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Integrated Design of Structural and Control Systems with a Homotopy Like Iterative Method

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***Abstract*— We consider an integrated design problem of structural and control systems. It is well known that even the simplest formulation of this problem results in a kind of a BMI problem. In this paper, a homotopy-like iterative design method based on LMIs is proposed to obtain an optimal plant and a controller *simultaneously* for a full-order output feedback problem. We can deal with a multiobjective problem, e.g., $\mathscr{H}_2/\mathscr{H}_\infty$ control problem etc. in the proposed design method. We can also optimize structural design parameters appearing nonlinearly in the coefficient matrices of the plant state-space form using the first order approximation of the nonlinear dependence in the proposed algorithm. Several design examples show that the proposed algorithm works quite effectively for various integrated design problems.**

I. Introduction

The control system design traditionally has been divided into two phases: the design of control object and the controller design, based on each design specification having not been necessarily cooperative each other. The integrated design framework aims to integrate the above two step design scheme. We can expect that the integrated deign methodology is able to achieve the higher performance in both of the structural and the control senses because the structure of the integrated design problem is clearly more reasonable than that of the above two step design.

The study on the integrated design methodology has been started since last two decades to achieve a tradeoff between the strict specification about the weight and the required property of damping, which is provided actively, for large space structures[1].

Recently several iterative design methods guaranteeing the convergence to a local optimal solution have been proposed[2], [3], [4], [5]. Those studies also has shown the negative result that the general integrated design problem becomes a kind of BMI problem. This fact means that we cannot obtain the global optimal solution with a reasonable amount of computation[6]. This difficulty is the biggest difference between the integrated design problem and the simple structural or the controller design problem. However, those integrated design methods can guarantee the convergence to the local optimal solution only when the coefficient matrices of the state-space or descriptor form of the control objects are linear functions on the structural design parameters. This limitation confines the range of the applications of the integrated design scheme to the general and the practical control system design.

In this paper we propose an integrated design technique which guarantees the local convergence even when the coefficient matrices appearing in the state-space form of the control objects are not linear functions on the structural design parameters. The proposed design method is an iterative LMI based synthesis procedure which is employed in so-called homotopy method [7], [8]. The LMI conditions are obtained by approximating the nonlinear matrix inequality describing the closed-loop norm constraints by neglecting the second or the higher order terms on the structural and the control design parameters. The result will greatly enlarge the class of the problem to which we can apply the integrated design method.

The rest of the paper is organized as follows: In Section II, the integrated design problem of structural and control systems is formulated. The homotopy like design algorithm is presented in Section III. Two design examples to show the effectiveness of the proposed design method are given in Section IV. In Section V, the conclusion of this paper is presented.

II. Integrated Design Problem

The control object is represented as the following general state-space form:

$$\begin{cases} \dot{x}(t) = A(p)x(t) + B_1(p)w(t) + B_2(p)u(t) \\ z(t) = C_1(p)x(t) + D_{11}(p)w(t) + D_{12}(p)(t) \\ y(t) = C_2(p)x(t) + D_{21}(p)w(t) \end{cases} \tag{1}$$

where $x(t) \in \mathbf{R}^{nx}$, $w(t) \in \mathbf{R}^{nw}$, $u(t) \in \mathbf{R}^{nu}$, $z(t) \in \mathbf{R}^{nz}$ and $y(t) \in \mathbf{R}^{ny}$ are the state vector, the disturbance, the control effort, controlled output and the measurement respectively. Matrices $A(p)$, $B_1(p)$, $B_2(p)$, $C_1(p)$, $C_2(p)$, $D_{11}(p)$, $D_{12}(p)$ and $D_{21}(p)$ have conformable dimensions and are (not necessarily linear) functions of np dimensional structural design parameter vector denoted as $p := [p_1, \ldots, p_{np}]^T$. In mechanical systems to be controlled, the damping, the stiffness and the sensor/actuator placement, etc. are supposed to be an element of the vector p. In this paper the vector p is assumed to be a real vector in a set $\mathscr{P}$ defined as the following:

$$\mathscr{P} := \{p \in \mathbf{R}^{np} : \ p_l \le p \le p_u, \ p_l, \ p_u \in \mathbf{R}^{np}\} \tag{2}$$

where p_l and p_u are the lower and the upper bounds of the vector p respectively.

For the control object possessing the adjustable design parameters given in (1) a following full order feedback

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controller is synthesized:

$$\begin{cases} \dot{x}_K(t) = A_K x_K(t) + B_K y(t) \\ u(t) = C_K x_K(t) \end{cases} \tag{3}$$

where $x_K(t) \in \mathbf{R}^{nx}$ is the state vector of the controller and all coefficient matrices in (3) have appropriate dimensions. Let the transfer function matrix of the controller as $K(s)$. Note that those coefficient matrices also can be considered as functions on the structural design parameter vector p.

