2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Stabilization and Tracking Control of Friction Dynamics of A One-Dimensional Nanoarray

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Abstract— **Friction can be manipulated by applying perturbations to accessible elements of a sliding system. The dynamics of a one dimensional nanoarray is described by a set of coupled nonlinear equations. We discuss the feedback control problem of the nanoparticle dynamics that is subjected to a simple periodic form of the substrate potential and linear inter-particle forces. Specifically, we design two control laws which render the closed-loop system globally asymptotically stable and globally tracking any constant velocity respectively. The controls are functions of physically accessible variables, the position and velocity of the center mass of the nanoarray. Simulations show satisfactory closed-loop responses.**

I. INTRODUCTION

Fundamental tribology has been an active research area due to its application significance. The modern world depends on satisfactory operation of tribology systems. However, many aspects of friction are still not well understood, particularly on an atomic and molecular level. As pointed out in [1], better understanding of friction at the nanoscale is expected to provide the required tools to control friction, reduce unnecessary wear, and predict mechanical faults and failure of lubrication in microelectromechanical systems and nanodevices.

Friction can be manipulated by applying perturbations to accessible elements and parameters of a sliding system ([6]). A control problem is formulated in [1] to address some fundamental issues related to targeting and control of friction in nanoscale driven nonlinear particle arrays. A global feedback control scheme is presented therein to render the system output, the velocity of center mass of the nanoarray, to a given targeted value. The control law is essentially a bounded control law and does not perform in a local region of the equilibrium. In this paper, we follow the same formulation of the friction control problem, and present a systematic control analysis. Specifically, we discuss the system stability without an external force, and then provide two control laws to stabilize the system globally and to track any constant targeted velocity. Lyapunov's direct method is used in the control design. Simulation results are given to demonstrate satisfactory performances of closed-loop systems. It should be noted that the current control problem proposes new challenge in controller design due to physical constraints on accessible variables which are average quantities of the particles. Existing results on decentralized control (for example, [3]) do not apply to the current problem.

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II. MODEL AND CONTROL PROBLEM FORMULATION

The basic equations for the driven dynamics of a one dimensional particle array of N identical particles moving on a surface are given by a set of coupled nonlinear equations of the form ([1], [2]):

$$m\ddot{x}_i + \gamma\dot{x}_i = -\frac{\partial U(x_i)}{\partial x_i} - \frac{\partial W(x_i - x_j)}{\partial x_i} + f_i + \eta(t) \tag{1}$$

where $i = 1, \ldots, N$, x_i is the coordinate of the ith particle, m is its mass, γ is the linear friction coefficient representing the single particle energy exchange with the substrate, f_i is the applied external force (which is between $f_{\min}$ and 1, and $f_{\min}$ is the minimal value of the force necessary to obtain sliding), $\eta(t)$ is Gaussian noise, $U(x_i)$ is the periodic potential, $U(x_i + a) = U(x_i)$, $W(x_i - x_j)$ is the potential through particle interactions.

As pointed out in [2], equation (1) provides a general framework of modelling friction although the amount of details and complexity varies in different studies from simplified 1D models through 2D and 3D models to a full set of molecular dynamics simulations. Under the simplifications, 1) the substrate potential is assumed to have a simple periodic form, 2) there is a zero misfit length between the array and the substrate, and 3) the same force is applied to each particle, and 4) there is a zero noise, the equation of motion reduces to the dynamic Frenkel-Kontorova model:

$$\ddot{\phi}_i + \gamma\dot{\phi}_i + \sin(\phi_i) = f + F_i \tag{2}$$

where ϕ_i is the dimensionless phase variable, $\phi_i = 2\pi x_i/a$, and F_i is the nearest-neighbor interaction force. A specific example that's often considered ([2], [1]) for F_i is the Morse-type inter-particle interaction in the following form:

$$\begin{aligned} F_i = & \frac{\kappa}{\beta}\left\{e^{-\beta(\phi_{i+1}-\phi_i)} - e^{-2\beta(\phi_{i+1}-\phi_i)}\right\} \\ & -\frac{\kappa}{\beta}\left\{e^{-\beta(\phi_i-\phi_{i-1})} - e^{-2\beta(\phi_i-\phi_{i-1})}\right\} \end{aligned} \tag{3}$$

where κ, β are positive constants. As $\beta \to 0$, (3) turns to the following linear interaction which is commonly used for its simplicity:

$$F_i = \kappa\left(\phi_{i+1} - 2\phi_i + \phi_{i-1}\right) \tag{4}$$

An illustration of the Frenkel-Kontorova model is shown in Figure 1.

