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Output-Feedback Control of Uncertain Nonlinear Systems Using Adaptive Fuzzy Observer

Yongfu WANG, Tianyou CHAI

Research Center of Automation, Northeastern University, Shenyang, 110004, China
Email: wyf3000@sohu.com

Abstract-- **In this paper, a robust adaptive controller based on fuzzy observer for a class of unknown nonlinear systems with bounded external disturbances is presented. Firstly, a fuzzy observer based on fuzzy-basis-functions is designed to estimate the system state variables, and the fuzzy observer can guarantee the uniform ultimate bounds of the observer errors by SPR theory. Secondly, the adaptive controller with a fuzzy observer which requires only the system output variable is showed to control a class of unknown nonlinear systems with external disturbances, and the adaptive controller can guarantee the uniform ultimate bounds of the tracking errors. Finally, this simulation results presented adaptive controller applied to a single link robot control system demonstrate and confirm the effectiveness of the proposed method.**

I. INTRODUCTION

Adaptive control is a technique of applying some system identification techniques to obtain a model of the process and its environment from input-output experiments and using to design a controller. In the past decade, there have been significant research efforts on adaptive control schemes for linear and nonlinear systems[1, 2, 3]. However, conventional adaptive control theory can only deal with the systems with known dynamic structure, but unknown (constant or slowly-varying) parameters. Furthermore, conventional adaptive controllers cannot make use of human operators' experience, which is usually expressed in linguistic terms.

The number of successful industrial fuzzy logic control applications has tremendously increased over the last decades**.** Theoretically founded by Zadeh [4] and explored by Mamdani [5] in the early 1970s the wide industrial application of fuzzy logic control began almost a decade later in Japan[6]. Many people have devoted a great deal of time and effort to both theoretical research and implementation techniques for fuzzy logic controllers. Adaptive fuzzy control have emerged in recent years as promising ways to approach nonlinear control problems. Fuzzy adaptive systems provide the advantage that both numerical and qualitative information are used in the construction stages. Furthermore, fuzzy adaptive systems are proven to be applied to approximate any continuous nonlinear function on a compact space (i.e., they are universal approximators). Recently, various fuzzy, direct and indirect, adaptive systems were proposed, and their stability is achieved using Lyapunov theory (see [7-11] for SISO systems, see [12] MIMO systems). In these adaptive fuzzy schemes, the controllers are generally composed of two main components. One is fuzzy logic system for the rough tuning. The other is one kind of robust compensator, such as supervisory control, H^{∞} control, sliding-mode control, or the combination of the latter two, for the fine-tuning. In the above algorithm, the system states are assumed to be available for measurement. In many practical systems (for example robot manipulators), it's the joint position measurement can be obtained by means of encoder, which gives very accurate measurement. The joint velocity is usually measured by velocity tachometer, which is expensive and often contaminated by noise. One possible solution is to implement a velocity observer. So fuzzy observer and fuzzy controller are very importance to the development of adaptive fuzzy control theory and the potential application.

This paper is organized as follows. First, the problem formulation is presented in Section II. Section III is devoted to the observer design. In Section IV, an observer-based tracking controller is developed. In Section V, a simulation with a single-link robot is presented. Section VI gives the conclusions of the advocated method.

II. PROBLEM FORMULATION

Consider the nth nonlinear dynamical system of the form

$$\begin{cases} x^{(n)} = f\left(x, \dot{x}, \ldots, x^{(n-1)}\right) + g\left(x, \dot{x}, \ldots, x^{(n-1)}\right)u + d \\ y = x \end{cases} \tag{1}$$

or equivalently of the form

$$\begin{cases} \dot{\underline{x}} = A\underline{x} + B\left[f(\underline{x}) + g(\underline{x})u + d\right] \\ y = C\underline{x} \end{cases} \tag{2}$$

where

$$A = \begin{bmatrix} 0 & 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}, \quad B = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix},$$

$$C = \begin{bmatrix} 1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}.$$

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$\underline{x} = (x_1, x_2, ..., x_n)^T = (x, \dot{x}, ..., x^{(n-1)})^T \in R^n$ is a vector of states, $u \in R$ is control input, $y \in R$ is system output, d is external bounded disturbance, $f(\underline{x})$ and $g(\underline{x})$ is unknown functions, Only the system output y is assumed to be measurable.

Control objectives: Design robust adaptive fuzzy controller based on fuzzy observer and adaptive laws for adjusting the parameter vectors such that the following conditions are met.

i) Develop a fuzzy observer so that the developed observer is guaranteed the performance of adaptive fuzzy observer and the state estimation errors $\underline{\tilde{x}}(t)$ is UUB.

ii) all the signals involved are uniformly bounded and the tracking errors $\underline{e}(t)$ is UUB.

