2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

OSNR Optimization in Optical Networks: Extension for Capacity Constraints

Yan Pan and Lacra Pavel

***Abstract*— This paper builds on the OSNR model in [13] and studies the optimization problem of optical signal to noise ratio (OSNR) in the case of single point-to-point links. An m-player noncooperative game is formulated and the cost function for each channel is introduced, with differentiated prices. The status of the link, i.e. such that the total input optical power will not exceed the link's capacity, is considered in the cost function. Conditions for existence and uniqueness of the Nash equilibrium solution are given. Some strategies for dynamic price setting are also discussed, in the context of dynamic channel-add.**

I. Introduction

Recently, there has been much interest in optical wavelength-division multiplexing (WDM) communication networks and their dynamic aspects, [4], [11], [12]. In optical communication systems, information is transmitted by modulating the optical power of light pulses of a specific wavelength, and multichannel optical systems are realized by WDM technology. Reconfigurable optical networks operate in a dynamic environment, with some existing channels remaining in service while network reconfiguration (channel add/drop) is being performed, [12]. Channel performance should be optimized and maintained after reconfiguration.

Channel performance typically depends on transmission parameters such as optical signal-to-noise ratio (OSNR), dispersion and nonlinear effects, [14]. In link optimization, OSNR is considered as the main performance parameter, while dispersion and nonlinear effects are kept low by proper link design, [13]. The dominant impairment is noise accumulation in chains of optical amplifiers and its effect on OSNR, [8]. By adjusting channel input power, the OSNR can be equalized. Some static approaches have been developed for single-link OSNR optimization of cascaded amplifiers, [7], [8].

[13] extends this problem to a general multi-link configuration, via a noncooperative game approach. An OSNR model is formulated for reconfigurable optical networks. Noncooperative games are characterized by assuming that cooperation between players is not possible, such that each play acts independently, [5]. This is an appropriate assumption in large-scale optical networks, where a centralized system for transmitting real-time information between all channels is difficult to maintain. In this sense, a network game is defined where each channel attempts to minimize its individual cost function by adjusting its transmission power, in response to the others' actions. A cost function with the utility function monotone in OSNR is defined and conditions for existence and uniqueness of the Nash equilibrium (NE) solution are given in [13].

Noncooperative game approaches have been suggested in recent studies of power control in wireless communication systems, [1], [3], [6], [15]. It is shown that if an appropriate cost function and pricing mechanism are used, one can find an efficient Nash equilibrium for a multi-user network, [2].

We extend the cost function in [13] to a more general case, motivated as follows. In optical networks, there exists a threshold (saturation power) of each link in the paths of channels, such that the nonlinear effects in the span following each amplifier are small, [8]. The total launched power has to be below or equal to that threshold, which can be interpreted as a capacity constraint. Each optical amplifier in a link is operated with a target optical power (less or equal to the threshold), therefore the total launched power in the span will be equal to it. This is the condition imposed and used in formulating the network game in [13], thereby ensuring it at any intermediary site in the link. However, this condition on the total launched power in a span was not yet imposed on the source of each link (Tx) in [13]. Therefore there is no guarantee for the span between Tx and the first amplifier that the total launched power will be equal or below the threshold of the link.

In this paper, we consider this capacity constraint and impose it indirectly at Tx by adding a new term on the cost function used in [13]. We not only assume that each channel pays a price proportional to the amount of optical power it uses, but also give a regulatory function by providing each channel with the link status, i.e., how much the total power is below the target. This is valuable for the case when the sum of channel optical power approaches the available capacity, as the price will increase without bound. Hence this preserves the power resources by forcing all channels to decrease their corresponding optical power. This is similar to the case in general networks: the network provides congestion indication signals to users and allows users to adapt their transmission rates in response to network congestion, [6]. As a first step extending the cost function in [13], We consider a single link accessed by m channels with all spans having equal length.

We characterize the NE solution and consider some dynamic properties of the pricing strategies. The organization of the paper is as follows. We present the OSNR model based on [13] for a single link in the next section. In

The authors are with Systems Control Group, Department of Electrical and Computer Engineering, University of Toronto, Toronto, Ontario, M5S 3G4, Canada {yanpan,pavel}@control.utoronto.ca

Section 3, the OSNR optimization problem is formulated. The existence and uniquness of NE solution is proven in Section 4. In Section 5 we give a discussion on dynamic pricing strategies, followed by conclusions in Section 6.