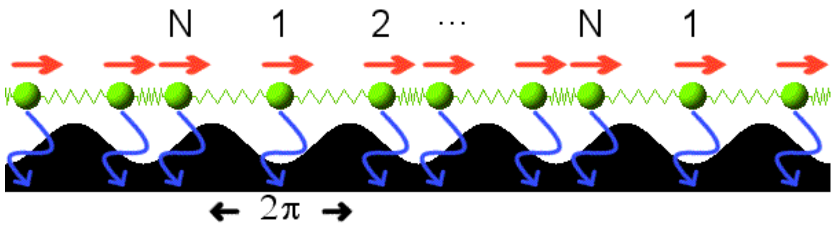

Fig. 1. The Frenkel-Kontorova model is a harmonic chain (mimic a layer of nano-particles) in a spatially periodic potential (mimic the substrate). The chain is driven by a constant force which is damped by a velocity-proportional damping.

System (2) is a nonlinear interconnected system and has rich dynamics depending on initial conditions, external force, and noises. It was revealed in [1] that the system with different sizes ($3 < N < 256$) exhibits four different regimes including rest (no motion), periodic sliding, periodic or chaotic stick-slip, where"stick"refers to the solution of $f - \sin x_{st} = 0$ and "slip" refers to the periodic solution $x(t) \approx \omega t + \psi(\omega t),\ \psi(\omega t + 2\pi) = \psi(\omega t)$.

Control can be applied to the nanoarray, so that the frictional dynamics of a small array of particles is controlled towards preassigned values of the average sliding velocity. A feedback control is added to system (2) as follows ([1]):

$$\ddot{\phi}_i + \gamma\dot{\phi}_i + \sin(\phi_i) = f + F_i + C(t) \tag{5}$$

where $C(t)$ is a function of three measurable quantities, v_{target}, the targeted velocity for the center of mass; $v_{cm} = \frac{1}{N}\sum_{i=1}^{N}\dot{\phi}_i$, average (center of mass) velocity; and $\phi_{cm} = \frac{1}{N}\sum_{i=1}^{N}\phi_i$, average (center of mass) position.

Since f and $C(t)$ are essential same (*i.e.*, external force), we rewrite (5) in the following form:

$$\ddot{\phi}_i + \gamma\dot{\phi}_i + \sin(\phi_i) = F_i + u(t) \tag{6}$$

We define our stabilizing control and tracking control problems for nonlinear system (6) as follows:

Stabilizing Control Problem: Design a feedback control law

$$u(t) = u(v_{cm}, \phi_{cm}), \tag{7}$$

where

$$\begin{aligned} v_{cm} &= \frac{1}{N}\sum_{i=1}^{N}\dot{\phi}_i, \\ \phi_{cm} &= \frac{1}{N}\sum_{i=1}^{N}\phi_i, \end{aligned} \tag{8}$$

such that system (6) is asymptotically stable.

Tracking Control Problem: Design a feedback control law

$$u(t) = u(v_{target}, v_{cm}, \phi_{cm}), \tag{9}$$

where v_{target} is a positive constant and v_{cm}, ϕ_{cm} are defined in (8), such that v_{cm} tracks v_{target}, and the tracking error tends to zero as t tends to ∞.

The above-defined problems propose new challenges in control of nonlinear interconnected systems. Different from decentralized control of large-scale systems ([3]) which employs local information in control, each subsystem's states are inaccessible and the control variables are the average quantities. We present our control design for the two problems in the next two sections.

