Model-based Networked Control for Nonlinear Systems with Stochastic Packet Dropout

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Abstract— In this paper we analyze model-based networked control systems for a discrete-time nonlinear plant model, operating in the presence of stochastic dropout of state observations. The dropout is modelled as a Markov chain, and sufficient conditions for stability are provided using the stochastic version of Lyapunov's second method.

I. INTRODUCTION

In several recent works, the problem of networked control systems (NCS) has been posed and partially investigated [2], [6], [9], [13], to [15]. This new problem deals with the possibility of controlling a system remotely via a communication network and as such, instantaneous and perfect signals between controller and plant are not achievable.

In [11] a model for the networked control of linear time invariant systems was proposed. The network is modelled as a sampler placed between the plant and sensors on one side, and the controller on the other side of the network. Utilizing an approximate model of the process at the controller's side, the controller can maintain stability while receiving only periodic updates of the actual state of the plant. Whenever a new update is received, the model plant is initialized with the new information. This idea was utilized in [1], where the system evolved in discrete-time, and state updates were either received or dropped at each sample due to the effect of the network. The characterization of such a dropout is achieved through the use of Markov chain that takes on values of 0 or 1 depending on whether a sample was lost or received, respectively. Recently in [12], the initial model for a continuous-time plant and a network modelled with a fixed rate sampler was extended to bounded yet random variable sample times driven by a Markov chain.

In this paper, we present an extension of the discrete-time systems in [1] into a nonlinear setting, i.e. our plant and the model used for state estimation are both nonlinear. We model the packets dropping as a Markov chain, and obtain results that guarantee stability in a stochastic Lyapunov setting.

The paper is organized as follows: In Section II, we reformulate the model-based networked control problem in the nonlinear setting with stochastic packet dropouts. Section III treats the case where the current state of the Markov chain is dependent on the previous state, while section IV treats the independent state case. Finally in Section V some

examples are presented to illustrate the results, and some concluding remarks are given in SectionVI.

II. PROBLEM FORMULATION

In [1] a discrete-time model-based control with observation dropouts is proposed for linear discrete-time systems. Our objective in this paper is to propose a similar framework in the case of nonlinear systems, and to study the stability of the closed-loop system. As depicted in Figure 1, discrete-time model-based control is comprised of a plant with the network residing between the sensors of the plant and the actuators.
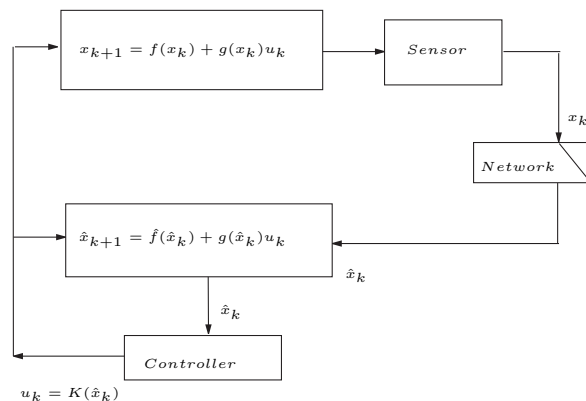

Fig. 1. Model-Based NCS

The network is modelled as a Markov chain θ_k, where a measurement is dropped if $\theta_k = 0$, and a measurement is received when $\theta_k = 1$. Due to our inability to receive an update of the plant's state at each discrete instant of time, we use an inexact model plant on the controllers side that provides us with the missing measurement. Such a model is given by

$$\hat{x}_{k+1} = \hat{f}(\hat{x}_k) + \hat{g}(\hat{x}_k)u_k. \tag{1}$$

In order to carry out the analysis, we define the estimation error as $e_k = x_k - \hat{x}_k$, and augment the state vector with e_k so that the closed-loop state vector is given by $z_k =$

