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Multi-rate Digital Design for Sliding-Mode-Observer-based Feedback Control

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Abstract— Implementation of Sliding-Mode-Observer-based feedback control is treated as a multi-rate digital signal processing problem. The oversampled delta-sigma modulation techniques is used to design a decimation filter that integrates high-frequency switching output of the sliding mode observer into the feedback control, which operates at much lower predetermined control update rate. Application of a multi-rate sliding mode observer in the function of a nonlinear uncertain system estimator in the terminal guidance loop is demonstrated. It's shown in this application that the correctly designed decimation filter for a multi-rate system delivers close-loop system performance superior to that of traditionally proposed low-pass filters of autoregressive type, which estimate "equivalent control" component in the digital sliding mode observer in the manner equivalent to their continuous-time analog implementations.

I. Introduction

Dynamic system state estimators, i.e., dynamic observers, have become an essential part of the integrated navigation-guidance-control (GNC) solutions for many aerospace flight control problems [10,11]. Stochastic design of a plant-model-based estimator, which is presented in numerous variations of Kalman filtering technique, is one of the popular choices for GNC integration. Another potentially powerful alternative or complementary technique to Kalman filtering in the domain of dynamic-observer-based control is the sliding mode technique [1-3]. However, one cannot deny that Kalman filter (KF) dominates the contemporary engineering solutions for GNC applications, and it can be attributed to a number of particularly attractive features of its digital design. Beside the fact that digital design of KF is absolutely rigorous, KF algorithms can easily accommodate signal input sources supplied at different rates, can utilize statistical redundancy not only from time-correlation within one signal source, but also from fusion of multiple measurements of the same signal. While processing rate in navigational KF [10] can match that in feedback control that provides tracking of guidance command computed from navigational data, sliding mode observer must have processing rate much higher than control update rate. Therefore, the downsampling problem is essential in digital implementation of sliding mode observers and is overlooked in the available literature on sliding mode observer based controllers [1-3]. The lack of practical guidelines and studies of digital implementation of sliding mode observers in a multiple-rate control-observation architecture downplays application of this potentially powerful method in some areas, including ones, where the closed-loop performance of the observer-based control rather than signal-to-noise ratio or RMS of the observer estimation error serve as the ultimate performance criterion for the complete integrated GNC system. Thus, this work concentrates on the problem of multi-rate digital processing in sliding mode observers, i.e. their digital implementation, namely decimation filter design that provides (speaking in digital signal processing terms) "delta-sigma demodulation" [4-6] and downsampling for high-frequency switching component of the sliding mode observer, which retrieves the uncertain part of the "equivalent control" for the sliding mode existing in the observer. The use of the contemporary oversampled-delta-sigma-modulation technique [4-6] is proposed to design the decimation filter.

The inherent feature of any practical implementation of sliding mode, i.e. finite frequency switching mode, – the oversampling delta-sigma modulation of the uncertain component of the equivalent control – puts an emphasis on designing the decimation filter that integrates the observer output to the feedback control operating at much lower control update rate. The goal of this decimator/demodulator is to achieve low-pass filtering and downsampling to a predetermined sampling rate of the input one-bit signal avoiding aliasing and delivering maximally efficient bit resolution of the output signal. The ultimate performance criterion of this decimation filter is in achieving accurate robust tracking performance for the closed-loop system. Therefore, only the closed-loop system behavior rather than RMS or spectrum of estimation error should be considered

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to justify a particular design for a sliding mode observer with chosen decimator/demodulator.

Because of practical emphasis of this work, only SISO case is considered, while MIMO cases with system structure reduced to vector relative degree [1,1,…1] or [2,2,…2] are straightforward and do not contribute any essential features that are worth considering from the standpoint of the chosen downsampling problem.

