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An observer and an integrated braking/traction and steering control for a cornering vehicle

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Abstract— **This paper presents an observer of the lateral velocity and an integrated control combining steering and braking/traction in order to improve vehicle dynamics performance and stability. The control is a multivariable regulation of the yaw rate, the longitudinal and lateral velocities. Standard specifications on vehicle dynamics are shown to be met by the proposed controller. The results are illustrated by means of simulations which use a quite complete nonlinear vehicle dyanamics model.**

I. Introduction

Car safety has been the object of many studies in the last decades. These works have produced many efficient safety components, including the "Direct Stability Control" (DSC). The latter aims aims at stabilizing the vehicle motion by controlling the yaw moment generated as a result of the difference in the braking tire forces between left and right. The performances of the DSC for straight-running stability, cornering stability and handling have been greatly enhanced. In general, the DSC is designed as a local function for the braking subsystem by ignoring the strong nonlinear multivariable coupling between the vehicle states. However, to effectively enhance the vehicle handling, performance and stability it is necessary to consider the aforementioned coupling. For the lateral dynamics control problem, two variables, namely the lateral velocity or side-slip angle and the yaw rate, must be taken into account by the control system. Using the DSC only, the control system gives priority to either one of the states. These are roughly some of the motivations of research on integrated chassis control systems, see for instance [1], [2], [3], [4]. Global multivariable strategies are investigated in order to efficiently control the various chassis control subsystems. There is a favorable situation used in the present paper: the combination of steering and braking/traction makes invertible some relationship between the inputs and outputs of the system. Regarding the recent developments on the mechatronics of steering and braking subsystems, it is possible to control the traction and braking forces more precisely and independently. These recent developments also offer the possibility of implementing sophisticated automatic control algorithms. Therefore, combining the active front steering (AFS) and DSC were proposed and provides superior performance. However, one of the main assumptions made was the accessibility to the lateral velocity measurement for the feedback, which is a difficult and a tricky task to do. In this paper we propose an observer of the lateral velocity based upon usually available measurements, namely the lateral acceleration and the yaw rate with an approximation of the longitudinal velocity to control the three states. A simple model of a vehicle with a mass distribution assumption is used for derivation of a generic integrated control for steering and braking. The sole nonlinearities considered in the simplified control model are those produced by the coupling of the different states In particular we use a linear model of the lateral forces dynamics. The feedback is designed such that tracking is satisfied and stability is improved. The actuators saturation limits are not explicitly used by the controller but the design keeps in mind their existence. Another main contribution is to show that for a class of nonlinear systems, the use of the observer to impose the dynamic of the system could be an interesting solution.

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The paper is organized as follows. The simplified vehicle dynamics model used throughout the paper is presented in the next section. In the section that follows we present the observer for the lateral velocity. In Section IV we provide details of the control. Numerical simulations with a complete nonlinear vehicle dynamics model are presented throughtout the previous two sections to show the effectiveness of the proposed integrated control.

II. The vehicle dynamics model

Complete vehicle dynamics models are well known to include hard nonlinearities which are difficult and to describe and which would result in extremely complex equations. These nonlinearities are essentially due to coupling terms between various variables and by contact forces acting between the wheels and the road. The expressions of the latter are known to be strongly nonlinear and complex. In this part we will present the equations of the model used in the synthesis and those used in simulations. The differential equations which describe the motion of the chassis with respect to a frame with origin at the car center of gravity are given by equations (1), where v_x, v_y and v_ψ are the states of our system, F_x the total longitudinal force applied to the center of gravity, F_yf and F_yr are the front and rear lateral tire forces, M the yaw moment generated by the longitudinal forces, m denotes the total mass of the vehicle, L_f and L_r are the distances of the front and rear axles to the center of gravity, I_z is the yaw moment of inertia.

$$\begin{cases} \dot{v}_{\mathrm{x}} & = & \dfrac{F_{\mathrm{x}}}{m} + v_{\mathrm{y}} v_{\psi} \, , \\ \dot{v}_{\mathrm{y}} & = & \dfrac{F_{\mathrm{yf}}}{m} + \dfrac{F_{\mathrm{yr}}}{m} - v_{\mathrm{x}} v_{\psi} \, , \\ \dot{v}_{\psi} & = & \dfrac{L_{\mathrm{f}}}{I_{\mathrm{z}}} F_{\mathrm{yf}} - \dfrac{L_{\mathrm{r}}}{I_{\mathrm{z}}} F_{\mathrm{yr}} + M \, . \end{cases} \tag{1}$$