The closed-loop system with the plant in (1) and the controller in (3) is given as

$$\begin{cases} \dot{x}_{cl}(t) = A_{cl} x_{cl}(t) + B_{cl} w(t) \\ z(t) = C_{cl} x_{cl}(t) + D_{cl} w(t) \end{cases}, \tag{4}$$

Define the transfer function matrix of the closed-loop system as $G_{cl}(s)$. For the closed-loop system given in (4) we define a scalar performance index J as

$$J := \|G_{cl}(s)\|_\bullet, \tag{5}$$

where $\|H(s)\|_\bullet$ denotes a norm of a transfer function matrix $H(s)$. We can assign the performance index J as various type of norms of $G_{cl}(s)$, e.g., $\mathscr{H}_2$, $\mathscr{H}_\infty$ and $\mathscr{L}_\infty$ norm etc. or the weighted sum of such closed-loop norms (multiobjective case). Now the integrated design problem of the structural and control systems is formulated as the following:

Integrated design problem of structural and control systems: *Find the structural design parameter $p_{opt} \in \mathscr{P}$ and the controller $K_{opt}(s)$ which minimize J or satisfy $J \le J_u$ ($J_u > 0$) where J_u is a scaler representing the performance specification determined by the designer.*

Remark 1: In the case of the multiobjective problem, it is often the case minimizing a norm of closed-loop transfer function from the components of $w(s) := \mathscr{L}(w(t))$ to those of $z(s) := \mathscr{L}(z(t))$ subject to inequality constraint on other norms of another closed-loop transfer matrix from (other) components of $w(s)$ to (other) ones of $z(s)$. The formulated problem is also able to handle this kind of problem.

III. Homotopy Like Design Algorithm

A. LMI conditions for controller design

The problem formulation in the previous section does not assume the specific norm of the closed-loop system. Actually we can deal with any kinds of norms if the optimal controller synthesis conditions are given as LMIs on controller design parameters, e.g., [9]. In this subsection we introduce LMI conditions for the optimal $\mathscr{H}_2$ and $\mathscr{H}_\infty$ controller as examples to describe the following homotopy like method concretely. Furthermore we show the principle for a design of a multiobjective controller, say, $\mathscr{H}_2/\mathscr{H}_\infty$ controller, in the sense of a sufficient condition by taking common Lyapunov matrices appearing in LMI conditions on single objective problem, i.e., each of $\mathscr{H}_2$ or $\mathscr{H}_\infty$.

$\mathscr{H}_2$ **norm:**[9] A controller $K_2(s)$ yielding the $\|G_{cl}(s)\|_2^2 < \gamma_2$ ($\gamma_2 > 0$) exists if and only if following LMIs have solution matrices $X_2 = X_2^T, Y_2 = Y_2^T \in \mathbf{R}^{nx\times nx}$, $Q = Q^T \in \mathbf{R}^{nz\times nz}$, $\hat{A}_2 \in \mathbf{R}^{nx\times nx}$, $\hat{B}_2 \in \mathbf{R}^{nx\times ny}$ and $\hat{C}_2 \in \mathbf{R}^{nu\times nx}$ exist:

$$\begin{bmatrix} \Theta_{11} & \Theta_{21}^T & \Theta_{31}^T \\ \Theta_{21} & \Theta_{22} & \Theta_{32}^T \\ \Theta_{31} & \Theta_{32} & \Theta_{33} \end{bmatrix} < 0, \tag{6}$$

$$\begin{bmatrix} \Lambda_{11} & \Lambda_{21}^T & \Lambda_{31}^T \\ \Lambda_{21} & \Lambda_{22} & \Lambda_{32}^T \\ \Lambda_{31} & \Lambda_{32} & \Lambda_{33} \end{bmatrix} > 0, \tag{7}$$

$$\mathrm{trace}(Q) < \gamma_2,\ D_{cl} = 0 \tag{8}$$

where

$$\begin{aligned}
&\Theta_{11} = AX_2 + X_2A^T + B_2\hat{C}_2 + (B_2\hat{C}_2)^T, \\
&\Theta_{21} = \hat{A}_2 + A^T, \\
&\Theta_{22} = A^TY_2 + Y_2A + \hat{B}_2C_2 + (\hat{B}_2C_2)^T, \\
&\Theta_{31} = B_1^T,\ \Theta_{32} = (Y_2B_1 + \hat{B}_2D_{21})^T,\ \Theta_{33} = -I, \\
&\Lambda_{11} = X_2,\ \Lambda_{21} = I,\ \Lambda_{22} = Y_2,\ \Lambda_{31} = C_1X_2 + D_{12}\hat{C}_2, \\
&\Lambda_{32} = C_1,\ \Lambda_{33} = Q
\end{aligned}$$

