = A_m e + B_p \Lambda(\tilde{K}_x x + \tilde{K}_r r + \tilde{f} + \Delta u)\,, \qquad (9)$$

where

$$\tilde{K}_x = K_x - K_x^*,\ \tilde{K}_r = K_r - K_r^*, \text{ and } \tilde{f} = \hat{f} - f^*.$$

TSH is implemented to compensate for the adverse effects of saturation. To this end, an auxiliary error is generated as in [16]:

$$\dot{e}_\Delta = A_m e_\Delta + B_p diag(\hat{\lambda})\Delta u\,,$$

where $\hat{\lambda} \in \Re^m$ is a vector, the terms of which are the current estimates of the diagonal terms of the failure matrix Λ. The effects due to saturation can be removed from the error in (9) by defining the augmented error as $e_u = e - e_\Delta$, which can be written

$$\dot{e}_u = A_m e_u + B_p \Lambda(\widetilde{K}_x x + \widetilde{K}_r r + \widetilde{f}) + B_p diag(\Delta u)\widetilde{\lambda}, \tag{10}$$

where $diag(\widetilde{\lambda}) = \Lambda - diag(\hat{\lambda})$, and exploiting the fact that $diag(\widetilde{\lambda})\Delta u = diag(\Delta u)\widetilde{\lambda}$.

Since Eq. (10) is in a standard form relevant to several adaptive systems, we choose the adaptive laws for adjusting the parameters K_x, K_r, $\hat{f}$, and $\hat{\lambda}$ as

$$\dot{K}_x = -\Gamma_1 B^T P e_u x^T \tag{11}$$

$$\dot{K}_r = -\Gamma_2 B^T P e_u r^T \tag{12}$$

$$\dot{\hat{f}} = -\Gamma_3 B^T P e_u \tag{13}$$

$$\dot{\hat{\lambda}} = \Gamma_4 diag(\Delta u) B_p^T P e_u, \tag{14}$$

where $A_m^T P + P A_m = -Q$, $Q > 0$, and Γ_i is diagonal and positive definite for $i = 1,...,4$. The right hand side of equation (15) intentionally has the opposite sign of the others.

Define a Lyapunov function candidate V as

$$V = e_u^T P e_u + Tr(\widetilde{K}_x^T \Gamma_1^{-1} \Lambda \widetilde{K}_x) + Tr(\widetilde{K}_r^T \Gamma_2^{-1} \Lambda \widetilde{K}_r) + \widetilde{f}^T \Gamma_3^{-1} \Lambda \widetilde{f} + \widetilde{\lambda}^T \Gamma_4^{-1} \widetilde{\lambda}. \tag{15}$$

Since Γ_i is diagonal, V is positive definite in $e_u, \Lambda\widetilde{K}_x, \Lambda\widetilde{K}_r, \Lambda\widetilde{f}$ and $\widetilde{\lambda}$. Taking the time derivative along the system trajectories leads to $\dot{V} = -e_u^T Q e_u \le 0$, which implies that the signals $e_u, \Lambda\widetilde{K}_x, \Lambda\widetilde{K}_r, \Lambda\widetilde{f}$ and $\widetilde{\lambda}$ are bounded.

Conjecture 1: There exist positive scalars x_{max} and K_{max} such that, for the system in (1) with the controller described in (4) and (11)-(14), $x(t)$, has bounded trajectories for all $t \ge t_0$ if

i) $\|x(t_0)\| < x_{\max} \dfrac{1}{\sqrt{\rho}}$,

ii) $\sqrt{V(t_0)} < K_{\max} \sqrt{\dfrac{\lambda_{\min}}{\gamma_{\min}}}$.