A. Pneumatic forces

The pneumatic forces represent a system which is extremely hard to model. There seems to be no definite analytical model. As common in the literature, we shall use the empirical Pacejka "magic formulae". The forces F_{x} and F_{y} are presented as functions of the slip σ and the sideslip β angles. In all simulations which will be presented, the model used is the one in (1) where F_{x}, F_{yf} and F_{yr} are as follows

$$\begin{aligned} F_{\mathrm{x}} \;=\; & D_{\mathrm{x}} \sin\Big(c_{\mathrm{x0}} \arctan\Big(B_{\mathrm{x}} (1 - E_{\mathrm{x}}) \sigma + \\ & E_{\mathrm{x}} \arctan\left(B_{\mathrm{x}} \, \sigma \right) \Big) \Big) \end{aligned} \tag{2}$$

$$\begin{aligned} F_{\mathrm{yf}} \;=\; & D_{\mathrm{yf}} \sin\Big(c_{\mathrm{yf0}} \arctan\Big(B_{\mathrm{yf}} \left(1 - E_{\mathrm{yf}}\right) \beta_{\mathrm{f}} + \\ & E_{\mathrm{yf}} \arctan\left(B_{\mathrm{yf}} \, \beta_{\mathrm{f}} \right) \Big) \Big) \end{aligned} \tag{3}$$

$$\begin{aligned} F_{\mathrm{yr}} \;=\; & D_{\mathrm{yr}} \sin\Big(c_{\mathrm{yr0}} \arctan\Big(B_{\mathrm{yr}} \left(1 - E_{\mathrm{yr}}\right) \beta_{\mathrm{r}} + \\ & E_{\mathrm{yr}} \arctan\left(B_{\mathrm{yr}} \, \beta_{\mathrm{r}} \right) \Big) \Big) \end{aligned} \tag{4}$$

where D, B, E, c_0 are pneumatic parameters, σ is the longitudinal slip which is define by

$$\sigma = \frac{r\omega - v_{\mathrm{x}}}{v_{\mathrm{x}}} \tag{5}$$

where r and ω are the radius and the angular velocity of the wheels. The quantities β_{f} and β_{r} are the sideslip angles of the car and their expressions are

$$\beta_{\mathrm{f}} = \delta - \arctan \frac{v_{\mathrm{y}} + L_{\mathrm{f}} v_{\psi}}{v_{\mathrm{x}}}$$

$$\beta_{\mathrm{r}} = - \arctan \frac{v_{\mathrm{y}} - L_{\mathrm{r}} v_{\psi}}{v_{\mathrm{x}}}$$

where δ the steering angle.

B. The simplified control model

The observer and the controller which are designed in this work use a simplified model based upon a linearisation of the lateral forces with respect to the sideslip angle. The lateral forces have the following expressions

$$\begin{cases} F_{\mathrm{yf}} = C_{\mathrm{f}} \beta_{\mathrm{f}} \\ F_{\mathrm{yr}} = C_{\mathrm{r}} \beta_{\mathrm{r}} \end{cases}$$

We also assumed small deviations such that

$$\tan(\beta_{\mathrm{f}} - \delta) = \beta_{\mathrm{f}} - \delta$$

and

$$\tan(\beta_{\mathrm{r}}) = \beta_{\mathrm{r}} \, .$$

This is true for a small steering input, and for a complete steering angle

$$\delta = \delta_{\mathrm{d}} + \Delta\delta,$$

which is the sum of the driver steering angle and the additional steering angle contributed by the control. We thus have a model that we write as follows

$$\begin{cases} \dot{x} = A(v_{\mathrm{x}}) x + B_1 \, \delta_{\mathrm{d}} + B_2 \, u \\ y = C(v_{\mathrm{x}}) x + D_1 \, \delta_{\mathrm{d}} + D_2 u \end{cases} \tag{6}$$

Here,

$$x = \begin{pmatrix} v_{\mathrm{x}} \\ v_{\mathrm{y}} \\ v_{\psi} \end{pmatrix}, \quad u = \begin{pmatrix} F_{\mathrm{x}} \\ \Delta\delta \\ M_{\mathrm{x}} \end{pmatrix}, \quad y = \begin{pmatrix} v_{\mathrm{x}} \\ a_{\mathrm{y}} \\ v_{\psi} \end{pmatrix}$$

$$A(v_{\mathrm{x}}) = \begin{pmatrix} 0 & v_{\psi} & 0 \\ 0 & -\dfrac{C_{\mathrm{f}} + C_{\mathrm{r}}}{m v_{\mathrm{x}}} & -v_{\mathrm{x}} - \dfrac{C_{\mathrm{f}} L_{\mathrm{f}} - C_{\mathrm{r}} L_{\mathrm{r}}}{m \, v_{\mathrm{x}}} \\ 0 & -\dfrac{C_{\mathrm{f}} L_{\mathrm{f}} - C_{\mathrm{r}} L_{\mathrm{r}}}{I_{\mathrm{z}} v_{\mathrm{x}}} & -\dfrac{C_{\mathrm{f}} L_{\mathrm{f}}^2 + C_{\mathrm{r}} L_{\mathrm{r}}^2}{I_{\mathrm{z}} v_{\mathrm{x}}} \end{pmatrix}$$