II. Single Link OSNR Model

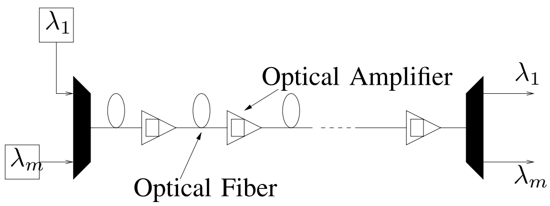

Fig. 1. Block Diagram of Optical Link

In this section, we review the OSNR model in [13] for a single link (Fig. 1). This link is composed of N cascaded optically amplified spans. There are m channels transmitted across the link. For a channel i, corresponding to wavelength λ_i, we denote by $u_i, n_{0,i}$ the input signal optical power and the input noise power. Let $\mathbf{u} = [u_1, ..., u_m]^T$ denote the vector of channel input power and $\mathbf{u}_{-i}$ be the vector obtained by deleting the i^{th} element from $\mathbf{u}$,

$$\mathbf{u}_{-i} = [u_1, ..., u_{i-1}, u_{i+1}, ..., u_m]^T.$$

Optical amplifiers are used to amplify the optical power of all channels in a link simultaneously. The gain of each channel can be compensated by equalizing channel optical powers when a different gain is experienced, because of the amplifier's non-uniform wavelength-dependent gain profile [10]. We assume that all the amplifiers in a link have the same spectral shape and are operated in automatic power control (APC) mode [14], with the total power target P_0, which we take as the optical power capacity of this link. We note that P_0 is selected to be bellow the threshold for nonlinear effects [8]. Because of the amplified spontaneous emission (ASE) noise which is also wavelength-dependent, different channels will have different OSNR. While the dispersion and nonlinearity effects are considered to be limited, the ASE noise accumulation will be the dominant impairment in our model. We make the same assumption as in [8], [13] that ASE noise power does not participate in amplifier gain saturation.

From Lemma 2 in [13], the OSNR γ_i for the i^{th} channel in a single link is given as

$$\gamma_i = \frac{u_i}{n_{0,i} + \sum_j \Gamma_{ij} u_j}, \tag{1}$$

where $\mathbf{\Gamma} = [\Gamma_{ij}]$ is the full $(m \times m)$ system matrix and Γ_{ij} may be viewed as the system gain from channel j to channel i. Or we can rewrite γ_i as

$$\gamma_i = \frac{u_i}{X_{-i} + \Gamma_{ii} u_i}, \tag{2}$$

where

$$X_{-i} = \sum_{j \neq i} \Gamma_{ij} u_j + n_{0,i}. \tag{3}$$

III. Noncooperative Game Formulation

In this section, we formulate a noncooperative game, with the cost function $J_i(u_i, \mathbf{u}_{-i})$ for each channel i being defined as the difference between a pricing and a utility function: $J_i(u_i, \mathbf{u}_{-i}) = P_i(u_i, \mathbf{u}_{-i}) - U_i(u_i, \mathbf{u}_{-i})$. Such a cost function not only sets the dynamic prices, but also captures the demand of a channel for the optical power. Each channel i minimizes its cost function J_i by adjusting the optical power u_i.

Note that in our case, both the pricing function and the utility function depend not only on u_i but also on all other channel powers.

We consider the same utility function as in [13]:

$$U_i(u_i, \mathbf{u}_{-i}) = q_i \ln(1 + k_i \frac{u_i}{X_{-i}}), \tag{4}$$

where $q_i > 0$ is channel specific parameter and $k_i > 0$ is introduced for scalability of the term $\frac{u_i}{X_{-i}}$.

This utility function U_i is twice continuously differentiable, monotone increasing and strictly concave in u_i, which is in accordance with the economic principle, law of diminishing returns, [1].

Note that we can express the utility function U_i as:

$$U_i = q_i \ln \frac{1 + (k_i - \Gamma_{ii})\gamma_i}{1 - \Gamma_{ii}\gamma_i},$$

such that U_i is monotone in γ_i. Here q_i indicates the strength of the channel's desire to maximize its OSNR, γ_i. Maximizing utility is related to maximizing channel OSNR.

The pricing function P_i indicates the current state of networks, [2]. We define this function as:

$$P_i(u_i, \mathbf{u}_{-i}) = p_i u_i + \frac{1}{P_0 - \sum_j u_j}, \tag{5}$$

where $p_i > 0$ is the pricing parameter of channel i and is determined by the network.

The pricing term not only sets the actual price for each channel, but also compared to [13], considers the link status via the extra term $\frac{1}{P_0 - \sum_j u_j}$. When the sum of optical power of channels approaches the target power P_0, the price increases without bound. Hence the power resources are preserved by forcing all channels to decrease their input power.