II. Problem Formulation

Consider n^{th} order nonlinear SISO plant under scalar control and uncertain disturbance inputs. State flow is hyperbolic at the origin, globally Lipshitz, and satisfies all nice properties of input-output feedback linearizable system. For a given measured sliding variable y of the plant, the input-output dynamics is of the first order

$$\begin{cases} \dot{x} = f(x) + u(t) + d(t), \\ y = x, \end{cases} \tag{1}$$

where y, x is the output and the state of the first order input-output sliding variable dynamics (1), $f(x)$ is known nonlinear term, $u(t)$ is known input to system (1), $\left(u(t) = ZOH(u[n]), u[n] = u(t_o + nT) = const\right)$ with ZOH standing for zero-order hold, and $d(t)$ is uncertain bounded and low-pass band-limited disturbance input to be estimated. Conventional sliding mode observer of a continuous form is a tracking system that follows system (1) output $\hat{x} \to y$

$$\dot{\hat{x}} = f(x) + u(t) + \rho \cdot u_d\,, \tag{2}$$

where $\rho > 0$ is a constant that satisfies inequality $\rho > \max|d(t)| + \max|f(\hat{x}(0)) - f(x(0))|$ or adaptive feedback gain. The switching control function u_d is a *signum* of sliding quantity σ

$$u_d = sign(\sigma),\ \sigma = e = y - \hat{x}, \tag{3}$$

which is equal to tracking error for observer (2). Stability of system (2) and the existence condition of sliding mode $\sigma = 0$ is well known [1,2]. According to [1,2], equivalent control for system (2) in the sliding mode $\sigma = 0$ is obtained from condition $\dot{\sigma} = 0$,

$$(\rho\, u_d)_{eq} = d(t)\,. \tag{4}$$

In the ideal sliding mode, the left part in (4) can be considered as the estimate of uncertain input $d(t)$ to the system (1). This estimate is obtained for the ideal sliding mode, which is featuring infinite frequency switching of (3) while the switching frequency in any practical digital system is limited to processing cycle frequency.

Now we proceed to formulating the problem considered in this work. One possible digital implementation of observer (2) employing Euler digital integration is used

$$\begin{aligned} &\hat{x}[k] = \hat{x}[k-1] + T_s\left(f(\hat{x}[k-1]) + u[k-1]\right) + \rho T_s u_d[k-1], \\ &u_d[k-1] = sign\left(y[k-1] - \hat{x}[k-1]\right), \end{aligned} \tag{5}$$

where k is sample number, T_s is sampling/processing time of observer (5). It is assumed that update rate of the known signal $u(t)$ is much slower than that for observer (5) processing rate, therefore $u[k]$ is actually equal to a constant $u[n]$ for many processing cycles k, where n is control update sample number corresponding to update time $t = t_o + nT$, $T >> T_s$ for control update in system (1). Also, sampling rate of measurement input is usually slower than processing rate $1/T_s$. Thus, processing in (5) delivers significant oversampling with respect to the inputs u and y.

An estimate of the disturbance $d(t)$ in (1) is deduced from the previous discussion as

$$\hat{d} = LPE\left(\rho\, T_s u_d\right), \tag{6}$$

where *LPE* stands for low-pass equivalent of the agrument. In order to complete digital design of the disturbance estimating system observer (5), the following problem is formulated.

The problem is in designing a digital linear filter-decimator that extracts a given low-pass-band component, $f \in [0, f_B]$, *with a given output rate* $1/T$ *from the binary wave form* $u_d[k], k = 0,1,2,...$ *given in real time, scaled by the factor* ρT_s *and supplied at the rate* $1/T_s$, $1/T_s >> 1/T$.

Remarks:

1) Linearity of the filter is required to provide linear phase-shift in the output, which could be a typical requirement in many practical systems, and the possibility to carry out a multiplicative scaling term while working with only one-bit numbers $u_d[k]$, which is really handy for circuit-level design.

2) The objective of this decimator/demodulator is to achieve low-pass filtering and downsampling to a predetermined sampling rate of the input one-bit signal avoiding aliasing and delivering maximum to the effective bit resolution of the output signal.