$\left(x_k^T;\, e_k^T\right)^T$. The closed-loop system evolves according to

$$z_{k+1} = \begin{pmatrix} f(x_k) \\ (f(x_k) - \hat{f}(x_k)) + .. \\ (1 - \theta_k)((\hat{f}(x_k) - \hat{f}(\hat{x}_k))) \end{pmatrix}$$
$$+ \begin{pmatrix} g(x_k)K(\hat{x}_k) \\ (g(x_k) - \hat{g}(x_k))K(\hat{x}_k) + .. \\ (1 - \theta_k)(\hat{g}(x_k) - \hat{g}(\hat{x}_k))K(\hat{x}_k) \end{pmatrix}. \quad (2)$$

In the above model $\theta_k \in \{0, 1\}$ is a Markov chain that indicates the reception ($\theta_k = 1$) or the loss ($\theta_k=0$) of the packet containing the state measurement x_k. If a packet is received, it is used as an initial condition for the next time step in the model, otherwise the previous state of the model is used. We then classify the NCS errors as follows:

(I) Model structure errors

$$e_{f1}(x_k) = f(x_k) - \hat{f}(x_k) \quad (3)$$
$$e_{g1}(x_k) = g(x_k) - \hat{g}(x_k). \quad (4)$$

These are the errors between the plant and the model evaluated at the plant's state, and are therefore dependent on the system's structure.

(II) State dependent errors

$$e_{f2}(x_k, \hat{x}_k) = \hat{f}(x_k) - \hat{f}(\hat{x}_k) \quad (5)$$
$$e_{g2}(x_k, \hat{x}_k) = \hat{g}(x_k) - \hat{g}(\hat{x}_k). \quad (6)$$

These represent the errors between the model evaluated at the plant's state and at its own state, i.e. the error introduced by the difference in the states.

(III) Structure and state dependent errors

$$e_{f3}(x_k, \hat{x}_k) = f(x_k) - \hat{f}(\hat{x}_k) \quad (7)$$
$$e_{g3}(x_k, \hat{x}_k) = g(x_k) - \hat{g}(\hat{x}_k), \quad (8)$$

which include both model structure and state dependent errors.

With the new notation, the system (2) becomes

$$z_{k+1} = \begin{pmatrix} f(x_k) + g(x_k)K(\hat{x}_k) \\ e_{f1}(x_k) + e_{g1}(x_k)K(\hat{x}_k) + .. \\ (1 - \theta_k)(e_{f2}(x_k, \hat{x}_k) + e_{g2}(x_k, \hat{x}_k)K(\hat{x}_k)) \end{pmatrix}$$

Based on the value of θ_k we have two possible situations:

1. for $\theta_k = 1$ the system will be

$$z_{k+1} = \begin{pmatrix} f(x_k) + g(x_k)K(\hat{x}_k) \\ e_{f1}(x_k) + e_{g1}(x_k)K(\hat{x}_k) \end{pmatrix} \quad (9)$$

2. for $\theta_k = 0$

$$z_{k+1} = \begin{pmatrix} f(x_k) + g(x_k)K(\hat{x}_k) \\ e_{f3}(x_k, \hat{x}_k) + e_{g3}(x_k, \hat{x}_k)K(\hat{x}_k) \end{pmatrix} \quad (10)$$

For the remainder of this paper we use the following compact form to represent the system above, which also highlights the fact that θ_k represents packet dropouts,

$$z_{k+1} = H_1(z_k) + H_2(z_k)(1 - \theta_k), \; k \geq 0 \quad (11)$$

with

$$H_1(z_k) = F_1(z_k) + G_1(z_k)K(\hat{x}) \quad (12)$$
$$H_2(z_k) = F_2(z_k) + G_2(z_k)K(\hat{x}). \quad (13)$$

$$F_1(z_k) = \begin{pmatrix} f(x_k) \\ e_{f1}(x_k) \end{pmatrix} \quad (14)$$

$$F_2(z_k) = \begin{pmatrix} 0 \\ e_{f2}(x_k, \hat{x}_k) \end{pmatrix} \quad (15)$$

$$G_1(z_k) = \begin{pmatrix} g(x_k) \\ e_{g1}(x_k) \end{pmatrix} \quad (16)$$

$$G_2(z_k) = \begin{pmatrix} 0 \\ e_{g2}(x_k, \hat{x}_k) \end{pmatrix} \quad (17)$$

While the control law has no access to the plant's state, we assume in the analysis of the global system full-state availability (i.e. both x_k and $\hat{x}_k$ available). Moreover, we assume that the control law $u_k = K(\hat{x}_k)$ stabilizes the model plant and in the case of full-state availability, it also stabilizes the plant.