III. ADAPTIVE FUZZY OBSERVER DESIGN

Consider the following fuzzy observer that estimates the state vector x in (2)

$$\begin{cases} \dot{\hat{\underline{x}}} = A\hat{\underline{x}} + B[\hat{f}(\hat{\underline{x}}) + \hat{g}(\hat{\underline{x}})u - v] + K(y - C\hat{\underline{x}}) \\ \hat{y} = C\hat{\underline{x}} \end{cases} \tag{3}$$

where $K = [k_1 \quad k_2 \quad \cdots \quad k_n]^T$ is the observer gain vector, chosen such that the characteristic polynomial of $A - KC$ is strictly Hurwitz because (C, A) is observable. The robust control term v is employed to compensate the external disturbance d, the observation errors and the approximation error.

Defining the system state and output observation errors as $\underline{\tilde{x}} = \underline{x} - \hat{\underline{x}}$ and $\tilde{y} = y - \hat{y}$, subtracting (3) from (2), we have

$$\begin{cases} \dot{\underline{\tilde{x}}} = (A - KC)\underline{\tilde{x}} + B[\tilde{f}(\underline{x}, \hat{\underline{x}}) + \tilde{g}(\underline{x}, \hat{\underline{x}})u + v + d] \\ \tilde{y} = C\underline{\tilde{x}} \end{cases} \tag{4}$$

where the functional estimate errors $\tilde{f}(\underline{x}, \hat{\underline{x}})$ and $\tilde{g}(\underline{x}, \hat{\underline{x}})$ are given by

$$\begin{cases} \tilde{f}(\underline{x}, \hat{\underline{x}}) = f(\underline{x}) - \hat{f}(\hat{\underline{x}}) \\ \tilde{g}(\underline{x}, \hat{\underline{x}}) = g(\underline{x}) - \hat{g}(\hat{\underline{x}}) \end{cases} \tag{5}$$

Because the fuzzy logic systems Based on FBF are universal approximators, then an ideal estimate of $f(\underline{x})$ and $g(\underline{x})$ in (2) can be given by

$$\begin{cases} f(\underline{x}) = \underline{\theta}_f^{*\,T} \underline{\xi}_f(\underline{x}) + \varepsilon_f(\underline{x}) \\ g(\underline{x}) = \underline{\theta}_g^{*\,T} \underline{\xi}_g(\underline{x}) + \varepsilon_g(\underline{x}) \end{cases} \tag{6}$$

where the optimal approximation error $\varepsilon_f(\underline{x})$ and $\varepsilon_g(\underline{x})$ are bounded by known constants $\varepsilon_{f,N}$ and $\varepsilon_{g.N}$. The $\underline{\theta}_f^*$ and $\underline{\theta}_g^*$ are the following optimal parameter vectors

$$\begin{cases} \underline{\theta}_f^* = \arg\min_{\underline{\theta}_f \in \Omega_f} [\sup_{\underline{x} \in \Omega_x} \| \hat{f}(\underline{x} \mid \underline{\theta}_f) - f(\underline{x}) \|] \\ \underline{\theta}_g^* = \arg\min_{\underline{\theta}_g \in \Omega_g} [\sup_{\underline{x} \in \Omega_x} \| \hat{g}(\underline{x} \mid \underline{\theta}_g) - g(\underline{x}) \|] \end{cases} \tag{7}$$

By using product inference, center_average and singleton fuzzifier, there exists a fuzzy logic systems based on states estimation in the form of (8) such that it can uniformly approximate the $f(\underline{x})$ and $g(\underline{x})$

$$\begin{cases} \hat{f}(\hat{\underline{x}} \mid \hat{\underline{\theta}}_f) = \hat{\underline{\theta}}_f^T \underline{\xi}_f(\hat{\underline{x}}) \\ \hat{g}(\hat{\underline{x}} \mid \hat{\underline{\theta}}_g) = \hat{\underline{\theta}}_g^T \underline{\xi}_g(\hat{\underline{x}}) \end{cases} \tag{8}$$

where $\hat{\underline{\theta}} = (\hat{\theta}_1, \hat{\theta}_2, ..., \hat{\theta}_k)^T$ is the adjustable parameter vector, and $\hat{\underline{\xi}} = (\hat{\xi}_1, \hat{\xi}_2, ..., \hat{\xi}_k)^T$ is the vector of fuzzy basis functions.

The expression for the functional estimate errors $\tilde{f}(\underline{x}, \hat{\underline{x}})$ defined in (5) is given by

$$\tilde{f}(\underline{x}, \hat{\underline{x}}) = \underline{\theta}_f^{*\,T} \underline{\xi}_f(\underline{x}) + \varepsilon_f(\underline{x}) - \hat{\underline{\theta}}_f^T \underline{\xi}_f(\hat{\underline{x}}) \tag{9}$$

The fuzzy basis function errors is defined as

$$\underline{\tilde{\xi}}_f(\underline{x}, \hat{\underline{x}}) = \underline{\xi}_f(\underline{x}) - \underline{\xi}_f(\hat{\underline{x}}) \tag{10}$$