III. Stabilizing Control Design

Define $x_{i1} = \phi_i, x_{i2} = \dot{\phi}_i$. We re-write equation (6) in the following state space form:

$$\begin{aligned} \dot{x}_{i1} &= x_{i2} \\ \dot{x}_{i2} &= -\sin x_{i1} - \gamma x_{i2} + F_i + u \end{aligned} \tag{10}$$

where F_i is the linear interaction

$$F_i = \kappa\left(x_{i+1,1} - 2x_{i1} + x_{i-1,1}\right) \tag{11}$$

and $x_{01} \approx x_{11},\ x_{N+1,1} \approx x_{N1}$.

Define Lyapunov function candidate for the whole interconnected system (10) to be:

$$V(x) = \sum_{i=1}^{N} V_i = \sum_{i=1}^{N}\left\{\int_0^{x_{i1}} \sin y dy + \frac{1}{2}x_i^T P x_i\right\} \tag{12}$$

where $x_i = [x_{i1}\ x_{i2}]^T$, and

$$P = \begin{bmatrix} P_{11} & P_{12} \\ P_{12} & P_{22} \end{bmatrix}.$$

For P to be a positive definite matrix, it must satisfy

$$P_{11} > 0, \qquad P_{11}P_{22} - P_{12}^T P_{12} > 0. \tag{13}$$

Its time derivative along the system dynamics is:

$$\begin{aligned} \dot{V}(x) &= \sum_{i=1}^{N}\{\sin x_{i1}x_{i2} + (P_{11}x_{i1} + P_{12}x_{i2})x_{i2} \\ &+ (P_{12}x_{i1} + P_{22}x_{i2})(-\sin x_{i1} - \gamma x_{i2} + F_i + u)\} \\ &= \sum_{i=1}^{N}\{(1 - P_{22})\sin x_{i1}x_{i2} + (P_{11} - \gamma P_{12})x_{i1}x_{i2} \\ &- P_{12}x_{i1}\sin x_{i1} + (P_{12} - \gamma P_{22})x_{i2}^2 \\ &+ (P_{12}x_{i1} + P_{22}x_{i2})(F_i + u)\} \end{aligned} \tag{14}$$

Let

$$P_{22} = 1, \quad P_{11} = \gamma P_{12} \quad P_{12} = \lambda, \tag{15}$$

where λ is a positive constant. From (13), we get

$$\lambda \quad < \quad \gamma. \tag{16}$$

By choosing the elements of P as above, we obtain

$$\begin{aligned} \dot{V}(x) \quad = \quad & \sum_{i=1}^{N} \{-\lambda x_{i1} \sin x_{i1} \\ & +(\lambda-\gamma)x_{i2}^2 + (\lambda x_{i1} + x_{i2})(F_i + u)\} \end{aligned} \tag{17}$$

Considering the term caused by particle interconnections in the above equation, we have:

$$\begin{aligned} & \sum_{i=1}^{N} (\lambda x_{i1} + x_{i2}) F_i \\ = \quad & \kappa\lambda \sum_{i=1}^{N} x_{i1}(x_{i+1,1} - 2x_{i1} + x_{i-1,1}) \\ & +\kappa \sum_{i=1}^{N} x_{i2}\left(x_{i+1,1} - 2x_{i1} + x_{i-1,1}\right) \\ = \quad & \kappa\lambda \left[\sum_{i=1}^{N} x_{i1}(x_{i+1,1} - x_{i1}) \right. \\ & \left. - \sum_{i=0}^{N-1} x_{i+1,1}(x_{i+1,1} - x_{i1}) \right] \\ & +\kappa \sum_{i=1}^{N} x_{i2}\left(x_{i+1,1} - 2x_{i1} + x_{i-1,1}\right) \\ = \quad & -\kappa\lambda \sum_{i=1}^{N} (x_{i+1,1} - x_{i1})^2 \\ & +\kappa \sum_{i=1}^{N} x_{i2}\left(x_{i+1,1} - 2x_{i1} + x_{i-1,1}\right) \\ = \quad & -\kappa\lambda \left(\sum_{i=1}^{N-1} x_{i1}^2 + \sum_{i=2}^{N} x_{i1}^2 - \sum_{i=1}^{N-1} 2x_{i+1,1}x_{i1} \right) \\ & +\kappa \sum_{i=1}^{N} x_{i2}\left(x_{i+1,1} - 2x_{i1} + x_{i-1,1}\right) \\ = \quad & -\kappa\lambda X_1^T A X_1 + \kappa X_1^T Q X_2 \end{aligned} \tag{18}$$