Therefore the cost function that the i^{th} channel will seek to minimize is

$$J_i(u_i, \mathbf{u}_{-i}) = p_i u_i + \frac{1}{P_0 - \sum_j u_j} - q_i \ln(1 + k_i \frac{u_i}{X_{-i}}). \tag{6}$$

Hence the underlying m-player noncooperative game is defined here in terms of the cost functions $J_i(u_i, \mathbf{u}_{-i})$, $i = 1, ..., m$, and the constraints

$$u_i \geq 0 \tag{7}$$

$$\sum_j u_j \leq P_0 \tag{8}$$

We denote by $\mathbf{U}$ the subset of $\mathbb{R}^m$ where power vector $\mathbf{u}$ belongs in view of the foregoing constraints (7, 8). Then the constraint set $\mathbf{U}$ is closed and bounded (therefore compact).

Note that in such a form, the term $\frac{1}{P_0 - \sum_j u_j}$ in (6), ensures that $u_i = P_0 - \sum_{j \neq i} u_j$ is not a solution to the minimization of J_i, i.e.,

$$J_i(\mathbf{u} : u_i = P_0 - \sum_{j \neq i} u_j) > J_i(\mathbf{u}), \forall \mathbf{u}, u_i \neq P_0 - \sum_{j \neq i} u_j.$$

This is a necessary condition for the NE solution to be an inner one. We make another assumption in order to guarantee that the NE solution is inner.

(A1) The i^{th} channel's cost function has the following property: at $u_i = 0$, $J_i(\mathbf{u} : u_i = 0) > J_i(\mathbf{u})$, $\forall \mathbf{u}$, $u_i \neq 0$.

We can rewrite **(A1)** as

$$q_i \ln(1 + k_i \frac{u_i}{X_{-i}}) + \frac{1}{P_0 - \sum_{j \neq i} u_j} > p_i u_i + \frac{1}{P_0 - \sum_j u_j}.$$

Hence, if q_i and p_i are selected such that $q_i \gg p_i$, **(A1)** is satisfied. This can be reached since q_i and p_i are determined by the channel and the network, respectively.

IV. Existence and Uniqueness of NE Solution

In this section, we prove existence and uniqueness of the NE solution. We will use the following mathematical preliminary results.

Definition 1 *Let A be a $m \times m$ matrix, $A = [a_{ij}]$. A is said to be diagonally dominant if*

$$|a_{ii}| \geq \sum_{j \neq i} |a_{ij}|, \forall i.$$

It is said to be strictly diagonally dominant if

$$|a_{ii}| > \sum_{j \neq i} |a_{ij}|, \forall i.$$

Lemma 1 *Let A be a $m \times m$ real matrix. The diagonal elements of A are all positive. Then A is positive definite if A and A^T are both strictly diagonally dominant.*

Proof. By Definition 1, we have $a_{ii} > \sum_{j \neq i} |a_{ij}|$ and $a_{ii} > \sum_{j \neq i} |a_{ji}|$. Thus for matrix $(A + A^T)$, we have $2a_{ii} > \sum_{j \neq i}(|a_{ij}| + |a_{ji}|) \geq \sum_{j \neq i} |a_{ij} + a_{ji}|$. Hence $2a_{ii} > \sum_{j \neq i} |a_{ij} + a_{ji}|$, such that $(A + A^T)$ is strictly diagonally dominant. From Gershgorin's Theorem, [9], it follows that all the eigenvalues of $(A + A^T)$ are real and positive, i.e., $(A + A^T)$ is positive definite. Therefore matrix A is positive definite. ∎

Lemma 2 *Let A, B, C, D and E be $m \times m$ real positive matrices with $A = B + C + D + E$. Then matrix A is full rank if B is positive semidefinite and C, D, E are positive definite.*

Now we give the main results.

Theorem 1 *Consider the m-player noncooperative game problem with individual cost function (6) and constraints (7)-(8). Such a problem admits a unique inner NE solution if k_i, q_i and p_i are selected such that, $\forall i, j$:*

$$k_i > (m - 1)\Gamma_{ij}, \, j \neq i, \tag{9}$$

$$q_{min} \leq q_i < \frac{q_{min}}{\sum_{j \neq i} \frac{\Gamma_{ji}}{k_j}}, \tag{10}$$

$$p_{max} \sqrt{q_i \sum_{j \neq i} \frac{\Gamma_{ji}}{k_j q_j}} < p_i \leq p_{max}, \tag{11}$$

where

$$q_{min} = \min_i q_i \tag{12}$$

$$p_{max} = \max_i p_i. \tag{13}$$

Proof. The proof follows the approach of [2]. We note that $\mathbf{U}$ is closed and bounded. Let $\mathbf{u}^1, \mathbf{u}^2 \in \mathbf{U}$ be two power vectors and $0 < \lambda < 1$ be a real number. We have, for any $\mathbf{u}^\lambda = \lambda \mathbf{u}^1 + (1 - \lambda)\mathbf{u}^2$, $\mathbf{u}^\lambda \leq P_0$. Thus $\mathbf{U}$ is convex, although not rectangular. Each $J_i(u_i, \mathbf{u}_{-i})$ is continuous and bounded except on the hyperplane defined by (8). Hence for a given sufficiently small $\epsilon > 0$, if we replace (8) with $\sum_j u_j \leq P_0 - \epsilon$ and denote the corresponding constraint set by $\mathbf{U}_\epsilon$. Then $\mathbf{U}_\epsilon$ is clearly compact, convex and non-rectangular. On $\mathbf{U}_\epsilon$, differentiating (6) with respect to u_i, we have