III. Oversampling Delta-Sigma Modulation and Digital Implementation of Sliding Modes

The closest resemblance can be observed between the sliding-mode observer described in Section 2 and the pulse modulators called delta (Δ) modulators and delta-sigma ($\Delta\Sigma$) modulators [4-7]. *Pulse modulation communication* *systems* (Δ-modulators) and *feedback relay control systems* operate in high-frequency auto-oscillation mode called the sliding mode. In the field of communication systems, the principles of a feedback processing [8] and utilization of robust limit cycle oscillations (auto-oscillations) [9] for pulse modulation techniques have been studied since the beginning of the sixties. A pulse-modulation system as a sampled-data system relies on the uniform sampling theorem for low-pass signals and its immediate corollary that continuous band-limited signals can be encoded by a sequence of discrete samples and reconstructed from it with negligible error provided the sampling rate is sufficiently high [4-6]. This property of signal reconstruction and the structure of the feedback loop are common to Δ-modulators and sliding mode observers; therefore, they can benefit from each other in design solutions. One equivalent sampled-data representation of the oversampling delta-sigma modulator can be given as

$$\begin{aligned} &\hat{x}[k] = \hat{x}[k-1] + T_s \cdot m[k-1] + \rho \cdot T_s \cdot u_d[k-1], \\ &u_d[k-1] = sign\left(0 - \hat{x}[k-1]\right), \end{aligned} \tag{7}$$

where k is sample number, $\rho \geq \max|m(t)|$, T_s is sampling/processing time, and oversampling means that $1/T_s$ is significantly higher that Nyquist rate for the message signal $m(t)$ encoded by a binary signal $u_d[k]$. Delta-sigma modulation is a popular method of analog-to-digital conversion where the processing rate of modern systems reaches GHz band, which delivers significant oversampling to many physical signal sources. The problem of digital demodulation of binary encoded signal is equivalent to the problem formulated in Section 2 for sliding mode disturbance observer, which in this case is to convert delta-sigma modulated signal back to the digital form sampled at Nyquist rate with maximal possible bit resolution in magnitude for a given dynamic range.

As we have mentioned, the tendency to increase the oversampling rate in delta-sigma modulator and sliding mode observer is motivated by the immediate corollary from the uniform sampling theorem for low-pass signals: the higher is the sampling rate, the lower is the signal reconstruction error, which is valid for both the known message signal to be encoded or the unknown disturbance signal to be estimated in the dynamic observer. Effectively, delta-sigma modulation technique trades resolution in time for resolution in amplitude. However, the demodulation process, i.e., abrupt low-pass filtering and decimation at elevated sampling rates, presents a significant circuit-level design challenge and added cost, while simple design solutions always carry significant noise penalty because of aliasing.

Fortunately, the discussed problem for delta-sigma modulation has been in the focus of research community for many years [4-6], and practical design solutions have been developed and implemented in digital electronics. Therefore, the solution of the problem in Section 2 formulated for sliding mode observer (5) and (6) can directly benefit from its communication system counterpart. One of the proven design solutions, multistage decimation, will be presented and applied to sliding mode observer in Section 4. The simulations and analysis of Section 5 will gauge the benefit of the selected design for closed-loop performance of the observer-based control system.

IV. MULTISTAGE DECIMATION FOR SLIDING MODE OBSERVER

One-bit code $u_d[k], k = 0,1,...$ in observer (5) is a representative of delta-sigma modulation of the uncertain disturbance $d(t)$ acting on system (1). However, due to measurement noise in the sensed system (1) output y and modeling error in observer (5), signal $u_d[k]$ also represents these components plus quantization noise. Therefore, the problem formulated in Section 2 includes the band limit f_B for estimating the low-frequency disturbance $d(t)$ while all out-of-band components should be eliminated. The ideal solution to the problem is to apply ideal low-pass filter followed by decimator to the required frequency. Practical low-pass filter with abrupt characteristic for elevated sampling rates would be of very high order. Therefore, decimation and filtering is usually performed as a two-stage procedure [4].

According to one design approach in [4], the first stage decimates from high processing rate to some intermediate sampling rate f_D. The filter for this stage is designed to attenuate noise components in the vicinity of f_D and its harmonics, which fold into the signal band. One well-suited digital filter having necessary characteristic is $sync^2$ filter

$$H(z) = \frac{1}{N^2}\left(\frac{1 - z^{-N}}{1 - z^{-1}}\right)^2, \tag{8}$$

where $N = f_s / f_D$ is the decimation ratio of the first stage. It can be implemented as a cascade of two accumulators providing $1/(1 - z^{-1})^2$ function followed by resampling at rate f_D, and followed by a cascade of two digital differentiators working at this lower rate f_D and generating total signal transformation according to (8).