Next we define a particular class of NCS for which we characterize the accuracy of the model in representing the plant's dynamics, and describe how the model discrepancy affects the NCS structure.

Definition 1: A model-based NCS of the form (11), belongs to a class C_{B-NCS} with the bounds $(B_f, B_g, B_{efi}, B_{egi}; B_{h_i})$, $i = 1, 2$ if for all $k \in \mathbb{N}$ and for all $x_k \in S \subset \mathbb{R}^n$, the system structure and error norms are bounded as follows

$$||f(x_k)|| \leq B_f \quad (18)$$
$$||g(x_k)u(\hat{x}_k)|| \leq B_g(\hat{x}_k)$$
$$||e_{f1}(x_k)|| \leq B_{ef1}$$
$$||e_{f2}(x_k, \hat{x}_k)|| \leq B_{ef2}(\hat{x}_k)$$
$$||e_{g1}(x_k)u(\hat{x}_k)|| \leq B_{eg1}(\hat{x}_k)$$
$$||e_{g2}(x_k, \hat{x}_k)u(\hat{x}_k)|| \leq B_{eg2}(\hat{x}_k)$$

where B_f, B_{ef1} are constant bounds and $B_g(\hat{x}_k)$, $B_{ef2}(\hat{x}_k)$, $B_{eg1}(\hat{x}_k)$, $B_{eg2}(\hat{x}_k)$ are bounds that depend on the model state. Such NCS are called bounded model-based NCS (B-MB-NCS).

The above definition describes the class of NCS, for which it is possible to define bounds on the plant and the NCS errors, and where such bounds depend only on the model's state. Introducing such class of system overcomes the problem of unknown plant in the stability analysis of NCS, we can now reformulate the stability analysis problem, in term of accuracy of the model and amount of information lost. We want to recall the reader attention on the fact that definition (1), is not itself very restrictive, in fact is more or less restrictive depending on the choice of the bounds associated with it and on the domain S. Next we state a lemma that describes properties of class C_{B-NCS}. In particular the lemma describes how bounds on the norm of the B-MB-NCS errors imply bounds on the norm of the NCS dynamics.

Lemma 1: Consider the NCS (11) and assume the system belongs to class C_{B-NCS}. Then the following bounds hold on the norm of the NCS dynamics for $i, j = \{1, 2\}, j \neq i, k \in \mathbb{N}$ and for all $x_k \in S \subset \mathbb{R}^n$,

$$H_i^T H_j \leq B_{H_{i,j}}(\hat{x}_k) \qquad (19)$$
$$H_i^T H_i \leq B_{H_i}(\hat{x}_k)$$

where the bounds on the vector functions are related to the bounds on the errors as follows:

$$
\begin{aligned}
B_{H_1}(\hat{x}_k) = \quad & (B_f + B_g(\hat{x}_k)) + (B_{ef1} + B_{eg1}(\hat{x}_k)) \\
& + 2(B_f B_g(\hat{x}_k)) + 2(B_{ef1} B_{eg1}(\hat{x}_k)) \\
B_{H_{1,2}}(\hat{x}_k) = \quad & (B_{ef1} B_{ef2}(\hat{x}_k) + B_{eg1}(\hat{x}_k) B_{eg2}^T(\hat{x}_k)) \\
& + B_f B_g(\hat{x}_k) + (B_{ef1} B_{eg2}(\hat{x}_k) \\
& + B_{eg1}(\hat{x}_k) B_{ef2}^T(\hat{x}_k)) \\
B_{H_2}(\hat{x}_k) = \quad & (B_{ef2} + B_{eg2}(\hat{x}_k)) + 2((B_{ef2} B_{eg2}(\hat{x}_k))
\end{aligned}
$$

The proof of the above lemma can be found in [10].