Further,

$$\|x(t)\| < x_{\max} \quad \forall t \ge t_0,$$

and the output error e is of the order

$$\|e\| = O\left[\sup_{\tau \le t} \|\Delta u(\tau)\|\right].$$

Two theorems similar to conjecture 1 are proved in [16], one for a scalar adaptive system and one for a single-input higher order adaptive system. It is envisioned that a similar strategy can be used to prove conjecture 1, however one encounters an important difficulty in extending the proof to the multi-input case. The proofs in [16] treat two separate conditions based on the sign of the saturated input u. Of course, in the multiple input case, one must find a generalization of these conditions in terms of vector quantities. This extension is currently under investigation.

The above formulation of an adaptive reconfigurable flight controller has two principle improvements over other similar formulations. First, the controller does not rely on a fixed control allocation, and consequently the range of allowable actuator failures is expanded. In particular, previous direct-adaptive strategies have focused on designing $u_a \in \Re^l$, where

$$u_c = G u_a,$$

and where $G \in \Re^{m \times l}$ is a fixed control allocation matrix. This requires that the failed system (A_p, $B_p \Lambda G$) be controllable. The formulation in this paper requires that (A_p, $B_p \Lambda$) be controllable, which is a less restrictive requirement. Second, this formulation includes multivariable TSH, which, it is conjectured, will give bounded solutions for certain initial conditions. It should also be noted that recent extensions along the lines of [19] can be used to improve the transient performance of the adaptive system further.

IV. Simulation Results

While the results presented in this paper are applicable to any system of the form (1) and (2), our focus is primarily on the 6 degree of freedom (6-DoF) aircraft dynamics. For this system, the state vector is of the form

$$x = [\alpha \quad q \quad \beta \quad p \quad r]^T,$$

whose elements are the angle of attack, body pitch rate, angle of sideslip, body roll rate, and body yaw rate, respectively. The first two and the last three states are nearly decoupled in a linearized, trimmed system. We have implicitly used a common time-scale separation approximation. The 5 states considered here are typical of models of "fast" aircraft dynamics for stability augmentation, or "inner loop" control. The "slow" dynamics, consisting of airspeed, 3 Euler angles, and 3 earth referenced positions, are typically used in models for aircraft guidance and navigation, or "outer loop" control, and will not be considered here.

Using a linear model representative of a large transport aircraft [20], the direct-adaptive reconfigurable flight controller is designed according to the equations in (4) and (11)-(14). In this simulation $n = 5$, $m = 10$, and $l = 4$. The aircraft, has 4 power plants, 2 independent elevator panels, 2 independent aileron panels, and 2 independent rudder panels. The input vector is defined as

$$u = [t1 \quad t2 \quad t3 \quad t4 \quad e1 \quad e2 \quad a1 \quad a2 \quad r1 \quad r2]^T.$$

The states, actuator deflections, and pilot inputs are denoted as shown in Table 1.

States (x)	Description
α	Angle of Attack
q	Body Pitch Rate
β	Side Slip Angle
p	Roll Rate
r	Yaw Rate

Actuators (u)	Description	Saturation Limits $u_{i,min} \le u_i \le u_{i,max}$
t1	Left Outboard Throttle	$t1_{min}$ = -.4331 $t1_{max}$ = .5669
t2	Left Inboard Throttle	$t2_{min}$ = -.4331 $t2_{max}$ = .5669
t3	Right Inboard Throttle	$t3_{min}$ = -.4331 $t3_{max}$ = .5669
t4	Right Outboard Throttle	$t4_{min}$ = -.4331 $t4_{max}$ = .5669
e1	Left Elevator	$e1_{min}$ = -.2797rad $e1_{max}$ = .3313rad
e2	Right Elevator	$e2_{min}$ = -.2797rad $e2_{max}$ = .3313rad
a1	Left Aileron	$a1_{min}$ = -.436rad $a1_{max}$ = .436rad
a2	Right Aileron	$a2_{min}$ = -.436rad $a2_{max}$ = .436rad
r1	Lower Rudder	$r1_{min}$ = -.524rad $r1_{max}$ = .524rad
r2	Upper Rudder	$r2_{min}$ = -.524rad $r2_{max}$ = .534rad

Pilot Inputs (r)	Description
T	Virtual Throttle
E	Virtual Elevator
A	Virtual Aileron
R	Virtual Rudder

Table 1: Chart showing the aircraft states, actuators with saturation limits, and pilot inputs used in the simulation. Note that the saturation limits are expressed as variations from a trimmed position.