The second stage consists in the low-pass filter designed to meet specifications for band limit f_B followed by the accumulate-and-dump block decimating from f_D to $2f_B$. Thus, the utility and benefits of this approach consist in the fact that only one switching element and a number of digital accumulators should work at elevated sampling rate in a dedicated but very simple hardware module while low-pass filter/demodulator of any complexity works at regular sampling rate of main guidance computer that carries on all the other GNC processing. Without the proposed multi-rate design, sliding mode observer in a GNC system has two choices: either working at the available processing rate that has been established for, say, Kalman filters and definitely failing the competition, or requiring elevated processing rate for the whole system, which is not cost effective. The presented important structural solution based on proven techniques in communication field and applied to digital sliding mode observer may, in our opinion, bring robust techniques of sliding mode control theory close to the position competing against Kalman filter based observers or complementing them in a hybrid design for contemporary GNC systems.

V. CASE STUDY: HOMING GUIDANCE SYSTEM

The following example of a real-world closed-loop control problem, terminal (homing) guidance loop under Augmented Proportional Navigation (APN) law [10,11], is selected to illustrate direct benefits in applying multi-rate digital sliding mode observer for the purpose of estimating target acceleration as a component of APN guidance.

Considering only planar motion without account for gravity, one can derive planar engagement kinematics in polar coordinate system where the relative position is presented by $\mathbf{R}=(r,\lambda)$, $r=$ range along line-of-sight (LOS), and $\lambda=$ LOS angle, as

$$\begin{cases} \ddot{r}-\dot{r}\dot{\lambda}^2 = a_{T\,r}-a_{M\,r}, \\ r\ddot{\lambda}+2\dot{r}\dot{\lambda} = a_{T\,\lambda}-a_{M\,\lambda}, \end{cases} \tag{9}$$

or, accepting range rate as V_r, LOS rate as ω_λ, and missile normal acceleration as a_n

$$\begin{cases} \dot{r}=V_r, \\ \dot{V}_r = r\omega_\lambda^2 + a_{T\,r} - \sin(\lambda-\gamma_M)a_n, \\ \dot{\lambda}=\omega_\lambda, \\ \dot{\omega}_\lambda = \frac{1}{r}\left(-2V_r\omega_\lambda + a_{T\,\lambda} - \cos(\lambda-\gamma_M)a_n\right), \end{cases} \tag{10}$$

where γ_M is the direction of missile velocity in this 2-dimensional system; let it be, for example, vertical plane, and γ_M angle is missile flight path angle. Acceleration projections for target and missile, $a_{T\,r}, a_{M\,r}$, are along LOS, and $a_{T\,\lambda}, a_{M\,\lambda}$ are transversal to LOS. Missile airframe compensated dynamics, as a transfer function for normal acceleration, will be presented by a first-order lag block with time constant 0.5[sec], rate saturation $\left|\dot{a}_n\right| \le 100[g/\sec]$, and magnitude saturation for APN guidance command $\left|a_n^c\right| \le 30[g]$.

Augmented Proportional Navigation, APN, which is optimal for zero-lag guidance system in presence of step-constant target maneuver, is computed as

$$a_{M\,\lambda}^c = N'\left(V_c\omega_\lambda + \tfrac{1}{2}a_{T\,\lambda}\right), \tag{11}$$

where N' is the navigation gain, $V_c=-V_r$ is closing velocity. Guidance command, $a_{M\,\lambda}^c$, will be supplied to the input of flight control system (11) as a piecewise constant signal with 50[Hz] update rate.

Missile seeker (target tracking head) as a LOS rate estimator is assumed to have a single-lag low-pass filter for LOS servo-tracking error measurements such that LOS rate measurement process is generated as

$$\dot{\omega}_\lambda^s = \frac{1}{T_s}\left(\omega_\lambda + \upsilon - \omega_\lambda^s\right), \tag{12}$$

where $T_s = 0.1[sec]$, input noise υ to the filter is uncorrelated zero-centered Gaussian process with $\sigma = 0.01[rad/sec]$. Measurements of LOS rate, ω_λ^s, are available to the guidance computer at the rate of $500[Hz]$.