$$f_i(\mathbf{u}) := \frac{\partial J_i}{\partial u_i} = p_i + \frac{1}{(P_0 - \sum_j u_j)^2} - \frac{q_i k_i}{X_{-i} + k_i u_i}; \tag{14}$$

and differentiating $f_i(\mathbf{u})$ with respect to u_i and u_j, $j \neq i$, yields

$$A_{ii}(\mathbf{u}) := \frac{\partial^2 J_i}{\partial u_i^2} = \frac{2}{(P_0 - \sum_j u_j)^3} + \frac{q_i k_i^2}{(X_{-i} + k_i u_i)^2}, \tag{15}$$

$$A_{ij}(\mathbf{u}) := \frac{\partial^2 J_i}{\partial u_j \partial u_i} = \frac{2}{(P_0 - \sum_j u_j)^3} + \frac{q_i k_i \Gamma_{ij}}{(X_{-i} + k_i u_i)^2}, \tag{16}$$

From (15), it follows directly that $\frac{\partial^2 J_i}{\partial u_i^2}$ is well-defined and positive on $\mathbf{U}_\epsilon$. By a standard theorem of the game theory (Theorem 4.4, p.176 in [5]), this game admits a NE solution.

Furthermore, the solution has to be inner from **(A1)**, i.e. the NE solution is independent of ϵ for $\epsilon > 0$ sufficiently small. Thus it provides also a NE solution to the original game on $\mathbf{U}$.

We next show the uniqueness of the NE solution. Note that the inner solution satisfies the first-order optimality conditions, i.e., the set of equations:

$$f_i(\mathbf{u}) = 0, \; i = 1, \ldots, m.$$

Suppose that there are two Nash equilibria, represented by two optical power vectors $\mathbf{u}^1$ and $\mathbf{u}^0$, respectively. Let

$\triangle \mathbf{u} = \mathbf{u}^0 - \mathbf{u}^1$. Define the pseudo-gradient vector:

$$g(\mathbf{u}) := \begin{pmatrix} \nabla_{u_1} J_1(\mathbf{u}) \\ \vdots \\ \nabla_{u_m} J_m(\mathbf{u}) \end{pmatrix} = \begin{pmatrix} f_1(\mathbf{u}) \\ \vdots \\ f_m(\mathbf{u}) \end{pmatrix}, \qquad (17)$$

where the notation in (14) is used.

Since the NE solution is necessarily an inner solution, from the first-order optimality condition for $\mathbf{u}^1$ and $\mathbf{u}^0$, it follows that $g(\mathbf{u}^1) = 0$ and $g(\mathbf{u}^0) = 0$, i.e. $\forall i$,

$$n_{0,i} + \sum_{j \neq i} \Gamma_{ij} u_j^0 + k_i u_i^0 = \frac{q_i k_i}{p_i + \frac{1}{(P_0 - \sum_j u_j^0)^2}}, \qquad (18)$$

$$n_{0,i} + \sum_{j \neq i} \Gamma_{ij} u_j^1 + k_i u_i^1 = \frac{q_i k_i}{p_i + \frac{1}{(P_0 - \sum_j u_j^1)^2}}, \qquad (19)$$

Define an optical power vector $\mathbf{u}(\theta)$ as a convex combination of the two equilibrium points $\mathbf{u}^1$ and $\mathbf{u}^0$:

$$\mathbf{u}(\theta) = \theta \mathbf{u}^0 + (1-\theta)\mathbf{u}^1,\ 0 < \theta < 1. \qquad (20)$$

Differentiating $g(\mathbf{u}(\theta))$ with respect to θ, we get

$$\frac{dg(\mathbf{u}(\theta))}{d\theta} = G(\mathbf{u}(\theta)) \frac{d\mathbf{u}(\theta)}{d\theta} = G(\mathbf{u}(\theta)) \triangle \mathbf{u}, \qquad (21)$$

where $G(\mathbf{u})$ is the Jacobian of $g(\mathbf{u})$, with respect to $\mathbf{u}$. Using (17) together with the notation in (15, 16) yields

$$G(\mathbf{u}) := [A_{ij}] = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m} \\ A_{21} & A_{22} & \cdots & A_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ A_{m1} & A_{m2} & \cdots & A_{mm} \end{pmatrix}. \qquad (22)$$