$\mathscr{H}_\infty$ **norm:**[9] A controller $K_\infty(s)$ yielding the $\|G_{cl}(s)\|_\infty < \gamma_\infty$ ($\gamma_\infty > 0$) exists if and only if following LMIs have solution matrices $X_\infty = X_\infty^T, Y_\infty = Y_\infty^T \in \mathbf{R}^{nx\times nx}$, $\hat{A}_\infty \in \mathbf{R}^{nx\times nx}$, $\hat{B}_\infty \in \mathbf{R}^{nx\times ny}$ and $\hat{C}_\infty \in \mathbf{R}^{nu\times nx}$ exist:

$$\begin{bmatrix} \Xi_{11} & \Xi_{21}^T & \Xi_{31}^T & \Xi_{41}^T \\ \Xi_{21} & \Xi_{22} & \Xi_{32}^T & \Xi_{42}^T \\ \Xi_{31} & \Xi_{32} & \Xi_{33} & \Xi_{43}^T \\ \Xi_{41} & \Xi_{42} & \Xi_{43} & \Xi_{44} \end{bmatrix} < 0, \tag{9}$$

$$\begin{bmatrix} X_\infty & I \\ I & Y_\infty \end{bmatrix} > 0 \tag{10}$$

where

$$\begin{aligned}
&\Xi_{11} = AX_\infty + X_\infty A^T + B_2\hat{C}_\infty + (B_2\hat{C}_\infty)^T, \\
&\Xi_{21} = \hat{A}_\infty + A^T, \\
&\Xi_{22} = A^TY_\infty + Y_\infty A + \hat{B}_\infty C_2 + (\hat{B}_\infty C_2)^T, \\
&\Xi_{31} = B_1^T,\ \Xi_{32} = (Y_\infty B_1 + \hat{B}_\infty D_{21})^T,\ \Xi_{33} = -\gamma_\infty I, \\
&\Xi_{41} = C_1X_\infty + D_{12}\hat{C}_\infty,\ \Xi_{42} = C_1,\ \Xi_{43} = D_{11},\ \Xi_{44} = -\gamma_\infty I
\end{aligned}$$

In both of the above cases, the coefficient matrices of the controller $K_i(s) := \left[\begin{array}{c|c} (A_K)_i & (B_K)_i \\ \hline (C_K)_i & 0 \end{array}\right]$ ($i = 2$ or ∞) is given as

$$\begin{aligned}
&(C_K)_i = \hat{C}_iM_i^{-T}, \\
&(B_K)_i = N_i^{-1}\hat{B}_i, \\
&(A_K)_i = N_i^{-1}(\hat{A}_i - N_i(B_K)_iCX_i - Y_iB(C_K)_iM_i^T - Y_iAX_i)M_i^{-T},
\end{aligned} \tag{11}$$

where matrices $M_i \in \mathbf{R}^{nx\times nx}$ and $N_i \in \mathbf{R}^{nx\times nx}$ are nonsingular square solutions to a following decomposition problem:

$$I - X_iY_i = M_iN_i^T,\ i = 2 \text{ or } \infty \tag{12}$$

Note that the above decomposition problem always has solution matrices M_i and N_i ($i = 2$ or ∞) if LMIs for each controller design problem is feasible. Both of the above LMIs can be solved efficiently with a standard LMI solver [10].

In the case of multiobjective problem of the above two criteria, i.e., $\mathscr{H}_2/\mathscr{H}_\infty$ problem, the necessary and sufficient condition for the existence of the controller $K_{2/\infty}(s)$ yielding $\|G_{cl}(s)\|_2^2 < \gamma_2$ and $\|G_{cl}(s)\|_\infty < \gamma_\infty$ is given by taking the common matrix variables as $\hat{A}_2 = \hat{A}_\infty := \hat{A}_{2/\infty}$, $\hat{B}_2 = \hat{B}_\infty := \hat{B}_{2/\infty}$ and $\hat{C}_2 = \hat{C}_\infty := \hat{C}_{2/\infty}$ in (6)-(8) and (9)[1]. However those matrix inequality conditions are no longer LMIs in this case and the optimal $\mathscr{H}_2/\mathscr{H}_\infty$ controller design problem is still an open problem. As a sufficient condition to exist the controller satisfying $\|G_{cl}(s)\|_2^2 < \gamma_2$ and $\|G_{cl}(s)\|_\infty < \gamma_\infty$ are obtained with the above variables changing and taking common Lyapunov matrices, i.e., $X_2 = X_\infty = X_{2/\infty}$ and $Y_2 = Y_\infty = Y_{2/\infty}$ [5], [9]. Then matrix inequality conditions in (6)-(8) and (9) are LMIs on each matrix variable and can be solved with the LMI solver. The resulted controller may be conservative depending on the given problem, however, this method currently can be said to be a reasonable way to obtain the multiobjective controller because of following reasons:

- We can obtain the controller taking multiple closed-loop criteria (although it is not necessarily the optimal controller) efficiently because the conditions are given as LMIs on variable matrices.
- The order of the controller is always same as the plant in contrast to the method based on coprime factor technique [11]. The property of the controller order is favorable in real applications.