Adding and substracting $\underline{\theta}_f^{*\,T} \underline{\xi}_f(\hat{\underline{x}})$ from (9) yield

$$\tilde{f}(\underline{x}, \hat{\underline{x}}) = \underline{\tilde{\theta}}_f^T \underline{\xi}_f(\hat{\underline{x}}) + \varepsilon_f(\underline{x}) + \omega_f \tag{11}$$

with the adaptive parameter estimation error $\underline{\tilde{\theta}}_f = \underline{\theta}_f^* - \hat{\underline{\theta}}_f$ and $\dot{\underline{\tilde{\theta}}}_f = -\dot{\hat{\underline{\theta}}}_f$, the disturbance term $\omega_f(t)$ is given by

$$\omega_f(t) = \underline{\theta}_f^{*\,T} \underline{\tilde{\xi}}_f(\underline{x}, \hat{\underline{x}}) \tag{12}$$

Following the same arguments for $\tilde{f}(\underline{x}, \hat{\underline{x}})$, we have an expression for $\tilde{g}(\underline{x}, \hat{\underline{x}})$

$$\tilde{g}(\underline{x}, \hat{\underline{x}}) = \underline{\tilde{\theta}}_g^T \underline{\xi}_g(\hat{\underline{x}}) + \varepsilon_g(\underline{x}) + \omega_g \tag{13}$$

where the disturbance term $\omega_g(t)$ is given by

$$\omega_g(t) = \underline{\theta}_g^{*\,T} \underline{\tilde{\xi}}_g(\underline{x}, \hat{\underline{x}}) \tag{14}$$

The observation errors is defined as

$$\omega_o = \omega_f(t) + \omega_g(t)u = \underline{\theta}_f^{*\,T} \underline{\tilde{\xi}}_f(\underline{x}, \hat{\underline{x}}) + \underline{\theta}_g^{*\,T} \underline{\tilde{\xi}}_g(\underline{x}, \hat{\underline{x}})u \tag{15}$$

The approximation errors is defined as

$$\varepsilon_a = \varepsilon_f(t) + \varepsilon_g(t)u \tag{16}$$

Then the proposed observer system (3) become

$$\begin{cases} \dot{\hat{\underline{x}}} = A\hat{\underline{x}} + B[\hat{\underline{\theta}}_f^T \underline{\xi}_f(\hat{\underline{x}}) + \hat{\underline{\theta}}_g^T \underline{\xi}_g(\hat{\underline{x}})u - v] + K(y - C^T \hat{\underline{x}}) \\ \hat{y} = C\hat{\underline{x}} \end{cases} \tag{17}$$

and the observation error dynamics (4) become

$$\begin{cases}\dot{\underline{\tilde{x}}}=(A-KC)\underline{\tilde{x}}+B[\underline{\tilde{\theta}}_f^T\underline{\xi}_f(\underline{\hat{x}})+\underline{\tilde{\theta}}_g^T\underline{\xi}_g(\underline{\hat{x}})u+v+\omega]\\ \tilde{y}=C\underline{\tilde{x}}\end{cases} \quad (18)$$

where $\omega=\omega_o+\varepsilon_a+d$ is the overall disturbance term. In order to apply SPR theory, the output estimation error in (18) $\tilde{y}$ may be written as

$$\tilde{y}=H(s)[\underline{\tilde{\theta}}_f^T\underline{\xi}_f(\underline{\hat{x}})+\underline{\tilde{\theta}}_g^T\underline{\xi}_g(\underline{\hat{x}})u+v+\omega] \quad (19)$$

where $H(s)$ is a known proper transfer function with stable poles, and is realized by $(A-KC,B,C)$.

The output estimation error (19) can be written in the form of

$$\tilde{y}=H(s)L(s)[\underline{\tilde{\theta}}_f^T\underline{\xi}_f^l(\underline{\hat{x}})+\underline{\tilde{\theta}}_g^T\underline{\xi}_g^l(\underline{\hat{x}})u+v^l+\omega^l] \quad (20)$$

where $\underline{\xi}_f^l(\underline{\hat{x}})=L^{-1}(s)\underline{\xi}_f(\underline{\hat{x}}),\underline{\xi}_g^l(\underline{\hat{x}})=L^{-1}(s)\underline{\xi}_g(\underline{\hat{x}})$, $v^l=L^{-1}(s)v$,

$\omega^l=L^{-1}(s)(\omega+\delta),\delta=\underline{\tilde{\theta}}_f^T\underline{\xi}_f+\underline{\tilde{\theta}}_g^T\underline{\xi}_g u-L(s)\left(\underline{\tilde{\theta}}_f^T\underline{\xi}_f^l+\underline{\tilde{\theta}}_g^T\underline{\xi}_g^l u\right)$.

and $L(s)$ is chosen so that $L^{-1}(s)$ is a proper stable transfer and $H(S)L(s)$ is SPR transfer function.