Lemma 2: Consider the NCS (11), belonging to class $C_{B-NCS}(B_f, B_g, B_{efi}, B_{egi}; B_{h_i})$, $i = 1, 2$ then for all $x_k \in S \subset \mathbb{R}^n, \forall k \in \mathbb{N}$

$$\|x_k\| \leq B_x(\hat{x}) \qquad (20)$$
$$\|e_k\| \leq B_e(\hat{x})$$
$$\|z_k\| \leq B_z(\hat{x})$$

where

$$
\begin{aligned}
B_x(\hat{x}) &= B_f + B_g(\hat{x}_k) & (21) \\
B_e(\hat{x}) &= B_{ef1} + B_{eg1}(\hat{x}_k) + B_{ef2}(\hat{x}_k) + B_{eg2}(\hat{x}_k) \\
B_z(\hat{x}) &= B_x(\hat{x}) + B_e(\hat{x})
\end{aligned}
$$

Proof: The first two inequalities just follow from (2), (18). The second part trivially follows from

$$\|z_k\| = (\|x_k\| + \|e_k\|) \leq$$
$$(B_x(\hat{x}) + B_e(\hat{x})) = B_z(\hat{x}) \qquad (22)$$

■

III. STOCHASTIC DEPENDENT MODEL FOR PACKET DROP

Consider the discrete-time jump nonlinear system or Markovian Jumping System [3]

$$z_{k+1} = H(z_k, \theta_k), \ k \geq 0 \qquad (23)$$

where $\{\theta_k\}$ is a finite-state time-homogeneous Markov chain with state space S of dimension N, transition probability matrix $P = (p_{i,j})_{N \times N}$ and initial distribution $p = (p_1, \ldots, p_N)$. If the Markov chain is ergodic then it will have a unique invariant probability distribution.

Definition 2: [4] Consider the probability space (Ω, F, P), in which Ω is the sample space, F is a σ-algebra, and P is a probability measure. Let Ξ be the collection of all probability distributions on S. Then the Markovian Jumping system (23) with Markovian process $\{\theta_k\}$ described above is said to be *(asymptotically)*

mean square stable (or second moment stable) if for any $z_0 \in \mathbb{R}^n$ and $\theta_0 \in \Omega$

$$\lim_{k \to \infty} \mathbb{E}[\|z_k(z_0, \theta_0)\|^2] = 0 \qquad (24)$$

Now we proceed to analyze the stability of the system based on the described model (11), in which the variable θ_k is a Markov chain that models packet dropping. We consider two possible scenarios for the packet dropouts: in the first scenario, packet dropout is at each step dependent on the previous step according to the following conditional probabilities

$$p_{ij} = P\{\theta_k = i \mid \theta_{k-1} = j\}, \ \forall k \geq 0, \ i, j = 0, 1 \qquad (25)$$

In the second scenario, independent increments are used, i.e. $p_{10} = p_{11} = p$, $p_{00} = p_{01} = (1 - p)$, where p is the probability of getting a packet. In the following sections we will separately deal with the two described scenarios.

A. Mean Square Stability of Nonlinear NCS

We are now ready to study mean-square stability of the networked discrete-time nonlinear jump system

$$z_{k+1} = H_1(z_k) + H_2(z_k)(1 - \theta_k), \ k \geq 0 \qquad (26)$$

We will use the stochastic version of Lyapunov's second method to study stochastic stability [5], [7].

Theorem 1: Assume $\theta_k \in \Omega = \{0, 1\}$ is a two-state time homogenous Markov chain with probability transition matrix $P = (p_{i,j})$, then the discrete-time nonlinear jump system (26) is mean-square stochastically stable if there exists positive definite matrices $Q(0), Q(1)$ such that, for $i = 0, 1$,

$$
\begin{aligned}
z^T Q(i) z \quad &> \quad \sum_{j=0}^{1} p_{i,j} [H_{12}(z, i)]^T Q(j) [H_{12}(z, i)] \\
H_{12}(z, i) \quad &= \quad H_1(z) + H_2(z)(1 - i)
\end{aligned}
$$

Proof: To prove the above result we use as Lyapunov function candidate the quadratic form $V(z_k, \theta_k) = z_k^T Q(\theta_k) z_k$. Then, the discrete increment of the function will be

$$\Delta V(z_k, \theta_k) = V(z_{k+1}, \theta_{k+1}) - V(z_k, \theta_k) \qquad (27)$$