In this paper we take the above common Lyapunov matrix strategy in the case of a multiobjective problem even when other closed-loop norm constraints (e.g., $\mathscr{L}_\infty$ or other quadratic constraints) are considered.

B. Homotopy like design algorithm

In this subsection we propose a homotopy like integrated design method based on the LMI conditions in the previous subsection. We show the algorithm only in the case of the $\mathscr{H}_\infty$ problem. The method itself is totally same when we consider the $\mathscr{H}_2$ problem or the mutiobjective one.

Define $\Delta p := [\Delta p_1,\ \ldots,\ \Delta p_{np}]^T \in \mathbf{R}^{np}$, $\Delta\hat{A}_\infty \in \mathbf{R}^{nx\times nx}$, $\Delta\hat{B}_\infty \in \mathbf{R}^{nx\times ny}$, $\Delta\hat{C}_\infty \in \mathbf{R}^{nu\times nx}$, $\Delta X_\infty = \Delta X_\infty^T \in \mathbf{R}^{nx\times nx}$, $\Delta Y_\infty = \Delta Y_\infty^T \in \mathbf{R}^{nx\times nx}$ and a scalar $\Delta\gamma_\infty$ as perturbations of p, $\hat{A}_\infty$, $\hat{B}_\infty$, $\hat{C}_\infty$, X_∞, Y_∞ and γ_∞ in (9) and (10) respectively. With those perturbations define the perturbed version of (9) and (10) as follows:

$$\begin{bmatrix} \Phi_{11} & \Phi_{21}^T & \Phi_{31}^T & \Phi_{41}^T \\ \Phi_{21} & \Phi_{22} & \Phi_{32}^T & \Phi_{42}^T \\ \Phi_{31} & \Phi_{32} & \Phi_{33} & \Phi_{43}^T \\ \Phi_{41} & \Phi_{42} & \Phi_{43} & \Phi_{44} \end{bmatrix} < 0, \tag{13}$$

$$\begin{bmatrix} X_\infty + \Delta X_\infty & I \\ I & Y_\infty + \Delta Y_\infty \end{bmatrix} > 0, \tag{14}$$

where

$$\begin{aligned}
\Phi_{11} &= A(p+\Delta p)(X_\infty+\Delta X_\infty) + (X_\infty+\Delta X_\infty)A(p+\Delta p)^T \\
&\quad + B_2(p+\Delta p)(\hat{C}_\infty+\Delta C_\infty) + (\hat{C}_\infty+\Delta C_\infty)^T B_2(p+\Delta p)^T, \\
\Phi_{21} &= \hat{A}_\infty + \Delta\hat{A}_\infty + A(p+\Delta p)^T, \\
\Phi_{22} &= A(p+\Delta p)^T(Y_\infty+\Delta Y_\infty) + (Y_\infty+\Delta Y_\infty)_\infty A(p+\Delta p) \\
&\quad + (\hat{B}_\infty+\Delta\hat{B}_\infty)C_2(p+\Delta p)^T + C_2(p+\Delta p)^T(\hat{B}_\infty+\Delta\hat{B}_\infty)^T, \\
\Phi_{31} &= B_1(p+\Delta p)^T,\ \Phi_{32} = \{(Y_\infty+\Delta Y_\infty)B_1(p+\Delta p) \\
&\quad + (\hat{B}_\infty+\Delta\hat{B}_\infty)D_{21}(p+\Delta p)\}^T,\ \Phi_{33} = -(\gamma_\infty+\Delta\gamma_\infty)I, \\
\Phi_{41} &= C_1(p+\Delta p)(X_\infty+\Delta X_\infty) + D_{12}(p+\Delta p)(\hat{C}_\infty+\Delta\hat{C}_\infty), \\
\Phi_{42} &= C_1(p+\Delta p),\ \Phi_{43} = D_{11}(p+\Delta p), \\
\Phi_{44} &= -(\gamma_\infty+\Delta\gamma_\infty)I
\end{aligned} \tag{15}$$