From Kalman-Yacu-Popov theorem, because $H(s)L(s)$ is SPR, there exists a positive-define symmetric matrix P such that

$$\begin{cases}(A-KC)^T P+P(A-KC)=-Q\\ PB_c=C^T\end{cases} \quad (21)$$

where Q a positive-define symmetric matrix, and $B_c=[b_1 \quad b_2 \quad \cdots \quad b_n]^T$.

Then the state-space realization of (20) is given by

$$\begin{cases}\dot{\underline{\tilde{z}}}=A_c\underline{\tilde{z}}+B_c[\underline{\tilde{\theta}}_f^T\underline{\xi}_f^l(\underline{\hat{x}})+\underline{\tilde{\theta}}_g^T\underline{\xi}_g^l(\underline{\hat{x}})u+v^l+\omega^l]\\ \tilde{y}=C_c\underline{\tilde{z}}\end{cases} \quad (22)$$

where $A_c=A-KC$ and $C_c=C$.

Assumption: ω and δ are assumed to satisfy $\|\omega\|\le k$ and $\|\delta\|\le\beta$,where k and β are positive constants.

Remark1: The assumption of $\|\omega\|\le k$ is reasonable because of the universal approximation theorem and the external bounded disturbance. According to (23) and (24), the proofs of $\|\underline{\theta}_f\|\le M_f$ and $\|\underline{\theta}_g\|\le M_g$ can be found in [7, 14]. By using triangle inequality, we have $\|\underline{\tilde{\theta}}_f\|\le 2M_f$ and $\|\underline{\tilde{\theta}}_g\|\le 2M_g$. The assumption of $\|\delta\|\le\beta$ is also reasonable because of the fact $\|\delta\|\le\beta_1\|\underline{\tilde{\theta}}_f\|+\beta_2\|\underline{\tilde{\theta}}_g\|$, and $\|\underline{\tilde{\theta}}_f\|\le 2M_f$ and $\|\underline{\tilde{\theta}}_g\|\le 2M_g$.

Remark2: A direct fuzzy observer (i.e., directly estimate system state variables) based on fuzzy basis functions is designed, whereas Leu designs an indirect fuzzy observer (i.e., directly estimate system tracking error variables) in order to obtain adaptive fuzzy controller based on system output in [14].

Remark3: A knotty problem that the KYP equation has a positive define symmetric solution by SPR theory is improved in [14].

Theorem 1: If system (1) satisfies assumption, we consider the observer system (17). Let $\underline{\hat{\theta}}_f$ and $\underline{\hat{\theta}}_g$ be adjusted by the adaptive laws (23) and (24), and let v be given by (25), then the state estimation errors $\underline{\tilde{x}}(t)$, adaptive parameter errors $\underline{\tilde{\theta}}_f$ and $\underline{\tilde{\theta}}_g$ are UUB.

$$\dot{\underline{\hat{\theta}}}_f=\begin{cases}\gamma_1\underline{\xi}_f^l(\underline{\hat{x}})\tilde{y} \ ,if \ \left(\|\underline{\hat{\theta}}_f\|<M_f\right)\\ \qquad or \ \left(\|\underline{\hat{\theta}}_f\|=M_f \ and \ \gamma_1\underline{\xi}_f^l(\underline{\hat{x}})\tilde{y}\ge 0\right)\\ P_f[\cdot] \ ,if \ \left(\|\underline{\hat{\theta}}_f\|=M_f \ and \ \gamma_1\underline{\xi}_f^l(\underline{\hat{x}})\tilde{y}<0\right)\end{cases} \quad (23)$$

$$\dot{\underline{\hat{\theta}}}_g=\begin{cases}\gamma_2\underline{\xi}_g^l(\underline{\hat{x}})\tilde{y}u \ ,if \ \left(\|\underline{\hat{\theta}}_g\|<M_g\right)\\ \qquad or \ \left(\|\underline{\hat{\theta}}_g\|=M_g \ and \ \gamma_2\underline{\xi}_g^l(\underline{\hat{x}})\tilde{y}u\ge 0\right)\\ P_g[\cdot] \ ,if \ \left(\|\underline{\hat{\theta}}_g\|=M_g \ and \ \gamma_2\underline{\xi}_g^l(\underline{\hat{x}})\tilde{y}u<0\right)\end{cases} \quad (24)$$

$$v=-\rho sign(\tilde{y}),\rho\ge k+\beta,sign(\tilde{y})=\begin{cases}1, & \tilde{y}\ge 0\\ -1, & \tilde{y}<0\end{cases} \quad (25)$$

where $P_f[\cdot]$ and $P_g[\cdot]$ are the projection operators in wang's [7].