Integrating (21) over θ, we obtain

$$0 = g(\mathbf{u}^1) - g(\mathbf{u}^0) = \left[\int_0^1 G(\mathbf{u}(\theta)) d\theta \right] \triangle \mathbf{u}. \qquad (23)$$

For simplicity, we define

$$\mathcal{G}(\mathbf{u}^1, \mathbf{u}^0) = \int_0^1 G(\mathbf{u}(\theta)) d\theta =: \left[\overline{A}_{ij} \right], \qquad (24)$$

where

$$\overline{A}_{ij} := \overline{A}_{ij}(\mathbf{u}^1, \mathbf{u}^0) = \int_0^1 A_{ij}(\mathbf{u}(\theta)) d\theta. \qquad (25)$$

Therefore, we can rewrite (23) as

$$0 = \mathcal{G}(\mathbf{u}^1, \mathbf{u}^0) \triangle \mathbf{u}. \qquad (26)$$

Note that $\triangle \mathbf{u}$ is a constant optical power vector.

We compute next $\overline{A}_{ii}$ and $\overline{A}_{ij}$ in (25).

Using (3, 15, 20) into (25) yields

$$\begin{aligned} \overline{A}_{ii} &= \int_0^1 A_{ii}(\mathbf{u}(\theta)) d\theta \\ &= \int_0^1 \Big[\frac{2}{(P_0 - \sum\limits_j u_j(\theta))^3} + \frac{q_i k_i^2}{(X_{-i}(\theta) + k_i u_i(\theta))^2}\Big] d\theta \\ &= \int_0^1 \Big[\frac{2}{(P_0 - \sum\limits_j u_j^1 - \theta \sum\limits_j \triangle u_j)^3} + \\ &\quad \frac{q_i k_i^2}{(n_{0,i} + \sum\limits_{j \neq i} \Gamma_{ij}(\theta \triangle u_j + u_j^1) + k_i u_i(\theta))^2}\Big] d\theta \end{aligned}$$

Recall (18, 19) and denote the left-hand side of (18, 19) as

$$a_i + b_i := n_{0,i} + \sum_{j \neq i} \Gamma_{ij} u_j^0 + k_i u_i^0, \qquad (27)$$

$$b_i := n_{0,i} + \sum_{j \neq i} \Gamma_{ij} u_j^1 + k_i u_i^1. \qquad (28)$$

With these notations, we can rewrite the foregoing as

$$\begin{aligned} \overline{A}_{ii} &= \int_0^1 \frac{2}{(P_0 - \sum\limits_j u_j^1 - \theta \sum\limits_j \triangle u_j)^3} d\theta \\ &\quad + \int_0^1 \frac{q_i k_i^2}{(a_i \theta + b_i)^2} d\theta \end{aligned} \qquad (29)$$

For the second integral, we get

$$\int_0^1 \frac{q_i k_i^2}{(a_i \theta + b_i)^2} d\theta = \frac{q_i k_i^2}{b_i(a_i + b_i)} \qquad (30)$$

With a_i, b_i defined in (27, 28) and using (18, 19), it follows

$$\int_0^1 \frac{q_i k_i^2 d\theta}{(a_i \theta + b_i)^2} = \frac{(p_i + \frac{1}{(P_0 - \sum_j u_j^1)^2})(p_i + \frac{1}{(P_0 - \sum_j u_j^0)^2})}{q_i} \qquad (31)$$

Therefore, computing the first integral and using (31), (29) can be written as

$$\begin{aligned} \overline{A}_{ii} &= \frac{(P_0 - \sum_j u_j^0) + (P_0 - \sum_j u_j^1)}{(P_0 - \sum_j u_j^0)^2 (P_0 - \sum_j u_j^1)^2} \\ &\quad + \frac{(p_i + \frac{1}{(P_0 - \sum_j u_j^1)^2})(p_i + \frac{1}{(P_0 - \sum_j u_j^0)^2})}{q_i} \\ &= \frac{(P_0 - \sum_j u_j^0) + (P_0 - \sum_j u_j^1)}{(P_0 - \sum_j u_j^0)^2 (P_0 - \sum_j u_j^1)^2} \\ &\quad + \frac{1}{q_i (P_0 - \sum_j u_j^0)^2 (P_0 - \sum\limits_j u_j^1)^2} \\ &\quad + \frac{p_i}{q_i} \frac{(P_0 - \sum_j u_j^0)^2 + (P_0 - \sum_j u_j^1)^2}{(P_0 - \sum_j u_j^0)^2 (P_0 - \sum_j u_j^1)^2} + \frac{p_i^2}{q_i}, \end{aligned}$$

or

$$\overline{A}_{ii} = W_1 + W_2 \cdot \frac{1}{q_i} + W_3 \cdot \frac{p_i}{q_i} + \frac{p_i^2}{q_i}, \qquad (32)$$

with

$$W_1 = \frac{(P_0 - \sum_j u_j^0) + (P_0 - \sum_j u_j^1)}{(P_0 - \sum_j u_j^0)^2 (P_0 - \sum_j u_j^1)^2},$$
$$W_2 = \frac{1}{(P_0 - \sum_j u_j^0)^2 (P_0 - \sum\limits_j u_j^1)^2},$$
$$W_3 = \frac{(P_0 - \sum_j u_j^0)^2 + (P_0 - \sum_j u_j^1)^2}{(P_0 - \sum_j u_j^0)^2 (P_0 - \sum_j u_j^1)^2}. \quad (33)$$