In the case that all coefficient matrices in (1) are linear functions on p, the perturbed coefficient matrices, say, $A(p+\Delta p)$, $B_1(p+\Delta p)$ and $B_2(p+\Delta p)$ etc. are given explicitly as

$$\bigstar(p+\Delta p) = \bigstar + \sum_{j=1}^{np} \Delta p_j \frac{\partial\bigstar}{\partial p_j}, \tag{16}$$

where the symbol $\bigstar$ denotes the component of each coefficient matrix in (1). If a component of those coefficient matrices are nonlinear function on p, the equation in (16) is satisfied approximately if Δp is sufficiently *small* such that the linear approximation obtained by taking the first order term of the infinite series expansion of the nonlinear function has a certain accuracy. The (approximated) perturbations of the coefficient matrices in (1) are denoted as ΔA, ΔB_1, ΔB_2, ΔC_1, ΔC_2, ΔD_{11}, ΔD_{12} and ΔD_{21} respectively. Then, (15) can be expanded in the form of the second order equations on the terms of the perturbations. Obviously (13) is a BMI problem on those perturbations and is difficult to solve. However, if we assume all perturbations are *small*, it is valid that we can approximate those BMIs as LMIs by ignoring the second order product of perturbation matrices. We take those approximated LMIs for the integrated design. The linear approximation of BMI in (13) is given as the following:

$$\begin{bmatrix} \Psi_{11} & \Psi_{21}^T & \Psi_{31}^T & \Psi_{41}^T \\ \Psi_{21} & \Psi_{22} & \Psi_{32}^T & \Psi_{42}^T \\ \Psi_{31} & \Psi_{32} & \Psi_{33} & \Psi_{43}^T \\ \Psi_{41} & \Psi_{42} & \Psi_{43} & \Psi_{44} \end{bmatrix} < 0, \tag{17}$$

where Ψ_{ij} $(i,j = 1,\ldots,4)$ is the linear approximation of Φ_{ij} in (15) obtained by neglecting the second-order terms on Δ. Equations (17) and (14) are LMIs on all perturbation matrices and we can obtain the feasible perturbation matrices satisfying (17) using the LMI solver.

As mentioned before the linear approximation of BMIs is reasonable only if the perturbations are small. Hence, we need to confine the amount of those perturbations in some senses. In this paper we take a norm constraint on the perturbation matrices given by

$$\|\Delta *\| < \varepsilon\|*\| \ (\varepsilon > 0) \tag{18}$$

[1] In this case the condition in (10) is automatically accomplished if the condition in (7) is feasible.

where the symbol $*$ denotes any variable matrices, e.g., p, X_∞ and Y_∞ etc.. The norm constraint in (18) is also given as following LMIs:

$$\begin{bmatrix} \varepsilon\|*\|I & \Delta * \\ (\Delta *)^T & \varepsilon\|*\|I \end{bmatrix} > 0 \tag{19}$$

Therefore, we can obtain the small amount of the optimal perturbations by combining LMI conditions (17), (14) and (19). With the above linearized LMIs we propose a following integrated design algorithm:

- Step 0:Define the iteration number $i = 0$. Set the initial value of the structural design parameter vector $p^0 \in \mathscr{P}$. In the following we define the symbols with the superscript ^{i} denote the one of the i-th iteration of this algorithm.
- Step 1:Set $\varepsilon > 0$ in (18). For the plant derived for the fixed p^i obtain the optimal $\mathscr{H}_\infty$ controller $K^i(s)$ and the performance index $J^i := \gamma_\infty^i$ for the fixed plant with LMI conditions (9) and (10).
- Step 2:The optimal perturbation matrices, e.g., Δp^i, $\Delta\hat{A}_\infty^i$ and $\Delta\hat{B}_\infty^i$ etc. minimizing the perturbation of the performance index $\Delta\gamma_\infty^i$ subject to constraints $\Delta\gamma_\infty^i < 0$ and $p^i + \Delta p^i \in \mathscr{P}$ satisfying (approximated) LMIs in (17), (14) and (19). If such perturbations can not be obtained, then set the optimal structural vector $p_{opt} = p^i$ and $K_{opt}(s) := K^i(s)$ and stop. Otherwise, update $p^{i+1} \leftarrow p^i + \Delta p^i$ and go to the next step.
- Step 3:Obtain the optimal $\mathscr{H}_\infty$ controller $K^{i+1}(s)$ and the performance index $J^{i+1} := \gamma_\infty^{i+1}$ again for the plant having the new structural design parameter vector p^{i+1}. If $J^{i+1} - J^i \geq 0$ then $\varepsilon \leftarrow \eta\varepsilon$ where $0 < \eta < 1$ and go to Step 2. Otherwise Update $p^i \leftarrow p^{i+1}$ and go to Step 1.