Closing velocity, range to the target, and the actuated missile acceleration orthogonal to LOS are assumed to be known, and the only quantity to be estimated is target acceleration $a_{T\,\lambda}$ that is used in APN guidance law (11). Taking ω_λ^s as measurement of LOS rate and omitting fast seeker dynamics, the following first-order sliding mode observer is designed using the last equation in system (10)

$$\begin{aligned} \hat{\omega}_\lambda[k] &= \hat{\omega}_\lambda[k-1] + \frac{T}{r[k-1]}(-(2V_r[k-1]\hat{\omega}_\lambda[k-1] + \\ &\quad a_{M\,\lambda}[k-1]) + \rho\cdot u_d[k-1], \\ u_d[k-1] &= sign\left(\omega_\lambda^s[k-1] - \hat{\omega}_\lambda[k-1]\right), \end{aligned} \tag{13}$$

where T is sampling time. Parameter ρ, if it is not adaptive but constant, should be equal to the known limit of $|a_{T\,\lambda}|$. Estimate of target acceleration, $\hat{a}_{T\,\lambda}$ is obtained by low-pass filtering of binary sequence $u_d[k]$ with scaling by ρT. We select 2nd order low-pass Butterworth filter with cut-off frequency of $0.15[rad/sec]$ implemented in digital form for each corresponding sampling rate. APN guidance command is computed using these estimates as $a_{M\,\lambda}^c = 4\left(V_c\hat{\omega}_\lambda + \frac{1}{2}\hat{a}_{T\,\lambda}\right)$ and decimated from processing rate 1/T to the rate 50[Hz].

One particular engagement scenario has been simulated and resulted in a direct hit where the range estimate of closest approach point is less than the distance traveled in sample time of measurement update. Engagement trajectory is given in Fig.1.

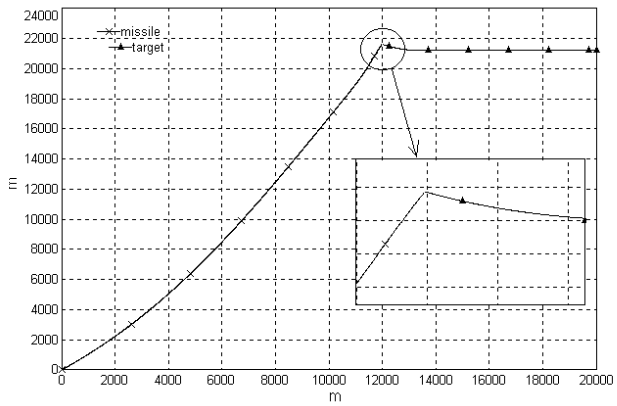

Fig.1 Engagement Trajectory: Incoming near-orthogonal intercept

Three simulation runs has been performed with different configurations of sliding mode observer for estimating target acceleration $\hat{a}_{T\lambda}$. All three engagements resulted in the target intercept, and the following target acceleration tracking performance has been demonstrated. In the first run, observer (13) and the following Butterworth filter has been working at the rate 100[*kHz*] with following downsampling to 50[*Hz*]; target estimation is shown in Fig. 2.

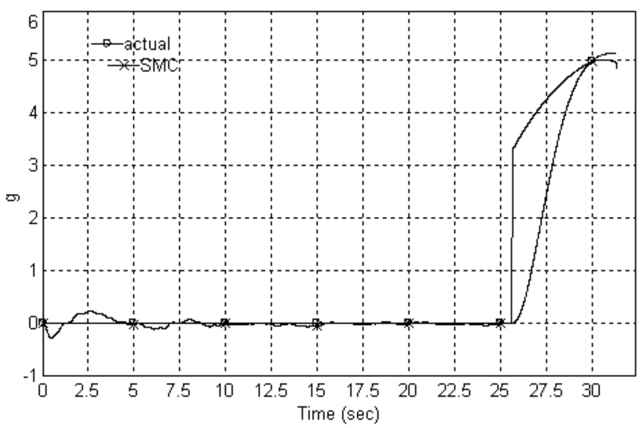

Fig. 2 Target acceleration estimation at processing rate of 100[*kHz*]