Using the stochastic version of Lyapunov's second method, the condition for the system (11) to be mean-square stochastic stable is that

$$\mathbb{E}[\Delta V(z_k, \theta_k) | z_k = z, \theta_k = i] < 0 \qquad (28)$$

or that

$$
\begin{aligned}
&\mathbb{E}[\Delta V(z_k, \theta_k); z_k = z, \theta_k = i] \\
&= \mathbb{E}[V(z_{k+1}, \theta_{k+1}) - V(z_k, \theta_k); z_k = z, \theta_k = i] \\
&= \mathbb{E}[z_{k+1}^T Q(\theta_{k+1}) z_{k+1} | z_k = z, \theta_k = i] - z^T Q(i) z \\
&= \mathbb{E}[(H_{12}(z_k, \theta_k))^T Q(\theta_{k+1})(H_{12}(z_k, \theta_k)); \\
&\quad z_k = z, \theta_k = i] - z^T Q(i) z \\
&= \mathbb{E}[(H_{12}(z, \theta_k))^T Q(\theta_{k+1})(H_{12}(z, \theta_k)); \theta_k = i] \\
&\quad - z^T Q(i) z \\
&= \sum_{j=0}^{1} p_{ij}[(H_{12})^T Q(j)(H_{12}(z, i))] \\
&\quad - z^T Q(i) z < 0 \qquad\qquad\qquad\qquad\qquad (29)
\end{aligned}
$$

which implies that

$$
\sum_{j=0}^{1} p_{ij}[(H_{12}(z, i))^T Q(j)(H_{12}(z, i))] < z^T Q(i) z,
$$
$$
i = 0, 1 \qquad\qquad\qquad\qquad\qquad\qquad (30)
$$

∎

Next we provide some sufficient conditions in order to guarantee stability in probability for system (26).

IV. STOCHASTIC INDEPENDENT MODEL FOR PACKET DROP

Next we consider the case in which the dropping of packets is independent from instant to instant so that the Markov chain θ_k is actually a binary independent random process. In this case we have $p_{10} = p_{11} = p$, $p_{00} = p_{01} = (1-p)$.

Theorem 2: Consider the system described in (11)

$$
z_{k+1} = H_1(z_k) + H_2(z_k)\varphi_k, \; k \geq 0 \qquad (31)
$$

Where H_1, H_2 are defined as before. Assume $\varphi_k = (1 - \theta_k)$ is an invariant random sequence of independent elements, with mean $\mu_\varphi = (1-p) = q$. Also assume that such NCS belongs to class C_{B-NCS} with the bounds $(B_f, B_g, B_{efi}, B_{egi}; B_{h_i})$, $i = 1, 2$. Then the discrete-time nonlinear system (26) is second moment stochastically stable if we have

$$
B_{h1} + B_{h2}q + (B_{h12} + B_{h21})q \leq B_z(\hat{x}) \qquad (32)
$$

Proof: To prove the above result we use once again the stochastic version of Lyapunov's second method [7] with a Lyapunov function candidate the quadratic form $V(z_k, \theta_k) = z_k^T z_k$. Then the discrete increment of the function will be

$$
\Delta V(z_k, \theta_k) = V(z_{k+1}, \theta_{k+1}) - V(z_k, \theta_k)
$$

The system (11) is mean-square stochastic stable if

$$
\mathbb{E}[\Delta V(z_k, \theta_k)] \leq 0, \; \forall k = 0, 1, 2, \ldots
$$

evaluating $V(z_k)$ we obtain

$$
\begin{aligned}
&\mathbb{E}[\Delta V(z_k, \theta_k)] = \mathbb{E}[V(z_{k+1}, \theta_{k+1}) - V(z_k, \theta_k)] \\
&= \mathbb{E}[z_{k+1}^T z_{k+1}] - z_k^T z_k \\
&= \mathbb{E}[(H(z_k, \theta_k))^T (H(z_k, \theta_k))] - z_k^T z_k \\
&= \mathbb{E}[(H_1(z_k))^T (H_1(z_k)) + (H_2(z_k))^T (H_2(z_k))\varphi_k^2 + \\
&\quad H_1(z_k)^T H_2(z_k) + H_2(z_k)^{(}H_1(z_k)\varphi_k] - z_k^T z_k
\end{aligned}
$$