Note that W_1, W_2, W_3 are positive constants. Similarly for $\overline{A}_{ij}$, $j \neq i$, we can obtain, using (16, 20) into (25), that

$$\overline{A}_{ij} = W_1 + W_2 \cdot \frac{\Gamma_{ij}}{q_i k_i} + W_3 \cdot \frac{p_i \Gamma_{ij}}{q_i k_i} + \frac{p_i^2 \Gamma_{ij}}{q_i k_i}, \quad (34)$$

where W_1, W_2, W_3 are defined in (33).

Let $B_{ii} = 1$, $C_{ii} = \frac{1}{q_i}$, $D_{ii} = \frac{p_i}{q_i}$ $E_{ii} = \frac{p_i^2}{q_i}$ and $B_{ij} = 1$, $C_{ij} = \frac{\Gamma_{ij}}{q_i k_i}$, $D_{ij} = \frac{p_i \Gamma_{ij}}{q_i k_i}$, $E_{ij} = \frac{p_i^2 \Gamma_{ij}}{q_i k_i}$, for $j \neq i$. Therefore, for any i, j, $\overline{A}_{ij}$ above in (32, 34) can be written as

$$\overline{A}_{ij} = W_1 \cdot B_{ij} + W_2 \cdot C_{ij} + W_3 \cdot D_{ij} + E_{ij},$$

and matrix $\mathcal{G}(\mathbf{u}^1, \mathbf{u}^0)$ in (24) can be expressed as

$$\mathcal{G}(\mathbf{u}^1, \mathbf{u}^0) = W_1 \cdot B + W_2 \cdot C + W_3 \cdot D + E. \quad (35)$$

It is obvious to check that matrix B with $B_{ij} = 1$ is positive semidefinite. If (9) holds, it follows $k_i > \sum_{j \neq i} \Gamma_{ij}$. Hence C, D and E are all strictly diagonally dominant. If (10) holds, i.e.,

$$\frac{1}{q_i} > \sum_{j \neq i} \frac{\Gamma_{ji}}{q_{min} k_j},$$

we can obtain

$$\frac{1}{q_i} > \sum_{j \neq i} \frac{\Gamma_{ji}}{q_j k_j}, \quad (36)$$

so that C^T is strictly diagonally dominant.

Now from (36), it follows $q_i \sum_{j \neq i} \frac{\Gamma_{ji}}{q_j k_j} < 1$, such that $\sqrt{q_i \sum_{j \neq i} \frac{\Gamma_{ji}}{q_j k_j}} > q_i \sum_{j \neq i} \frac{\Gamma_{ji}}{q_j k_j}$. From (11) and the foregoing, it follows that

$$p_{max} q_i \sum_{j \neq i} \frac{\Gamma_{ji}}{k_j q_j} < p_i \leq p_{max},$$

from which the following inequality can be obtained:

$$\frac{p_i}{q_i} > \sum_{j \neq i} \frac{p_j \Gamma_{ji}}{q_j k_j}. \quad (37)$$

The above shows that D^T is strictly diagonally dominant.

Following similar arguments, it can be shown that

$$\frac{p_i^2}{q_i} > \sum_{j \neq i} \frac{p_j^2 \Gamma_{ji}}{q_j k_j}, \quad (38)$$

so that E^T is also strictly diagonally dominant. From Lemma 1, matrix C, D and E are all positive definite, and from Lemma 2, we conclude that matrix $\mathcal{G}(\mathbf{u}^1, \mathbf{u}^0)$ in (35) is full rank. From (26), it readily follows that $\triangle \mathbf{u} = 0$, i.e., $\mathbf{u}^0 = \mathbf{u}^1$. Therefore, the NE solution is unique. ∎

Remark:

When $p_i = p$ and $q_i = q$, the same price is set for channels and each channel has the same preference for OSNR. Therefore the cost function is reduced to

$$J_i(u_i, \mathbf{u}_{-i}) = p u_i + \frac{1}{P_0 - \sum\limits_j u_j} - q \ln(1 + k_i \frac{u_i}{X_{-i}}).$$

In this case, the conditions in Theorem 1 are reduced to (9) only. Compared with the condition (28) in [13], (9) is more restrictive. However, unlike [13], the cost function above guarantees that the total launched power at Tx satisfies the capacity constraint, $\sum_j u_j < P_0$.