where $X_1 = [x_{11}\ x_{21}\ \ldots x_{N1}]^T, X_2 = [x_{12}\ x_{22}\ \ldots x_{N2}]^T$, and

$$A = \begin{bmatrix} 1 & -2 & 0 & \ldots & 0 \\ 0 & 2 & -2 & 0 & \ldots \\ & & \vdots & & \\ 0 & \ldots & 0 & 2 & -2 \\ 0 & \ldots & 0 & 0 & 1 \end{bmatrix}, \tag{19}$$

$$Q = \begin{bmatrix} -1 & 1 & 0 & \ldots & 0 \\ 1 & -2 & 1 & 0 & \ldots \\ & & \vdots & & \\ 0 & \ldots & 1 & -2 & 1 \\ 0 & \ldots & 0 & 1 & -1 \end{bmatrix}. \tag{20}$$

A. Stability In the Absence of Control

In the absence of control, that is, $u = 0$, the equilibrium points of the system are $[x_{i1}, x_{i2}] = [l\pi, 0], l = 0, 1, 2, \ldots$. In the following, we provide the asymptotical stability result in the local region of the equilibrium $[l\pi, 0]$ with $l = 0, 2, 4, \ldots$. for the system without control.

Lemma 1: For the interconnected nonlinear system (10), the system is locally asymptotically stable in the absence of control (that is, $u = 0$) if there exists a positive constant $\lambda < \gamma$, such that the following matrix is positive definite:

$$B \quad = \quad \begin{bmatrix} \lambda I_N + \kappa\lambda A & \frac{-\kappa}{2} Q \\ \frac{-\kappa}{2} Q & (\gamma - \lambda) I_N \end{bmatrix}, \tag{21}$$

where $X = [X_1^T\ X_2^T]^T$, $I_N \in \Re^{N \times N}$ is the identity matrix.

Proof: Substituting (18) into (17), and let $u = 0$, we obtain

$$\begin{aligned} \dot{V}(x) \quad \leq \quad & \sum_{i=1}^{N} \{-\lambda x_{i1} \sin(x_{i1}) - (\gamma - \lambda) x_{i2}^2\} \\ & -\kappa\lambda X_1^T A X_1 + \kappa X_1^T Q X_2. \end{aligned} \tag{22}$$

In the local region of the origin, we can approximate $\sin(x)$ by x, and (22) turns to

$$\begin{aligned} \dot{V}(x) \quad \leq \quad & \sum_{i=1}^{N} \{-\lambda x_{i1}^2 - (\gamma - \lambda) x_{i2}^2\} \\ & -\kappa\lambda X_1^T A X_1 + \kappa X_1^T Q X_2 \\ = \quad & -X^T B X, \end{aligned} \tag{23}$$

where B is give in (21). Since V is a positive definite function, and $\dot{V}$ is a negative definite function if B is positive definite, from Lyapunov's stability theory, the system is locally asymptotically stable.

B. Stabilizing Control Design

Since only center mass position and velocity variables are obtainable for control, we choose the following physically feasible controller:

$$u \quad = \quad -k_1 \left(\sum_{i=1}^{N} x_{i1} \right) - k_2 \left(\sum_{i=1}^{N} x_{i2} \right) \tag{24}$$

where k_1, k_2 are positive constants.