evaluating the expectation and using the bounds on the we obtain the following

$$
\begin{aligned}
&\mathbb{E}[\Delta V(z_k, \theta_k)] \leq \\
&B_{h1} + B_{h2}q + (B_{h12} + B_{h21})q - B_z(\hat{x}) \quad (33)
\end{aligned}
$$

which, along with the assumptions (32) gives

$$
\mathbb{E}[\Delta V(z_k, \theta_k)] \leq 0, \; \forall z_k \in \mathbb{R} \qquad (34)
$$

and therefore the mean-square stability of the system follows. ∎

V. NUMERICAL EXAMPLES

In this section we present three examples to illustrate the main results of this paper. The stabilizability of the system through the network using a model-based approach is mainly dependent on two factors: the number of packets received to reconstruct the state, and the precision with which the model approximates the original plant. Each of the examples studies a particular model, and for each model, different packets-dropping probability are considered. In the first example we use an exact model of the original plant, and study how the system reacts to different packets-dropping probabilities. In the second example the original model is perturbed in amplitude, and again the stability of the system is investigated for different values of the probability of dropping a packet. Finally, the last example analyzes the case of a polynomial model that approximates the original system locally around the origin. We will show how in this last case the performance of the system degenerates due to the approximation error between the model and the original plant.

A. Example 1: Exact Model

The first example is based on a model that exactly reproduces the plant. Our goal is to show how the dropped packets affect the controller performance. Consider the plant

$$
x_{k+1} = sin(x_k) + e^{(-x_k)} u_k \qquad (35)
$$

and the model

$$
\hat{x}_{k+1} = sin(\hat{x}_k) + e^{(-\hat{x}_k)} u_k \qquad (36)
$$

We study the stability of the system for different values of the packet-dropping probability. With the previous assumption and considering that $K(\hat{x}) = -.53\hat{x}_k$ stabilizes the system in case of full information, we apply theorem 2 to obtain that in case of exact model the errors between the model and the plant are zeros and the error on the state is zero as well. In this case it is irrelevant whether there is or not packet dropping as we can see in Figure 2.
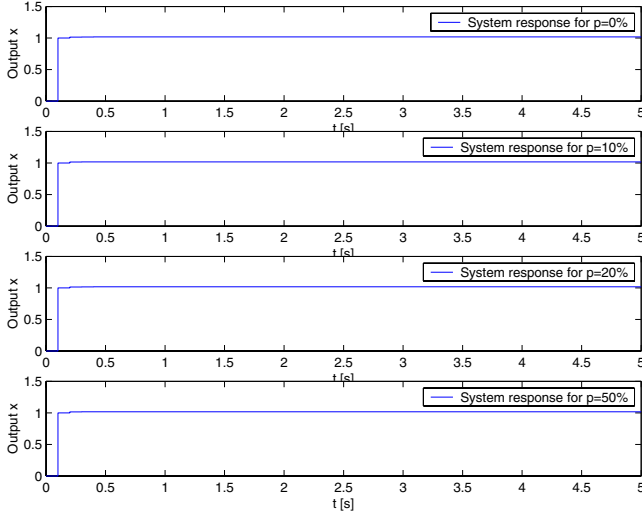
Fig. 2. System response for different values of p with exact model.

Fig. 3. System response for different value of p with non-polynomial model.

B. Example 2: Model with Parameter Variation

The next step is to investigate how imperfections in the model affect the stability of the system. At first we will consider a change only in the amplitude of the model. Consider the plant

$$x_{k+1} = sin(x_k) + e^{(-x_k)}u_k \qquad (37)$$

and the model

$$\hat{x}_{k+1} = 0.7sin(\hat{x}_k) + 0.3e^{(-\hat{x}_k)}u_k \qquad (38)$$

We then proceed to study the inequality $\mathbb{E}[\Delta V(z_k, \theta_k)] \leq 0$ for different values of q. In Figure 3 the plots resulting from the system simulation show how the system stability degenerate as the information became limited (50% of packets dropped).