V. Discussion

As indicated before, q_i is related to the preference for OSNR of i^{th} channel, and p_i is the pricing parameter determined by the network. We will discuss parameter selection in the following subsections.

A. Decentralized Pricing Strategy

Here, the network sets fixed prices for each channel, and channels decide their willingness (q_i) to pay to obtain satisfied OSNR levels.

First, we study the relation between q_i and the corresponding γ_i for each channel. From (2, 14), at each Nash equilibrium, we have

$$q_i(\gamma_i) = \frac{p_i}{k_i}(X_{-i} + k_i u_i(\gamma_i)) + \frac{X_{-i} + k_i u_i(\gamma_i)}{k_i (P_0' - u_i(\gamma_i))^2}, \quad (39)$$

where

$$P_0' = P_0 - \sum_{j \neq i} u_j,$$
$$u_i(\gamma_i) = \frac{X_{-i}}{1/\gamma_i - \Gamma_{ii}}.$$

For a given lower bound of OSNR, γ_i^*, we can show that if q_i is adjusted to satisfy the lower bound

$$q_i > [\frac{p_i}{k_i} \frac{1 + (k_i - \Gamma_{ii})\gamma_i^*}{1 - \Gamma_{ii}\gamma_i^*} + \frac{(1 - \Gamma_{ii}\gamma_i^*)(1 + (k_i - \Gamma_{ii})\gamma_i^*)}{k_i (P_0' - (P_0' \Gamma_{ii} + X_{-i})\gamma_i^*)^2}] X_{-i}, \quad (40)$$

each channel will achieve at least a certain OSNR level, i.e., $\gamma_i > \gamma_i^*$. Recall the lower bound in [13]: $\frac{p_i}{k_i} \frac{1 + (k_i - \Gamma_{ii})\gamma_i^*}{1 - \Gamma_{ii}\gamma_i^*} X_{-i}$, we note that the lower bound in (40) is more restrictive, due to the presence of the second term. However as mentioned above, the more general cost function considered here guarantees $\sum_j u_j < P_0$.

B. Dynamic Channel-Add

In reconfigurable optical networks, the number of channels can change dynamically. We will study the case of channel-add with two extreme strategies for channels to be added: worst-OSNR strategy and best-OSNR strategy. For simplicity, we consider $m = 2$ and $m = 3$ only.

(I) Worst-OSNR Strategy: For this case, each newly added channel i sets q_i as q_{min} and gets p_i as p_{max}, where q_{min} and p_{max} are defined in (12, 13).

Consider $m = 2$ and the 2nd channel is newly added. From (9, 10, 11), we can get the following conditions for selecting parameters of channel 2: $k_1 > \Gamma_{12}$, $k_2 > \Gamma_{21}$ and

$$\frac{\Gamma_{21}}{k_2} q_1 < q_2 \leq q_1,$$

$$p_1 \leq p_2 < p_1 \sqrt{\frac{k_2 q_2}{\Gamma_{21} q_1}}.$$

For the 3rd channel being added, we can obtain $k_1 > 2\max\{\Gamma_{12}, \Gamma_{13}\}$, $k_2 > 2\max\{\Gamma_{21}, \Gamma_{23}\}$, $k_3 > 2\max\{\Gamma_{31}, \Gamma_{32}\}$ for $m = 3$, and q_3, p_3 need to be selected such that

$$\max\{(\frac{\Gamma_{12}}{k_1} + \frac{\Gamma_{32}}{k_3}) q_2, (\frac{\Gamma_{21}}{k_2} + \frac{\Gamma_{31}}{k_3}) q_1\} < q_3 \leq \min_{i=1,2}\{q_i\}$$

$$\max_{i=1,2}\{p_i\} \leq p_3 < \min\{\sqrt{\frac{p_1^2/q_1}{\frac{\Gamma_{21}}{q_2 k_2} + \frac{\Gamma_{31}}{q_3 k_3}}}, \sqrt{\frac{p_2^2/q_2}{\frac{\Gamma_{12}}{q_1 k_1} + \frac{\Gamma_{32}}{q_3 k_3}}}\}$$

(II) Best-OSNR Strategy: For this case, each newly added channel i sets q_i as much as he/she can, and get the smallest p_i. The conditions for channel i to select k_i are same as those corresponding conditions in the worst-OSNR Strategy. So here we discuss parameter selection of q_i and p_i only.