We can clearly see that the design parameters converge at least to a local optimal solution with the proposed algorithm. Note also that we can deal with other closed-loop specifications, e.g., $\mathscr{H}_2$ or multiobjective specifications without any difficulties in the proposed algorithm.

IV. Design Examples

A. 2DOF System

Let us consider an active control of a 2DOF system depicted in Fig. 1, where $q_1(t)$, $q_2(t)$, $w(t)$ and $u(t)$ are the displacement of m_1, m_2, the disturbance force and the control force respectively. The spring and the damper denoted by k_i and d_i $(i = 1, 2)$ connect the fixed wall and m_1, and m_1 and m_2 respectively. The disturbance and the control force are applied to the mass m_1. The displacement of the mass m_2 is measured to suppress the vibration of the same mass. By taking the state vector $x(t)$ as $x(t) :=$

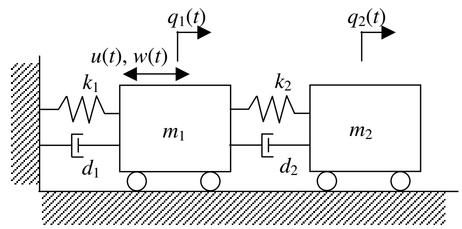

Fig. 1. Active control of 2DOF System

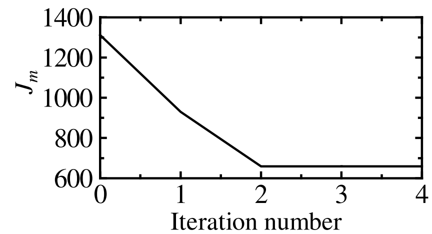

Fig. 2. The optimization history for the performance index J_m

$[q_1(t)\ q_2(t)\ \dot{q}_1(t)\ \dot{q}_2(t)]^T$, the state-space form in (1) is

$$\begin{cases} \dot{x}(t) = Ax(t) + [B_1^1\ B_1^2] \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} + B_2 u(t) \\ z(t) = \begin{bmatrix} z_1(t) \\ \hline z_2(t) \end{bmatrix} = \begin{bmatrix} C_1^1 \\ \hline 0_{1\times 4} \end{bmatrix} x(t) + \begin{bmatrix} 0 \\ \hline 1 \end{bmatrix} u(t) \\ y(t) = C_2 x(t) + [0\ 10^{-3}] \begin{bmatrix} w(t) \\ v(t) \end{bmatrix} \end{cases}, \tag{20}$$

$$B_1^2 := 0_{4\times 1},\ C_1^1 = [0\ 10^3\ 0\ 0],\ C_2 = [0\ 1\ 0\ 0].$$

where $v(t)$ is the measurement noise.

We consider a multiobjective problem where the performance index J_m is defined by

$$J_m := \overline{J_\infty} + \alpha\overline{J_2}, \alpha > 0, \tag{21}$$

where $\overline{J_\infty}$ and $\overline{J_2}$ are upper bounds of the closed-loop $\mathscr{H}_\infty$ and the square of $\mathscr{H}_2$ norms from $[w(t)\ v(t)]^T$ to $z_1(t)$ and $z_2(t)$ respectively. The scalar $\alpha > 0$ is a weighting parameter between the above two performance indeces. For the fixed plant, the controller optimizing J_m can be obtained by taking $X_2 = X_\infty = X$, $Y_2 = Y_\infty = Y$, $\hat{A}_2 = \hat{A}_\infty = \hat{A}$, $\hat{B}_2 = \hat{B}_\infty = \hat{B}$ and $\hat{C}_2 = \hat{C}_\infty = \hat{C}$ in LMIs given in (6)-(8) and (9) and minimizing the $\gamma_\infty + \alpha\gamma_2$.We take the structural design parameter vector p as the following:

$$p := \begin{bmatrix} d_1 & k_1 \end{bmatrix}^T \tag{22}$$

We set the lower and the upper bounds of the vector p as $p_l := 10^{-2} \times [d_1^n\ k_1^n]^T$ and $p_u := 10 \times [d_1^n\ k_1^n]^T$, where $d_1^n = 0.01$ [Ns/m] and $k_1^n = 1$ [N/m] respectively. In this example the matrix A is a linear function of each component of the vector p. The other physical design parameters are defined as $m_1 = m_2 = 1$ [kg], $d_2 = d_1^n$ and $k_2 = k_1^n$ respectively.