As we can see from the plots in Figure 3 the resulting closed-loop system with the chosen controller,is robust with respect to variations in the model, up to 20% of information loss.

C. Example 3: Polynomial Approximation Model

We explore next how the controller performs in the case of a polynomial approximation of the original system. Since the approximation is not global, it is not reasonable to expect global stability results. Consider the plant

$$x_{k+1} = sin(x_k) + e^{(-x_k)}u_k \qquad (39)$$

and the model

$$\hat{x}_{k+1} = \hat{x}_k + u_k \qquad (40)$$

As can be inferred from the plots in Figure 4, the system can be stabilized through the network with no packet dropouts by using a polynomial model. In the presence of 10% and 20% packet dropouts the stability of the system starts to degenerate and with 50% of packet dropouts
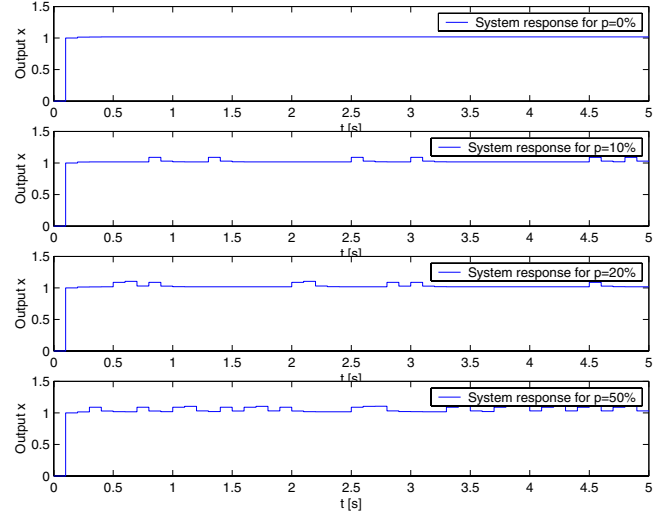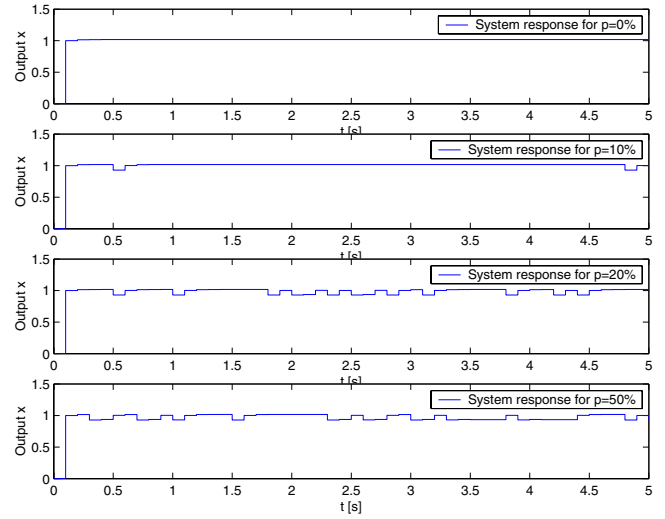

Fig. 4. System response for different value of p with polynomial model.

stability is lost before one second elapses. This result is expected since the polynomial model only approximates the real system in a neighborhood of the origin.

VI. CONCLUSION

We presented stochastic stability analysis results for discrete-time nonlinear model based NCS. The main result focuses on mean-square stability of the networked control system. In one case, we considered the packet dropouts modelled as a Markov chain and sufficient conditions for mean-square stability of the NCS were obtained by using a discrete stochastic version of Lyapunov second method. Then the case of independent packet dropout was considered and its stability analysis was carried out. Several examples were provided to illustrate our results. In future work we aim to extend the proposed results to the design of

stabilizing compensators, and to analytically show how the packets loss affects the stability of the closed-loop system. We also want to investigate the finite-time stability of the networked system. Finally, we will consider a deterministic model for the packet drop, so that the network dynamics may be included in the model and the effects of network traffic investigated.

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