Substituting (18) and (24) to (17), it becomes

$$\begin{aligned} & \dot{V}(x) \leq \sum_{i=1}^{N} \{-\lambda x_{i1} \sin(x_{i1}) - (\gamma - \lambda) x_{i2}^2\} \\ & -\kappa\lambda X_1^T A X_1 + \kappa X_1^T Q X_2 \\ & -\lambda k_1 \left(\sum_{i=1}^{N} x_{i1} \right)^2 - k_2 \left(\sum_{i=1}^{N} x_{i2} \right)^2 \\ & -(k_1 + \lambda k_2) \left(\sum_{i=1}^{N} x_{i1} \right) \left(\sum_{i=1}^{N} x_{i2} \right) \end{aligned} \tag{25}$$

Since

$$\left(\sum_{i=1}^{N} x_{i1}\right)^2 = \sum_{i=1}^{N} x_{i1}^2 + \sum_{i=1}^{N}\sum_{j=1,j\neq i}^{N} x_{i1}x_{j1} \quad (26)$$

and in view of $a\sin(a) + a^2 \geq 0$ for $a \in \Re$, we have

$$\begin{aligned}&\dot{V}(x) \leq \sum_{i=1}^{N}[-\lambda(x_{i1}\sin(x_{i1}) + x_{i1}^2)] \\ &-(\gamma-\lambda)X_2^TX_2 - \kappa\lambda X_1^TAX_1 + \kappa X_1^TQX_2 \\ &-\lambda k_1 X_1^T\Theta X_1 + \lambda X_1^TX_1 \\ &-k_2X_2^T\Theta X_2 - (k_1+\lambda k_2)X_1^T\Theta X_2 \\ &\leq -X^TCX \end{aligned} \quad (27)$$

where

$$C = \begin{bmatrix} C_1 & C_2 \\ C_3 & C_4 \end{bmatrix}, \quad (28)$$

where

$$C_1 = \lambda k_1\Theta - \lambda I_N + \kappa\gamma A,$$

$$C_2 = \frac{(k_1+\lambda k_2)\Theta - \kappa Q}{2},$$

$$C_3 = C_2^T,$$

$$C_4 = (\gamma-\lambda)I_N + k_2\Theta.$$

We claim our main result for stabilizing control:

Theorem 1: The system (10) with control (24) is asymptotically stable if there exist positive constants $\lambda < \gamma, k_1, k_2$ such that matrix C in (28) is positive definite.

The proof of the theorem is straight forward using Lyapunov's direct method ([4], [7]).

Remark 1: We showed in this section that the Frenkel-Kontorova model (6) can be stabilized to the origin globally applying a linear controller which is a function of accessible physical variables, the center mass position and velocity. Compared to the uncontrolled system, which is only locally asymptotically stable, an appropriate control can bring any initial states to its equilibrium. The control design involves carefully choosing parameters λ, k_1, k_2, and the existence of the controller depends on system parameters γ, κ.

To check the positive definiteness of matrix C, we use the following lemma from [5]:

Lemma 2 ([5]): An arbitrarily partitioned matrix

$$H = \begin{bmatrix} H_{11} & H_{12} \\ H_{12}^T & H_{22} \end{bmatrix}$$

is positive definite if and only if either

$$\begin{cases} H_{22} > 0 \\ H_{11} - H_{12}H_{22}^{-1}H_{12}^T > 0 \end{cases} \quad (29)$$

or

$$\begin{cases} H_{11} > 0 \\ H_{22} - H_{12}^TH_{11}^{-1}H_{12} > 0 \end{cases} \quad (30)$$

holds.

We can see that Lemma 1 gives conditions on lower dimensional matrix for checking positive definiteness, in spite of which, finding general conditions on positive definiteness of high dimensional matrix is not a trivial task. In light of Lemma 1, we have the following corollary:

Corollary 1: The system (10) with control (24) is globally asymptotically stable if there exist positive constants $\lambda < \gamma, k_1, k_2$ such that

$$C_1 - C_2C_4^{-1}C_2 > 0$$

holds.

Proof: It's easy to check that as long as $\lambda < \gamma$, C_4 is positive definite as all of its eigenvalues are positive. Then Corollary 1 follows directly from Lemma 1.