The matrices A_p, A_m, B_p, and B_m, the simulation initial conditions, and adaptive gains Γ_i and Q are shown in the appendix. The aircraft model is taken from a nonlinear 6-DoF aircraft simulation linearized at a speed of mach .3 and an altitude of 1000m. The reference model is chosen so as to decouple the aircraft roll and yaw desired dynamics.

The performance of the closed loop adaptive system is compared with that of a nominal controller using a control law with the same structure as in (4) but with fixed gains K_{nomx} and K_{nomr} designed to achieve model following for the unfailed plant. The nominal gains are given in the appendix. The results of four simulation runs with different control failures are shown in Figures 1-6. Failure 1 shows a locked left elevator scenario, failure 2 corresponds to a locked left aileron, failure 3 represents a left outboard engine-out scenario, and failure 4 is a failed left elevator locked at full deflection. These failure scenarios are discussed in detail below.

A. Failure 1

Figures 1 and 2 show the results of a failed left elevator locked at 20% of its maximum upward deflection, which is represented as

$$\Lambda(5,5) = 0 \text{ and } f(5) = .2 \times e1_{\min},$$

where negative elevator corresponds to upward deflection. The aircraft is commanded through two aggressive pitch doublets and the failure occurs at the beginning of the first maneuver at t = 6 seconds. With the nominal controller, the aircraft performance degrades quickly after the failure, as shown in figure 1. With the adaptive controller, the model is followed closely through the maneuvers after a transient of about 2 seconds. Figure 2 shows the control activity required to follow the model. In this example, the asymmetric elevator deflection due to failure causes a roll moment, which is compensated for by the ailerons. The lateral dynamics are not shown for the sake of brevity.

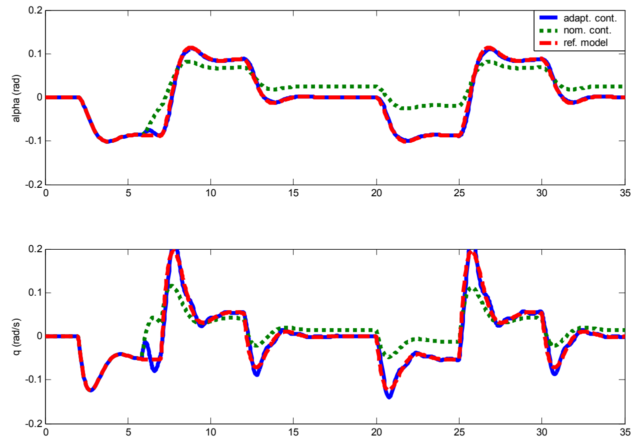

Figure 1: System response to failure 1, a failed left elevator locked at 20% of its upward travel. The aircraft is commanded through two aggressive pitch doublets and the failure occurs at t = 6s. Only the longitudinal states are shown.

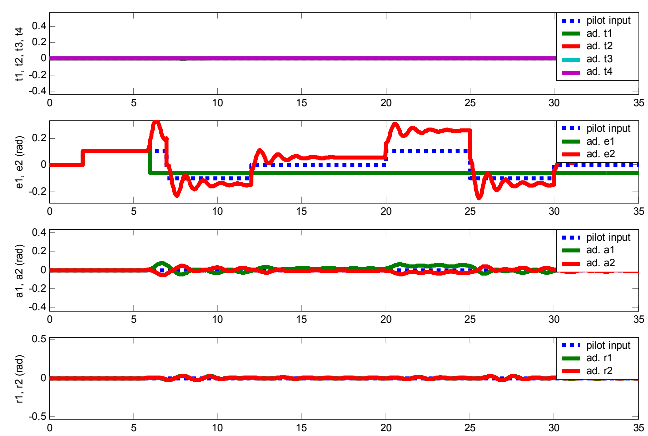

Figure 2: Actuator activity of the aircraft with failure 1, a failed left elevator locked at 20% of its upward travel. The aircraft is commanded through two aggressive pitch doublets and the failure occurs at t = 6s.