By taking $\varepsilon = 0.2$ (in (18)) and $\alpha = 1$, the proposed integrated design algorithm is applied to the problem. The initial value of the vector p is $p^0 := [d_1^n\ k_1^n]^T$. Optimization

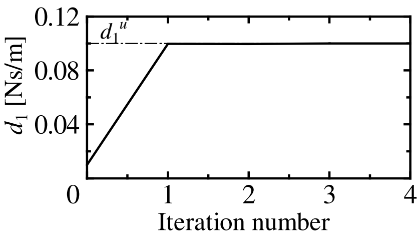

Fig. 3. The optimization history for the damping coefficient d_1

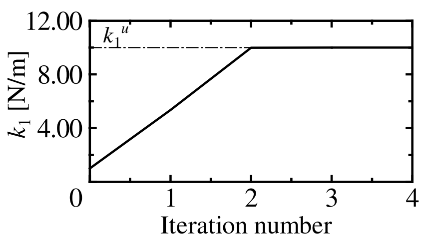

Fig. 4. The optimization history for the spring constant k_1

histories for the performance index J_m, the structural design parameters d_1 and k_1 are shown in Figs. 2, 3 and 4 respectively.

Those values are converged only four times iterations. The damping coefficient d_1 and the spring constant k_1 converge to their upper bounds. To check the quality of the obtained solution, we carry out the search for the whole design space $\mathscr{P} := \{p : p_l \leq p \leq p_u\}$ by computing the optimal controller for 2500 plants obtained by gridding of each range ($d_1 : [10^{-4}, 0.1]$, $k_1 : [10^{-2}, 10]$) of structural design parameters to 50 points. The 3D plot of the result of the exhaustive search is shown in Fig. 5. Clearly, in this case we could successfully find out the global optimal solution with the proposed homotopy based method. This result also means that we could find out the global optimal solution of the current BMI problem.

B. Sensor/Actuator Placement

Let us consider an active vibration control system of a simply supported beam with a circular cross section in Fig. 6. The length, the diameter, the density and Young's modulus of the beam are denoted by L [m], d [m], ρ [kg/m^3] and E [N/m^2] respectively. The moment of inertia of area is obtained by $I := \frac{\pi d^4}{64}$. Two actuators producing control forces $u_1(t)$ and $u_2(t)$ are installed at $\xi = \xi_a^1$, $\xi = \xi_a^2$, respectively. Two sensors are equipped for measuring displacements $q(\xi_s^1, t)$ and $q(\xi_s^2, t)$. A disturbance force $w(t)$ is injected at $\xi = \xi_w$. We obtain the optimal sensor and actuator placement in this problem with the proposed method. The structural design parameter vector and its lower and the upper bounds are defined as follows:

$$p := [\xi_a^1 \ \xi_a^2 \ \xi_s^1 \ \xi_s^2]^T, \ p_l = 0_{4\times 1}, \ p_u = L \times [1 \ 1 \ 1 \ 1]^T \quad (23)$$

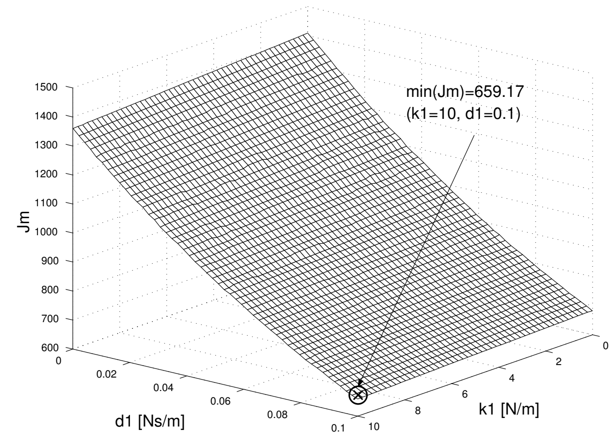

Fig. 5. The actual performance index J_m

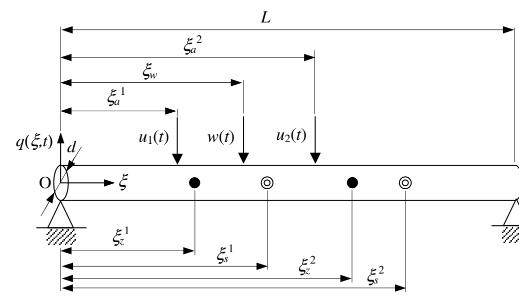

Fig. 6. Simply Supported Beam System

We assume that the displacement $q(\xi, t)$ can be approximated by

$$q(\xi, t) \simeq \sum_{j=1}^{3} q_j(t)\phi_j(\xi), \quad (24)$$

where $q_j(t)$ is the j-th modal displacement of the beam. The function $\phi_j(\xi)$ is given as the following normalized j-th modal shape of a simply supported beam given as

$$\phi_j(\xi) = \sqrt{\frac{2}{L}} \sin\left(\frac{j\pi\xi}{L}\right). \quad (25)$$