Proof: Consider the Lyapunov function candidate

$$V=\frac{1}{2}\underline{\tilde{z}}^T P\underline{\tilde{z}}+\frac{1}{2\gamma_1}tr\left(\underline{\tilde{\theta}}_f^T\underline{\tilde{\theta}}_f\right)+\frac{1}{2\gamma_2}tr\left(\underline{\tilde{\theta}}_g^T\underline{\tilde{\theta}}_g\right) \quad (26)$$

with $P=P^T>0$. Differentiating (26) with respect to time, we get

$$\dot{V}=\frac{1}{2}\dot{\underline{\tilde{z}}}^T P\underline{\tilde{z}}+\frac{1}{2}\underline{\tilde{z}}^T P\dot{\underline{\tilde{z}}}+\frac{1}{\gamma_1}tr\left(\underline{\tilde{\theta}}_f^T\dot{\underline{\tilde{\theta}}}_f\right)+\frac{1}{\gamma_2}tr\left(\underline{\tilde{\theta}}_g^T\dot{\underline{\tilde{\theta}}}_g\right) \quad (27)$$

Inserting (23), (24) and (25) in the above equation yields

$$\dot{V}=\frac{1}{2}\underline{\tilde{z}}^T\left(A_c^T P+PA_c\right)\underline{\tilde{z}}+\underline{\tilde{z}}^T PB_c\underline{\tilde{\theta}}_f^T\underline{\xi}_f^l+\underline{\tilde{z}}^T PB_c\underline{\tilde{\theta}}_g^T\underline{\xi}_g^l u$$

$$+\underline{\tilde{z}}^T PB_c v^l+\underline{\tilde{z}}^T PB_c\omega^l+\frac{1}{\gamma_1}tr\left(\underline{\tilde{\theta}}_f^T\dot{\underline{\tilde{\theta}}}_f\right)+\frac{1}{\gamma_2}tr\left(\underline{\tilde{\theta}}_g^T\dot{\underline{\tilde{\theta}}}_g\right) \quad (28)$$

because $PB_c\underline{\tilde{z}}=C\underline{\tilde{z}}=\tilde{y}$, $\dot{\underline{\tilde{\theta}}}_f=-\dot{\underline{\hat{\theta}}}_f$ and $\dot{\underline{\tilde{\theta}}}_g=-\dot{\underline{\hat{\theta}}}_g$, we get

$$\dot{V}=-\frac{1}{2}\underline{\tilde{z}}^T Q\underline{\tilde{z}}+\tilde{y}[\underline{\tilde{\theta}}_f^T\underline{\xi}_f^l+\underline{\tilde{\theta}}_g^T\underline{\xi}_g^l u+v^l+\omega^l] \quad (29)$$

$$+\frac{1}{\gamma_1}tr\left(\underline{\tilde{\theta}}_f^T\dot{\underline{\tilde{\theta}}}_f\right)+\frac{1}{\gamma_2}tr\left(\underline{\tilde{\theta}}_g^T\dot{\underline{\tilde{\theta}}}_g\right)$$

$$\dot{V}\le-\frac{1}{2}\lambda_{\min}(Q)\|\underline{\tilde{z}}\|^2+\tilde{y}[\underline{\tilde{\theta}}_f^T\underline{\xi}_f^l+\underline{\tilde{\theta}}_g^T\underline{\xi}_g^l u]$$

$$+\frac{1}{\gamma_1}tr\left(\underline{\tilde{\theta}}_f^T\dot{\underline{\tilde{\theta}}}_f\right)+\frac{1}{\gamma_2}tr\left(\underline{\tilde{\theta}}_g^T\dot{\underline{\tilde{\theta}}}_g\right) \tag{30}$$

$$\dot{V}\le-\frac{1}{2}\lambda_{\min}(Q)\|\underline{\tilde{z}}\|^2 \tag{31}$$

According to standard Lyapunov theorem, this demonstrates the UUB of $\|\underline{\tilde{z}}(t)\|$, $\left\|\underline{\tilde{\theta}}_f\right\|$ and $\left\|\underline{\tilde{\theta}}_g\right\|$. In order to show the boundedness of the state estimation error $\underline{\tilde{x}}(t)$, consider the estimation error dynamics (18). The trajectory can be expressed as

$$\underline{\tilde{x}}(t)=\phi(t,0)\underline{\tilde{x}}(0)+\int_0^t\phi(t,\tau)BU_1(\tau)d\tau \tag{32}$$

where $U_1=\underline{\tilde{\theta}}_f^T\underline{\xi}(\hat{\underline{x}})+\underline{\tilde{\theta}}_g^T\underline{\xi}(\hat{\underline{x}})u+v+\omega$, using the norm inequality and lemma 2.2 in [13] yields

$$\|\underline{\tilde{x}}(t)\|\le C_1+\left(C_2+C_3\left\|\underline{\tilde{\theta}}_f\right\|_2^\alpha+C_4\left\|\underline{\tilde{\theta}}_g\right\|_2^\alpha\right)\frac{1}{\sqrt{\alpha}} \tag{33}$$

where C_1 is a term decaying exponentially to zero owing to the initial condition, C_2, C_3 and C_4 are positive and computable constants. From (32), the trajectory $\|\underline{\tilde{x}}(t)\|$ is UUB by the adaptive parameter estimation errors $\left\|\underline{\tilde{\theta}}_f\right\|$ and $\left\|\underline{\tilde{\theta}}_g\right\|$, which are shown to be bounded.