B. Failure 2

Figures 3 and 4 show the results of a failed left aileron locked at 40% downward deflection, represented as

$$\Lambda(7,7) = 0 \text{ and } f(7) = .4 \times a1_{\max},$$

where positive left aileron corresponds to downward deflection. Similar to the previous case, the aircraft is commanded through two aggressive roll doublets and the failure occurs at the beginning of the first maneuver at t = 6 seconds. The nominal controller causes the performance of the aircraft to degrade after the failure. The adaptive system, on the other hand, follows the model closely throughout the maneuver after a short transient. Notice that yaw and roll have been successfully decoupled, even in the presence of failure and saturation, as can be seen by the clipped aileron signal in Figure 4. Also, it can be seen that the rudder must be actuated to achieve the yaw-roll decoupling, and the elevators are actuated anti-symmetrically to help produce the required roll moment.

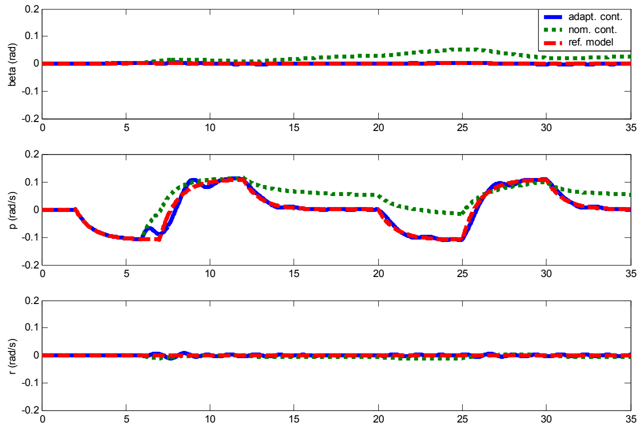

Figure 3: System response to failure 2, a failed left aileron locked at 40% of its downward travel. The aircraft is commanded through two aggressive roll doublets and the failure occurs at t = 6s. Only the lateral states are shown.

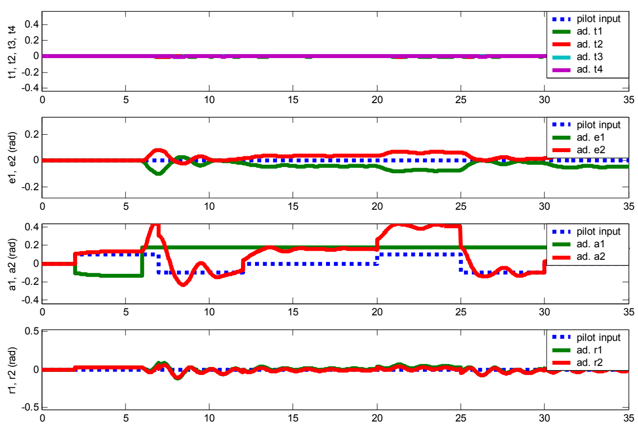

Figure 4: Actuator activity of the aircraft with failure 2, a failed left aileron locked at 40% downward travel. The aircraft is commanded through two aggressive roll doublets and the failure occurs at t = 6s.