In the second run, observer (13) and the following Butterworth filter has been working at the rate 1[*kHz*] with following downsampling to 50[*Hz*]; target estimation is shown in Fig. 3. One can observe in Fig. 3 that low switching frequency of 1[*kHz*] in observer (13) led to high quantization noise plus aliasing of the high frequency noise component in the input ω_{λ}^{s} to the base-band of target estimate $\hat{a}_{T\lambda}$. As a result, error in $\hat{a}_{T\lambda}$ (for the first 25[sec] of flight time, in absence of target maneuver) is about 1[g]. Although the second engagement has been successful, the APN guidance command that includes $\hat{a}_{T\lambda}$ term has caused unnecessary lateral motion of the missile during first 25[sec] of engagement, which could lead to loss of kinetic energy and diminish missile ability to perform intercept.

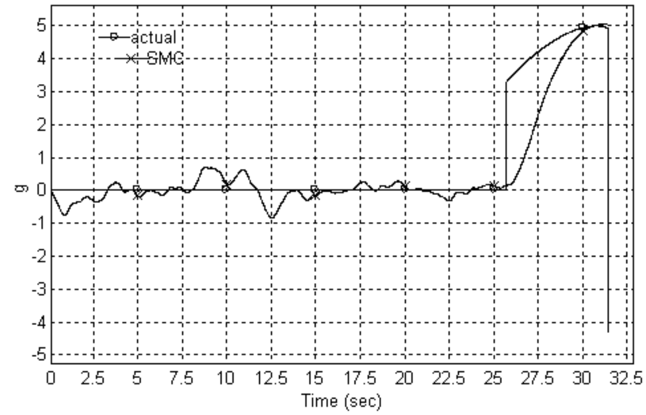

Fig. 3 Target acceleration estimation at processing rate of 1[*kHz*]

For the third run, multi-rate design to observer (13) has been applied. The switching frequency of sliding mode in observer (13) has been 100[*kHz*], binary output $u_d[k]$ passed through *sync*^{2} decimating filter (8) and scaled by ρT. Output rate has been 50[*Hz*], and the following digital Butterworth filter has been working at the rate 50[*Hz*]. Target estimation for multi-rate observer is shown in Fig.4.

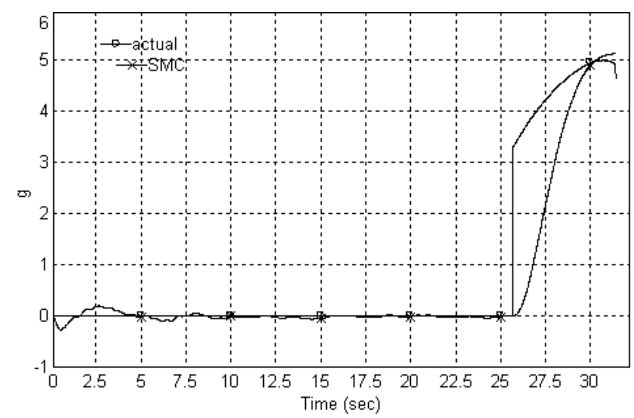

Fig. 4 Target acceleration estimation in multirate sliding mode observer

One can notice that target estimation performance in Figs. 3 and 4 is almost identical, which justifies the presented multi-rate design with application of elevated sampling rate only for delta-sigma modulation of target acceleration in observer (13) followed by correctly designed decimation filter to provide all subsequent processing at predetermined output sampling rate.

VII. Conclusions

The presented work carries two main ideas. The first one is that delta-sigma modulation techniques in communication field and sliding mode techniques in control field may benefit each other in great deal, many more design solutions can be borrowed in each direction. The other idea is that the practical success of sliding mode observers in modern digital world may be achieved only in multi-rate design solutions. The both ideas are illustrated on a practical example: terminal (homing) guidance system based on Augmented Proportional Navigation. Future work is to be dedicated to rigorous study of multi-rate digital implementation of sliding mode observers, including use of high order sliding modes.

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