IV. FUZZY CONTROLLER BASED ON FUZZY OBSERVER

If the function $f(\underline{x})$ and $g(\underline{x})$ in (1) are known, the state variables are measureable and there is no external disturbance, then the control law

$$u=\frac{1}{g(\underline{x})}\left[-f(\underline{x})+y_r^{(n)}+L\underline{e}\right] \tag{34}$$

can be applied to the nonlinear system (1) to achieve the following asymptotically error dynamic system

$$e^{(n)}+l_1e^{(n-1)}+...+l_ne=0 \tag{35}$$

where $L=[l_1,l_2\cdots,l_n]$ are constant design to specify the desired transient performance of the closed system. Therefore, the system tracking can be achieved with its transient performance specified by Eq. (35). This implies that starting from any initial conditions, we have $\lim_{t\to\infty}|e(t)|=0$,i.e., tracking of the reference trajectory is asymptotically achieved.

However, since the function $f(\underline{x})$ and $g(\underline{x})$ are unknown, the state variables are unavailable and there is external disturbance. Obtaining a control algorithm similar to (33) is impossible. In this situation, the approximation by fuzzy logic systems based on state estimation in section III are employed to treat this tracking control design problem. Therefore, in the following, we replace them by the fuzzy logic systems based on state estimation $\hat{f}(\hat{\underline{x}})$ and $\hat{g}(\hat{\underline{x}})$ to construct a self-tuning controller

$$u=\frac{1}{\hat{g}\left(\hat{\underline{x}}\mid\hat{\underline{\theta}}_g\right)}\left[-\hat{f}\left(\hat{\underline{x}}\mid\hat{\underline{\theta}}_f\right)+y_r^{(n)}+L\hat{\underline{e}}-u_\alpha\right] \tag{36}$$

Applying (36) to (1) and after some simple manipulations, we obtain

$$\begin{aligned}\dot{\underline{e}}=&A\underline{e}+B\left([\hat{f}\left(\hat{\underline{x}}\mid\hat{\underline{\theta}}_f\right)-f(\underline{x})]+[\hat{g}\left(\hat{\underline{x}}\mid\hat{\underline{\theta}}_g\right)-g(\underline{x})]u\right)+\\&Bu_\alpha-Bd-BL\hat{\underline{e}}\end{aligned} \tag{37}$$

From (9), (13), (15) and (16) can be rewritten as

$$\dot{\underline{e}}=A\underline{e}-BL\hat{\underline{e}}-B\left(\underline{\xi}^T(\hat{\underline{x}})\underline{\tilde{\theta}}_f+\underline{\xi}^T(\hat{\underline{x}})\underline{\tilde{\theta}}_g u\right)+Bu_\alpha-B\omega \tag{38}$$

Theorem 2: If system (1) satisfies assumption, we consider the observer system (17). Let $\hat{\underline{\theta}}_f$ and $\hat{\underline{\theta}}_g$ be adjusted by the adaptive laws (23) and (24), and controller be given by (39) and (40), then tracking errors $\underline{e}$ is UUB.

$$u=\frac{1}{\underline{\xi}_g^T(\hat{\underline{x}})\hat{\underline{\theta}}_g}\left[-\underline{\xi}_f^T(\hat{\underline{x}})\hat{\underline{\theta}}_f+y_r^{(n)}+L\hat{\underline{e}}-u_\alpha\right] \tag{39}$$

$$u_\alpha=v \tag{40}$$

we know

$$\underline{\tilde{e}}=\underline{e}-\hat{\underline{e}}=(\underline{y}_r-\underline{x})-(\underline{y}_r-\hat{\underline{x}})=-\underline{x}+\hat{\underline{x}}=-\underline{\tilde{x}} \tag{41}$$

Applying (41) to (37) and after some simple manipulations, we can obtain the error equation

$$\dot{\underline{e}}=(A-BL)\underline{e}-B\left(\underline{\xi}_f^T(\hat{\underline{x}})\underline{\tilde{\theta}}_f+\underline{\xi}_g^T(\hat{\underline{x}})\underline{\tilde{\theta}}_g u-L\underline{\tilde{e}}\right)+B(u_\alpha-\omega) \tag{42}$$