For $m = 2$ and the 2nd channel being newly added, the conditions for q_2 and p_2 can be written as

$$q_1 \leq q_2 < \frac{k_1}{\Gamma_{12}} q_1, \tag{41}$$

$$p_1 \sqrt{\frac{\Gamma_{12} q_2}{k_1 q_1}} < p_2 \leq p_1. \tag{42}$$

For the 3rd channel being added,

$$q_1 \leq q_3 < \frac{q_1}{\frac{\Gamma_{13}}{k_1} + \frac{\Gamma_{23}}{k_2}}, \tag{43}$$

$$p_1 \sqrt{q_3 (\frac{\Gamma_{13}}{q_1 k_1} + \frac{\Gamma_{23}}{q_2 k_2})} < p_3 \leq p_1. \tag{44}$$

Note that when the 3rd channel is added, the conditions for channel 2 will be changed to

$$q_1 \leq q_2 < \frac{q_1}{\frac{\Gamma_{12}}{k_1} + \frac{\Gamma_{32}}{k_3}}, \tag{45}$$

$$p_1 \sqrt{q_2 (\frac{\Gamma_{12}}{q_1 k_1} + \frac{\Gamma_{32}}{q_3 k_3})} < p_2 \leq p_1. \tag{46}$$

Comparing (45) with (41), we can find that (41) is more restrictive. So q_2 will not be reselected, neither p_2.

The above are two possible extreme strategies. Other strategies are possible, and the basic condition for a channel to select its strategy is to meet the requirement of OSNR.

VI. Conclusions

In this paper, the framework of noncooperative game theory was used to study the OSNR optimization problem, extending the results in [13]. Motivated by the fact that the capacity threshold of a link is not imposed on the total launched power at Tx in [13], we extended the cost function by considering the status of the link. This modified cost function considers implicitly the capacity constraint. We studied the case of a single link and obtained conditions for the existence and uniqueness of the NE solution. These conditions are more restrictive compared with those in [13], however this trade-off guarantees the total launched power will not exceed the capacity threshold of a link. Under such conditions, two possible extreme strategies for the case of dynamic channel-add were studied. There are several directions for future research. One possible extension of this work is to a general multi-link configuration. Another topic of research is to analyze other strategies of setting parameters of dynamic channel-add, as well as development of real-time iterative power control algorithms, with provable convergence for general configurations.

References

[1] T. Alpcan and T. Basar, “A game-theoretic framework for congestion control in general topology networks,” *Proc. Conference on Decision and Control*, pp. 1218–1224, December 2002.

[2] ——, “Distributed algorithms for nash equilibria of flow control games,” *to appear on Annals of the International Society of Dynamic Games*, 2004.

[3] T. Alpcan, T. Basar, R. Srikant, and E. Altman, “CDMA uplink power control as a noncooperative game,” *Proc. Conference on Decision and Control*, pp. 197–202, December 2001.

[4] K. Bala and C. Brackett, “Cycles in wavelength routed optical networks,” *Journal of Lightwave Technology*, vol. 14, no. 7, pp. 1585–1594, July 1996.

[5] T. Basar and G. Olsder, *Dynamic Noncooperative Game Theory*, 2nd ed. Society for Industrial and Applied Mathematics, 1999.

[6] T. Basar and R. Srikant, “Revenue-maximizing pricing and capacity expansion in a many-users regime,” *Proc. IEEE Infocom 2002*, pp. 294–301, June 2002.

[7] A. Chraplyvy, J. Nagel, and R. Tkach, “Equalization in amplified wdm lightwave transmission systems,” *IEEE Photonics Technology Letters*, vol. 4, no. 8, pp. 920–922, August 1992.

[8] F. Forghieri, R. Tkach, and D. Favin, “Simple model of optical amplifier chains to evaluate penalties in wdm systems,” *Journal of Lightwave Technology*, vol. 16, no. 9, pp. 1570–1576, September 1998.

[9] R. Horn and C. Johnson, *Matrix Analysis*. Cambridge University Press, 1999.

[10] L. Pavel, “Control design for transient power and spectral control in optical communication networks,” *Proc. IEEE Conference Control Application, CCA*, pp. 415–422, June 2003.

[11] ——, “a μ-analysis application to stability of optical networks,” *Proc. 2004 American Control Conference*, pp. 3956–3961, June 2004.

[12] ——, “Dynamics and stability in optical communication networks: A system theory framework,” *Automatica*, vol. 40, pp. 1361–1370, 2004.

[13] ——, “Power control for OSNR optimization in optical networks: A noncooperative game approach,” *Proc. 2004 Conference on Decision and Control*, pp. 3033–3038, December 2004.

[14] R. Ramaswami and K. Sivarajan, *Optical Networks: A Practical Perspective*. Academic Press, 2002.

[15] H. Shen and T. Basar, “Differentiated internet pricing using a hierarchical network game model,” *Proc. 2004 American Control Conference*, pp. 2322–2327, June 2004.