Then the (approximated) modal equation of motion of the beam system is obtained as

$$\ddot{q}_f(t) + 2Z\Omega\dot{q}_f(t) + \Omega^2 q_f(t) = L_w w(t) + L_a u(t), \quad (26)$$

where $q_f(t) := [q_1(t) \ q_2(t) \ q_3(t)]^T$ is the (approximated) modal displacement vector and $u(t) := [u_1(t) \ \ u_2(t)]^T$ respectively. Matrices $Z := \mathrm{diag}(\zeta_1, \zeta_2, \zeta_3)$ and $\Omega := \mathrm{diag}(\omega_1, \omega_2, \omega_3)$, ($\omega_j := (j\pi)^2\sqrt{\frac{EI}{\rho S L^4}}$, $j = 1, 2, 3$, $S = \frac{\pi d^2}{4}$) are modal damping matrix and normal frequency matrix respectively. We take a controlled output $z(t)$ as

$$z(t) := [q(0.3L, t) \ q(0.6L, t) \ r_1 u_1(t) \ r_2 u_2(t)]^T, \quad (27)$$

where r_1 and r_2 are the positive scalar weightings. By assuming the proportional damping, i.e., $Z = \beta\Omega$ ($0 < \beta \ll$

TABLE I

PARAMETER VALUES

	Value
Length L [m]	1.00
Diameter d [m]	5.00×10^{-3}
Density ρ [kg/m^3]	6.21×10^{3}
Young's modulus E[N/m^2]	6.06×10^{6}
Proportional constant β	10^{-3}
Weighting r_1 and r_2	1

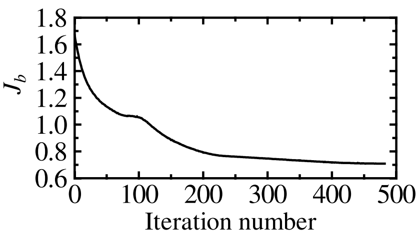

Fig. 7. The optimization history for the performance index J_b

1) and taking the state vector $x(t) := [q_f(t)\ \dot{q}_f(t)]^T$, we derive the state-space form of the beam system as (1).

In this case matrices B_2 and C_2 are nonlinear functions on the vector p in (23). We take the performance index (denoted by J_b) as the closed-loop $\mathscr{H}_\infty$ norm from $w(t)$ to $z(t)$. The optimal controller can be obtained for a fixed p by minimizing γ_∞ in LMIs in (9) and (10). The values of physical parameters are depicted in Table I.

By taking $\varepsilon = 5 \times 10^{-3}$, the proposed optimal design method is applied to the sensor/actuator placement problem. To obtain the *better* locally optimal solution, we conduct the optimization several times from different initial placements. In most cases both actuator placements converge to the place where the disturbance is applied ($\xi = 0.4L$). A typical result is presented in Figs. 7 and 8. In this problem, it is found that the performance index J_b is quite insensitive to the sensor placements ξ_s^1 and ξ_s^2. This result also suggests we do not need to equip two actuators but need only one actuator at the place where the disturbance is injected.

We do not check whether the obtained solutions are the globally optimal or not in this example because the amount of the computation is too large to obtain the global optimal solution with the extensive search of the whole design parameter space. However we can claim that the obtained result is quite reasonable from the physical viewpoint because it is the efficient way to suppress the effect of the disturbance force by collocating actuators.

The results in this section show that the proposed integrated design method works quite effectively even for the problems, which earlier proposed BMI based integrated design approaches [4], [5] cannot deal with. The results of design examples indicates the capability of the proposed design scheme for more general and complex design problems in real applications.

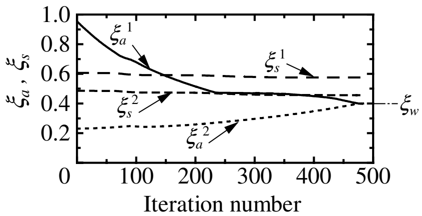

Fig. 8. The optimization history for the sensor/actuator ξ_a^i and ξ_s^i $(i = 1, 2)$

V. CONCLUSION

We have investigated the homotopy based integrated design method in this paper. An iterative design algorithm based on the homotopy method has been proposed. In this method we can utilize all LMI conditions for controller synthesis to the integrated design problem. This fact means that we can deal with the multiobjective problem, which currently LMI based approach is recognized as one of the most effective solution method, in the integrated design problem. The proposed method also can be applied to the case where the coefficient matrices of the plant state-space (descriptor) form are nonlinear functions on their structural design parameters by taking the linear approximation of the nonlinear dependence and limiting the amount of the updates of design parameters. The proposed iterative algorithm is guaranteed to converge at least to a local optimal solution. With design examples we could have shown the capability of the proposed design methodology.

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