Considering the tracking error dynamics (42). The system trajectory can be expressed as

$$\underline{e}(t)=\phi(t,0)\underline{e}(0)+\int_0^t\phi(t,\tau)BU_2(\tau)d\tau \tag{43}$$

where $U_2=\underline{\tilde{\theta}}_f^T\underline{\xi}_f(\hat{\underline{x}})+\underline{\tilde{\theta}}_g^T\underline{\xi}_g(\hat{\underline{x}})u-L\underline{\tilde{e}}+u_\alpha-\omega$, using the norm inequality and lemma 2.2 in [13] yields

$$\|\underline{e}(t)\|\le D_1+\left(D_2+D_3\left\|\underline{\tilde{\theta}}_f\right\|_2^\alpha+D_4\left\|\underline{\tilde{\theta}}_g\right\|_2^\alpha+D_5\|\underline{\tilde{e}}\|_2^\alpha\right)\frac{1}{\sqrt{\alpha}} \tag{44}$$

where D_1 is a term decaying exponentially to zero owing to the initial condition, D_2, D_3, D_4 and D_5 are positive and computable constants. From (44), trajectory $\|\underline{e}(t)\|$ is UUB by the adaptive parameter estimation errors $\left\|\underline{\tilde{\theta}}_f\right\|$, $\left\|\underline{\tilde{\theta}}_g\right\|$ and $\|\underline{\tilde{e}}\|$, which are shown to be bounded**.**

V. AN EXAMPLE OF SIMULATION

This section presents the simulation results of the proposed adaptive fuzzy control based on observer for single-link robot. The dynamic equation of the single-link robot system is given as follows [13]

$$\begin{cases} M\ddot{q} + \frac{1}{2}mgl\sin(q) = u \\ y = q \end{cases} \quad (45)$$

where $q, \dot{q}, \ddot{q} \in R$ denote the joint angular positions, velocity and acceleration respectively, l is link length, m is link mass, τ is the input torque.

Letting $x_1 = q_1$, $x_2 = \dot{q}_1$ then (45) can be written as the following state-space form including external disturbances:

$$\begin{cases} \begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}\left(\frac{u - 0.5mgl\sin x_1}{M} + d\right) \\ y = x_1 \end{cases} \quad (46)$$

where $m=1, l=1, M=0.5, g=9.8, d = square\ (t)$.

Step 1: The fuzzy model for the robotic systems is given by the following fuzzy rule and the membership functions are selected, i.e.,

$$\mu_{F_i^1}(x_i) = 1/(1+\exp(5(x_i+2))),$$

$$\mu_{F_i^2}(x_i) = \exp\left(-(x_i+1.5)^2\right),$$

$$\mu_{F_i^3}(x_i) = \exp\left(-(x_i+0.5)^2\right),$$

$$\mu_{F_i^4}(x_i) = \exp\left(-(x_i-0.5)^2\right),$$

$$\mu_{F_i^5}(x_i) = \exp\left(-(x_i-1.5)^2\right),$$

$$\mu_{F_i^6}(x_i) = 1/(1+\exp(-5(x_i-2))).$$

in which we have two state variables, i.e., x_1, x_2 fuzzy rules of the following form are included in the fuzzy bases.

$$R^{(j)}: if\ (x_1\ is\ F_1^j)\ and\ (x_2\ is\ F_2^j)$$

$$Then\ y\ is\ G^j \quad (j = 1, 2, \cdots, 6)$$

Denote $Sum = \sum_{j=1}^{6}\prod_{i=1}^{2}\mu_{F_i^j}(x_i)$,

Step 2: Fuzzy observer and fuzzy controller

Select the fuzzy observer and controller coefficients

$\gamma_1 = 5\times10^{-1}$, $\gamma_2 = 5\times10^{2}$, $L^{-1} = 1/(s+3)$, $K = [80 \quad 800]$,

$L = [2 \quad 1]$, $y_r = \sin(t), v = -26 sign(x_1 - \hat{x}_1)$,

$x_1 = 0.5, x_2 = 0.5, \hat{x}_1 = 0.1, \hat{x}_2 = 0$,

$\underline{\theta}_f = (-8, -6, -1, 3, 8, 1), \underline{\theta}_g = (5, 4, 2, 6, 5, 3)$.

Obtain adaptive laws, controller as

$$u = \frac{1}{\underline{\xi}_g^T(\hat{x})\hat{\underline{\theta}}_g}\left[-\underline{\xi}_f^T(\hat{x})\hat{\underline{\theta}}_f + \sin(t) + \cos(t) - 2\hat{x}_1 - \hat{x}_2 - u_\alpha\right]$$

$$\begin{cases} \dot{\underline{\theta}}_f(\hat{x}) = \gamma_1 \underline{\xi}_f^l(\hat{x})(x_1 - \hat{x}_1) \\ \dot{\underline{\theta}}_g(\hat{x}) = \gamma_2 \underline{\xi}_g^l(\hat{x})(x_1 - \hat{x}_1)u \end{cases}$$