C. Failure 3

Figure 5 shows the results of a failed left outboard power plant giving zero thrust, which is represented as

$$\Lambda(1,1) = 0 \text{ and } f(1) = 1 \times t1_{\min} .$$

In this case, the aircraft is commanded through two aggressive yaw doublets and, as before, the failure occurs at the beginning of the first maneuver at t = 6 seconds. Again, the nominal controller causes the aircraft trajectory to differ from the model, however the adaptive system follows the model despite failure. In addition, yaw and roll are successfully decoupled. The failed power plant creates a large yaw moment, which is counter acted by the rudders (the actuator activity is not shown for brevity). It should be noted that adaptive control with actuators with slow dynamics, such as power plants, should be undertaken cautiously. In these simulations, small adaptive gains, Γ_i , are used to discourage fast engine actuation (see appendix for Γ_i values), such that the commanded task does not exceed the engine bandwidth.

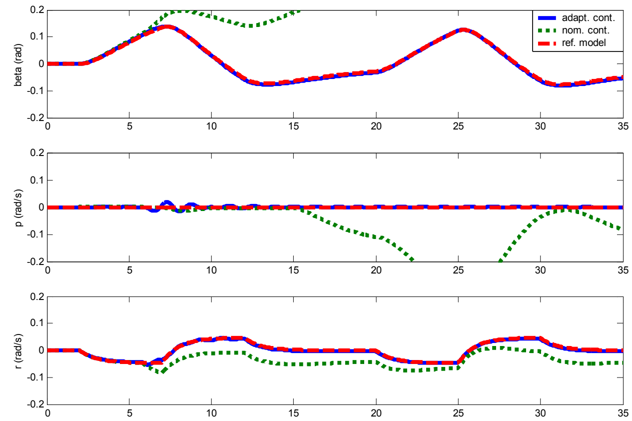

Figure 5: System response to failure 3, a failed left outboard engine giving zero thrust. The aircraft is commanded through two aggressive yaw doublets and the failure occurs at t = 6s. Only the lateral states are shown.

D. Failure 4

Figure 6 shows the stabilizing effect of the TSH in the presence of actuator saturation. The pitch rate and elevator deflections are shown for a failed left elevator locked at full upward deflection, which is represented as

$$\Lambda(5,5) = 0 \text{ and } f(5) = 1 \times e1_{\min} .$$

For this case, the aircraft is commanded through two aggressive pitch doublets and the failure occurs at t = 6 seconds, as before. In comparison with failure 1, this failure provokes the actuators to saturate because the disturbance due to the locked actuator $f(5)$ is greater. The plots show the undesirable effects of saturation on the adaptive system without TSH. With TSH, the plant follows the model more closely, with less control activity. In this case the open loop plant was, in fact, stable, so stability is preserved despite the large saturation disturbance. An open-loop unstable plant will be more easily provoked to instability by saturation.

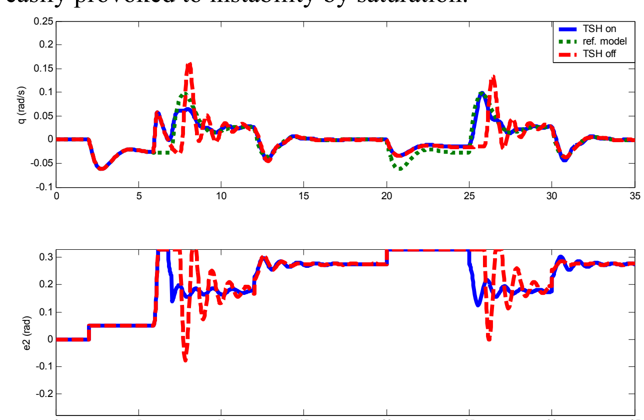

Figure 4: The stabilizing effect of TSH is illustrated for failure 4. The aircraft is commanded through two aggressive pitch doublets and the left elevator locks at 100% of its upward travel at t = 6s. Only the pitch rate and elevator activity are shown.