IV. TRACKING CONTROL DESIGN

Define error state $e_{i1} = \phi_i - v_t t, e_{i2} = \dot{\phi}_i - v_t$, where $v_t = v_{target}$. We have the following error dynamics:

$$\begin{aligned}\dot{e}_{i1} &= e_{i2} \\ \dot{e}_{i2} &= -\sin(e_{i1}+v_t t) - \gamma(e_{i2}+v_t) + F_i + u(t)\end{aligned} \quad (31)$$

where

$$F_i = \kappa\left(e_{i+1,1} - 2e_{i1} + e_{i-1,1}\right). \quad (32)$$

Define the following Lyapunov function candidate:

$$\begin{aligned}V(t,e) &= \sum_{i=1}^{N} V_i \\ &= \sum_{i=1}^{N}\left\{\frac{\varepsilon}{2}e_{i1}^2 + \frac{1}{2}(\lambda e_{i1} + e_{i2})^2\right\}\end{aligned} \quad (33)$$

where λ, ε are positive constants to be chosen later.

Its time derivative along (31) is

$$\begin{aligned}&\dot{V}(t,e) = \sum_{i=1}^{N}\{\varepsilon e_{i1}e_{i2} + (\lambda e_{i1} + e_{i2})[\lambda e_{i2} \\ &-\sin(e_{i1}+v_t t) - \gamma(e_{i2}+v_t) + F_i + u(t)]\} \\ &= \sum_{i=1}^{N}\{(\varepsilon + \lambda^2 - \lambda\gamma)e_{i1}e_{i2} - (\gamma-\lambda)e_{i2}^2 \\ &-(\lambda e_{i1} + e_{i2})\sin(e_{i1} + V_t t) \\ &+(\lambda e_{i1} + e_{i2})F_i + (\lambda e_{i1} + e_{i2})[-\gamma v_t + u(t)]\}\end{aligned} \quad (34)$$

For the sinusoidal term of the above equation, applying the inequality $2ab \leq \frac{1}{\tau}a^2 + \tau b^2$ (for $a, b \in \Re$) twice, and bounding $\|\sin(a)\|$ by $\|a\|$, we have

$$\begin{aligned}
&\sum_{i=1}^{N}\{-(\lambda e_{i1}+e_{i2})[\sin(e_{i1}+V_t t)-\sin v_t t]\}\\
\leq\ & \sum_{i=1}^{N}(\|\lambda e_{i1}\|+\|e_{i2}\|)2\|\sin\frac{e_{i1}}{2}\|\|\cos\frac{e_{i1}+2v_t t}{2}\|\\
\leq\ & \sum_{i=1}^{N}\left\{\frac{1}{\tau}(\lambda^2 e_{i1}^2+e_{i2}^2)+\frac{\tau}{2}e_{i1}^2\right\}\\
=\ & \sum_{i=1}^{N}\left\{\left(\frac{\lambda^2}{\tau}+\frac{\tau}{2}\right)e_{i1}^2+\frac{1}{\tau}e_{i2}^2\right\} \quad (35)
\end{aligned}$$

where τ is a positive constant.

Following the same derivation in (18) with x_{i1}, x_{i2} replaced by e_{i1}, e_{i2} respectively, we obtain

$$\sum_{i=1}^{N}(\lambda e_{i1}+e_{i2})F_i = -\kappa\lambda E_1^T A E_1 + \kappa E_1^T Q E_2 \quad (36)$$

where $E_1 = [e_{11}\ e_{21}\ \ldots e_{N1}]^T$, $E_2 = [e_{12}\ e_{22}\ \ldots e_{N2}]^T$, and A, Q are defined in (19) and (20) respectively.

We choose our control to be

$$u(t) = \gamma v_t + \sin v_t t - k_1\left(\sum_{i=1}^{N} e_{i1}\right) - k_2\left(\sum_{i=1}^{N} e_{i2}\right) \quad (37)$$

where k_1, k_2 are positive constants.