Step 3: Simulation results and analysis

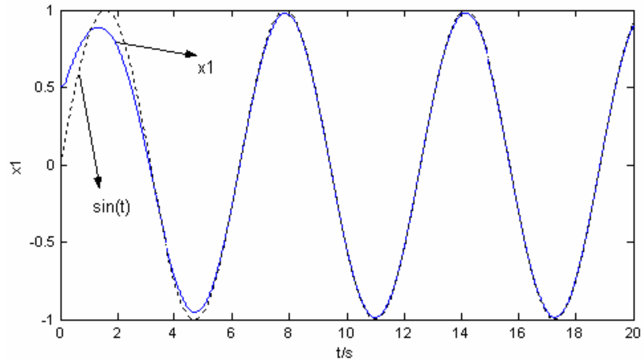

Fig.1 (a). Trajectories of the state of x_1 and y_r

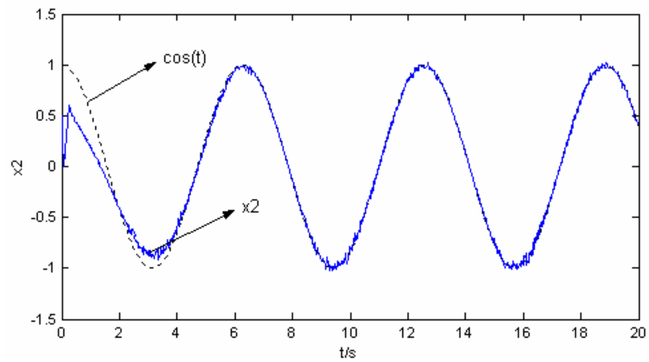

Fig.1 (b). Trajectories of the state of x_2 and $\dot{y}_r$

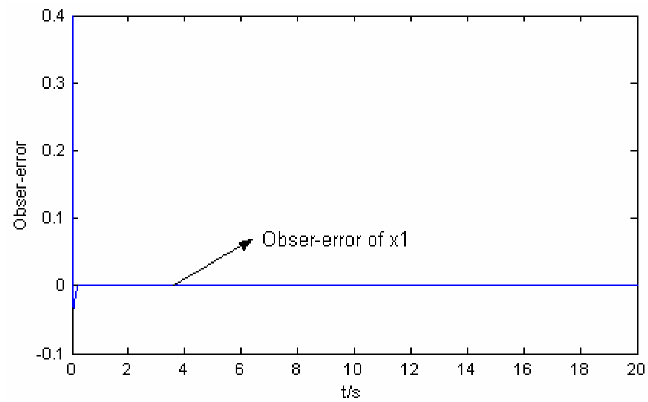

Fig.1 (c). Observation error of the state of x_1

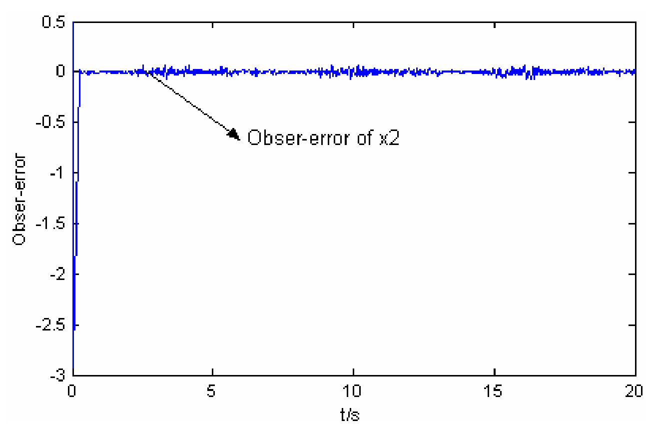

Fig.1 (d). Observation error of the state of x_2

The simulation results based on fuzzy observer are shown in Figs.1 (a-d). The tracking trajectories of the state of x_1 is shown in Figs.1 (a). The tracking trajectories of the state of x_2 is shown in Figs.1 (b). The tracking error trajectories of the state of x_1 is shown in Figs.1 (c). The tracking error trajectories of the state of x_2 is shown in Figs.1 (d).

From the simulation results, it is obvious that the trajectories for the case of adaptive controller based on fuzzy observer have achieved the desired results. The detail comparison with High-Gain observer [15, 16, 17] is omited in simulation because of pages limits.

Remark4: In single-link robot simulation, the tracking trajectories and the observation errors trajectories of all state are given. In [14], the only tracking trajectory of the state x_1 and observation error of the state x_1 can be seen, whereas the tracking trajectory of the state variable x_2 and observation error of the state variable x_2 can't be seen. It is well known the state variable x_2 should mainly be estimated because the state x_2 isn't usually measured.

Remark5: Using Ode45 to simulate, we regard the adaptive laws equation and the fuzzy observer as extension of the state-space.

VI. CONCLUSION

A robust adaptive controller based on fuzzy observer for a class of unknown nonlinear system with bounded external disturbances is presented. The parameters of adaptive controller based on fuzzy observer are tuned on-line in order to achieve the desired performance. Considering the estimation error, the approximation error and external disturbance, a robust control term and on-line adaptive laws are designed to guarantee the performance of fuzzy observer and adaptive controller. The adaptive controller with fuzzy observer is applied to a single-link robot system. The simulation results demonstrate and confirm the theoretical results.

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