V. Summary

The problem of reconfiguration in the presence of actuator failures and saturation was considered in this paper. A general linear model was formulated to adequately describe actuator failures of various types. An adaptive controller was proposed that incorporated two main improvements, which include an adaptive control allocation scheme and the use of a multi-input variation of TSH to preserve stability in the presence of saturation. It was conjectured that the controller provides bounded trajectories for a limited set of initial conditions and a possible proof was outlined. The controller was compared to a fixed nominal controller in a simulation of a large transport aircraft with three different failures. The simulations demonstrated that the proposed controller can achieve stable reconfiguration following actuator failures in the presence of saturation. In addition to aircraft, the method proposed here is applicable to any dynamic system with multiple actuators capable of reconfiguration.

APPENDIX

Adaptive Gains:

$\Gamma_i = 50diag([.1 \quad .1 \quad .1 \quad .1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1 \quad 1])$, for $i = 1,2,3,4$,

$Q = diag([1 \quad .5 \quad 1 \quad 1 \quad 1])$

Linearized Transport Aircraft Model:

$$A_p = \begin{bmatrix} -0.6582 & 0.9705 & 0 & 0 & 0 \\ -3.3105 & -1.4741 & 0 & 0 & 0 \\ 0 & 0 & -0.1706 & -0.0075 & -1 \\ 0 & 0 & -2.4792 & -1.3585 & 0.5897 \\ 0 & 0 & 0.8050 & 0.0559 & -0.5584 \end{bmatrix}$$

$$B_p =$$

$$\begin{bmatrix} 0.0001 & 0.0001 & 0.0001 & 0.0001 & -0.0367 & -0.0367 & -0.0107 & -0.0107 & 0 & 0 \\ 0.0067 & -0.0011 & -0.0011 & 0.0067 & -1.8382 & -1.8382 & -0.0672 & -0.0672 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.0128 & 0.0128 \\ 0.0067 & 0.0038 & -0.0038 & -0.0067 & 0.1276 & -0.1276 & 0.5462 & -0.5462 & 0.0410 & 0.0830 \\ 0.1276 & 0.0726 & -0.0726 & -0.1276 & -0.120 & 0.0120 & -0.0620 & 0.0620 & -.2366 & -.02339 \end{bmatrix}$$

Reference Model:

$$A_m = \begin{bmatrix} -0.6582 & 0.9705 & 0 & 0 & 0 \\ -3.3105 & -1.4741 & 0 & 0 & 0 \\ 0 & 0 & -0.1706 & -0.0075 & -1 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix} \quad B_m = \begin{bmatrix} 0.0004 & -0.0734 & 0 & 0 \\ 0.0112 & -3.6764 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & -1.0924 & 0 \\ 0 & 0 & 0 & -0.4705 \end{bmatrix}$$

Nominal Controller:

$K_{nomx} = B_a(B_pB_a)^+(A_m - A_p)$, $K_{nomr} = B_a(B_pB_a)^+B_m$,

where '$( )^+$' denotes pseudoinverse and where

$$B_a^T = \begin{bmatrix} 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & -1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 \end{bmatrix}$$

Initial Conditions:

$x(t_0) = \mathbf{0}$, $x_m(t_0) = \mathbf{0}$, $r(t_0) = \mathbf{0}$, $u_c(t_0) = B_a r(t_0)$, $K_x(t_0) = K_{nomx}$,

$K_r(t_0) = K_{nomr}$, $\hat{f}(t_0) = \mathbf{0}$, $\hat{\lambda}(t_0) = ones(10,1)$, $e_\Delta(t_0) = \mathbf{0}$,

REFERENCES

[1] Eslinger, R. A., and P. R. Chandler, "Self Repairing Flight Control System Program Overview," *Proc. of IEEE National Aerospace and Electronics Conf.*, Vol. 2, pp. 504-511, 1988.

[2] Gao, Z., and P. J. Antsaklis, "On the Stability of the Pseudo-Inverse Method for Reconfigurable Control Systems," *International Journal of Control*, Vol. 53, No. 3, pp. 717-729, 1991.