Choose the design parameter ε such that $\varepsilon+\lambda^2-\lambda\gamma = 0$. Substituting (37), (35) and (36) into (34), after simplification, we get

$$\begin{aligned}
&\dot{V}(t,e) \leq \left(\frac{\lambda^2}{\tau}+\frac{\tau}{2}\right)E_1^T E_1 - \kappa\lambda E_1^T A E_1\\
&+\left(\frac{1}{\tau}-\gamma+\lambda\right)E_2^T E_2 + \kappa E_1^T Q E_2\\
&-\lambda k_1 E_1^T \Theta E_1 - k_2 E_2^T \Theta E_2 - (k_1+\lambda k_2)E_1^T\Theta E_2\\
&\triangleq -E^T D E \quad (38)
\end{aligned}$$

where $E = [E_1^T\ E_2^T]^T$,

$$D = \begin{bmatrix} D_1 & D_2 \\ D_3 & D_4 \end{bmatrix}, \quad (39)$$

and

$$D_1 = \lambda k_1 \Theta + \kappa\lambda A - \left(\frac{\lambda^2}{\tau}+\frac{\tau}{2}\right)I_n,$$

$$D_2 = \frac{(k_1+\lambda k_2)\Theta - \kappa Q}{2},$$

$$D_3 = D_2^T,$$

$$D_4 = k_2\Theta - \left(\frac{1}{\tau}-\gamma+\lambda\right)I_n.$$

From the above discussion, we can obtain the following theorem:

Theorem 2: The tracking error system (31) with control (37) is asymptotically stable if there exist positive constants λ, τ, k_1, k_2 such that matrix D in (39) is positive definite.

The proof of the theorem is straight forward using Lyapunov's direct method.

Remark 2: We presented a global tracking control result for the Frenkel-Kontorova model (6) in this section. Similar to the stabilizing control design, the control law is a function of accessible physical variables. Using this controller, the center mass velocity can be rendered to any constant targeted velocity asymptotically. This is to contrast to the controller presented in [1], whose closed-loop system's response can only achieve bounded results. Note that the existence of the control parameters λ, τ, k_1, k_2 depends on the system parameters γ, κ as well.

V. Simulation Results

We performed simulations for the Frenkel-Kontorova model (6). We use the following set of parameters as given in [2]:

$$\begin{aligned}
\gamma &= 0.7\\
\kappa &= 0.022 \quad (40)
\end{aligned}$$

The stabilizing control law we use is

$$\begin{aligned}
u &= -k_1\phi_{cm} - k_2 v_{cm}\\
&= -k_1\left(\frac{1}{N}\sum_{i=1}^{N}x_{i1}\right) - k_2\left(\frac{1}{N}\sum_{i=1}^{N}x_{i2}\right), \quad (41)
\end{aligned}$$

where $k_1 = 1.5, k_2 = 2$. For a system with $N = 256$ nano-particles, the closed-loop system responses without and with control are shown in Figures 2 and 3 respectively. The initial conditions in both cases are $[\phi_{cm}, v_{cm}] = [3, 3]$. The control input history is shown in Figure 4. We can see that the system without control is not asymptotically stable with respect to the origin; while applying the designed feedback control law, system states are rendered to the origin asymptotically.

Using the same set of system parameters for $N = 256$ nano-particles, the tracking response of the closed-loop system is shown in Figure 5. The initial conditions are $E_1 = E_2 = 4$, and the targeted velocity is $v_t = 5$. The tracking control law we use is

$$\begin{aligned}
u &= \gamma v_t + \sin v_t t - k_1\phi_{cm} - k_2 v_{cm}\\
&= -k_1\left(\frac{1}{N}\sum_{i=1}^{N}x_{i1}\right) - k_2\left(\frac{1}{N}\sum_{i=1}^{N}x_{i2}\right). \quad (42)
\end{aligned}$$

The control parameters we chose are

$$\tau = 1.7,\ k_1 = 5.8,\ k_2 = 4.$$

The control input history is shown in Figure 6. We can see that the simulations demonstrate satisfactory performances for tracking.

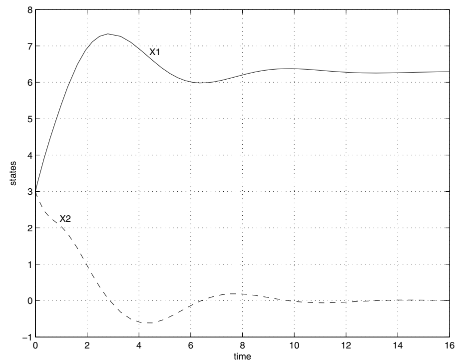

Fig. 2. System response without control.

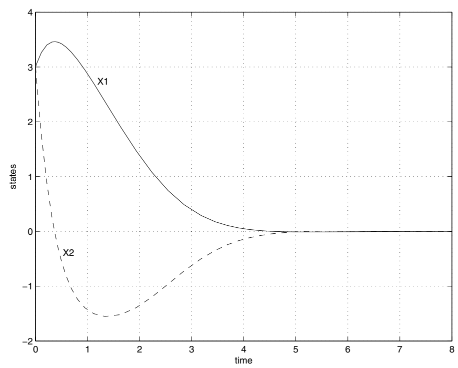

Fig. 3. Stabilizing response of the closed-loop system. $X_1 = \phi_{cm} = \frac{1}{N}\sum_{i=1}^{N}\phi_i,\ X_2 = v_{cm} = \frac{1}{N}\sum_{i=1}^{N} v_i$.

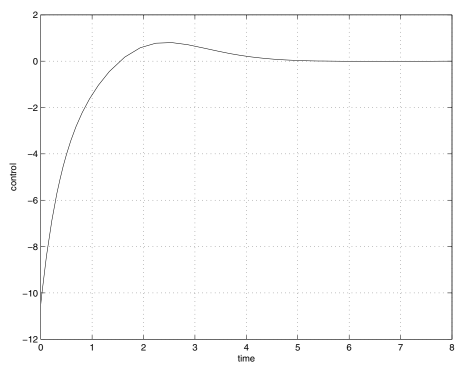

Fig. 4. Stabilizing input history.

VI. Conclusion

We discussed the stabilization and tracking control problem of a one dimensional nanoarray on a surface. The dynamics is described by a set of coupled nonlinear equations, and the accessible variables are center mass position and velocity. We designed two feedback control laws using only accessible variables. One globally asymptotically stabilizes the system, and the other globally asymptotically tracks any constant targeted velocity. Simulations are shown with satisfactory responses. Future work includes verification by physical experiments and analysis of performances.

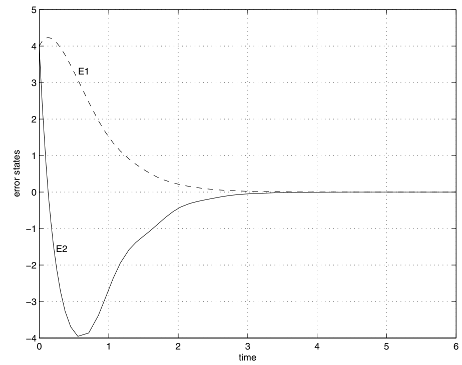

Fig. 5. Tracking error response of the closed-loop system. $E_1 = \phi_{cm} - v_t t = \frac{1}{N}\sum_{i=1}^{N}\phi_i - v_t t, E_2 = v_{cm} - v_t = \frac{1}{N}\sum_{i=1}^{N} v_i - v_t$.

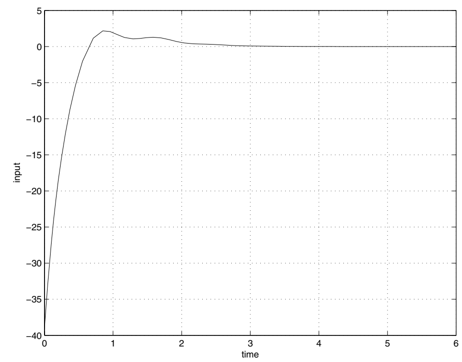

Fig. 6. Tracking input history.

Acknowledgment

The work is sponsored by Oak Ridge National Laboratory (ORNL) under contract 4000034419. We are grateful for technical discussions with Dr. Zhenyu Zhang in Condensed Matter Division at ORNL. We also thank Drs. Yehuda Braiman and Jacob Barhen in ORNL for providing us relevant papers.

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