[3] Noura, H., D. Sauter, F. Hamelin, D. Theilliol, "Fault-Tolerant Control in Dynamic Systems: Application to a Winding Machine," *IEEE Control Systems Magazine*, Vol. 20, pp. 33-49, 2000.

[4] Jiang, B., J. L. Wang, Y. C. Soh, "An Adaptive Technique for Robust Diagnosis of Faults with Independent Effects on System Outputs," *International Journal of Control*, Vol. 75, Iss. 11, pp. 792-802, 20 July 2002.

[5] Virnig, J., and D. Bodden, "Multivariable Control Allocation and Control Law Conditioning when Control Effectors Limit," *Proc. of the AIAA Guidance Navigation and Control Conf.*, Denver, CO, Aug. 2000.

[6] Burken, J., P. Lu, and Z. Wu, "Reconfigurable Flight Control Designs with Application to the X-33 Vehicle," *Proc. of AIAA Guidance, Control, and Dynamics Conf.*, Portland, OR, pp. 951-965, 1999.

[7] Enns, D., "Control Allocation Approaches," *Proc. of AIAA Guidance, Control, and Dynamics Conf.*, Boston, MA, pp. 98-108, 1998.

[8] Durham, W. C., "Constrained Control Allocation," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 16, No. 4, pp. 717-725, 1993.

[9] Page, A. B., M. L. Steinberg, "A Closed Loop Comparison of Control Allocation Methods," *Proc. of AIAA Guidance, Navigation, and Control Conf.*, Aug. 2000, Denver CO.

[10] Boskovic, D. J., and R. K. Mehra, "Multiple Model Adaptive Flight Control Scheme for Accommodation of Actuator Failures," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 25, No. 4, pp. 712-724, 2002.

[11] Wohletz, J. M., J. D. Paduano, A. M. Annaswamy, "Retrofit Systems for Reconfiguration in Civil Aviation," *Proc. of AIAA Guidance, Navigation, and Control Conf.*, pp. 995-1005, Portland, OR, 1999.

[12] Bodson, M. and J. E. Groszkiewicz, "Multivariable Adaptive Algorithms for Reconfigurable Flight Control," *IEEE Transactions on Control Systems Technology*, Vol. 5, No. 2, pp. 217 – 229, March 1997.

[13] Kim, K. S., K. J. Lee, and Y. Kim, "Model Following Reconfigurable Flight Control System Design Using Direct Adaptive Scheme," *Proc. of AIAA Guidance, Navigation, and Control Conf.*, Monterey CA, Aug. 5-8, 2002.

[14] Calise, A. J., S. Lee, and M. Sharma, "Development of a Reconfigurable Flight Control Law for Tailless Aircraft," *AIAA Journal of Guidance, Control, and Dynamics*, Vol. 24, No. 5, Sept.-Oct. 2001.

[15] Tao, G., S. M. Joshi, and X. Ma, "Adaptive State Feedback and Tracking Control of Systems with Actuator Failures," *IEEE Transaction on Automatic Control*, Vol. 46, No. 1, pp. 78-95, January 2001.

[16] Karason, S. P., and A. M. Annaswamy, "Adaptive Control in the Presence of Input Constraints," *IEEE Transactions on Automatic Control*, Vol. 39, No. 11, pp. 2325-2330, November 1994.

[17] Narendra, K. S., and A. M. Annaswamy, *Stable Adaptive Systems*, Prentice-Hall, 1989.

[18] Slotine, J. J. E., and W. Li, *Applied Nonlinear Control*, Prentice Hall, Englewood Cliffs, NJ, 1991.

[19] Lavretsky, E., and N. Hovakimyan, "Positive μ-modification for stable adaptation in a class of nonlinear systems with actuator constraints," *Proc. American Control Conf.*, Boston, MA, 2004.

[20] Stevens, B. L., and F. L. Lewis, *Aircraft Simulation and Control*, John Wiley & Sons, Inc, 1992.