$$B_1 = \begin{pmatrix} 0 \\ \dfrac{C_{\mathrm{f}}}{m} \\ \dfrac{C_{\mathrm{f}} L_{\mathrm{f}}}{I_{\mathrm{z}}} \end{pmatrix}, \quad B_2 = \begin{pmatrix} \frac{1}{m} & 0 & 0 \\ 0 & \dfrac{C_{\mathrm{f}}}{m} & 0 \\ 0 & \dfrac{C_{\mathrm{f}} L_{\mathrm{f}}}{I_{\mathrm{z}}} & 1 \end{pmatrix}$$

$$C(v_{\mathrm{x}}) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -\dfrac{C_{\mathrm{f}} + C_{\mathrm{r}}}{m v_{\mathrm{x}}} & -\dfrac{C_{\mathrm{f}} L_{\mathrm{f}} - C_{\mathrm{r}} L_{\mathrm{r}}}{m \, v_{\mathrm{x}}} \\ 0 & 0 & 1 \end{pmatrix},$$

$$D_1 = \begin{pmatrix} 0 \\ \dfrac{C_f}{m} \\ 0 \end{pmatrix}, \quad D_2 = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \dfrac{C_f}{m} & 0 \\ 0 & 0 & 0 \end{pmatrix}.$$

III. The lateral velocity observer

A. Design of the observer

In practice, it is very hard to obtain measurements of the lateral velocity which plays an important role in the stability of the vehicle. Moreover, this variable must be regulated to improve the performances of the car. In this section we propose an observer as a solution to the problem and we shall use it in the next section to control the motion of the chassis.

We now consider the system

$$\dot{z} = A(x)z + B_2 v \tag{7}$$

for which we note that $A(x)$ contains only the measurable variables, namely v_x and v_ψ, and that the matrix B_2 is invertible. This is one of the advantages of the integration of the braking and steering. In practice, the measurement of v_x is very hard to obtain too, but in this paper, we will use an approximation of this velocity with the measurement of the longitudinal acceleration and angular velocities of the wheels. We know that

$$a_x = \frac{1}{m} F_x = \sum_1^4 F_{xi}$$

and

$$F_{xi}(\sigma_i) = \frac{\partial F_{xi}}{\partial \sigma_i}\Big|_{\sigma_i = 0} \sigma_i = C_{xi}\sigma_i$$

which is true for a small value of the slip σ and from equation (5) we have the following approximation of the longitudinal velocity:

$$v_x \simeq \frac{\sum_1^4 C_{xi} r_i \omega_i}{a_x m + \sum_1^4 C_{xi}} \tag{8}$$

It should be noted that this approximation can only be used at small steering angle. We consider the following control law

$$v = u + B_2^{-1} K(y - D_1 \delta_d - D_2 u - C(x)z) \tag{9}$$

with $K = (A - A_d)C(x)^{-1}$ and A_d given by

$$A_d = \begin{pmatrix} -\lambda_1 & 0 & 0 \\ 0 & -\lambda_2 & 0 \\ 0 & 0 & -\lambda_3 \end{pmatrix} \tag{10}$$

Here, λ_1, λ_2, and λ_3 are positive numbers. We note that the matrix $C(x)$ is invertible because we consider the case where the longitudinal velocity is always different from zero, and we know that the sum of the stiffness coefficients is different from zero, too. Under this condition system (7) represents an observer of system (6) and the observation error $e = x - z$ has linear dynamics of the form:

$$\dot{e} = A_d e$$

We conclude that $z = \widetilde{x}$. Note that we use the measurements of the lateral acceleration and the yaw rate together to design the observer in order to simplify the design of the controller given in the next section, but we might have used only one of the two measurements to observe the components of the lateral motion, for more details, see [5], [6].

B. Simulations

The vehicle model used in these simulations is a quite complete nonlinear as introduced in Section II. In particular, the tire forces were computed using the Pacejka formulae (2), (3), and (4). We illustrate the performance of the observer through the next four figures: Fig. 1, 2, 3, and 4. The first one, Fig. 1, shows the simulated driver steering angle at the road wheel. Fig. 2 shows how well the estimation of the longitudinal velocity using the wheel angular speed and the longitudinal acceleration given by equation (8) is.

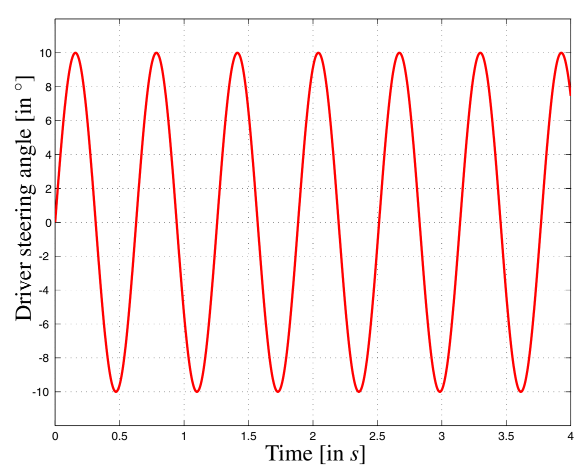

Fig. 1: The simulated driver steering angle is a sinusoid of amplitude 10 degrees.

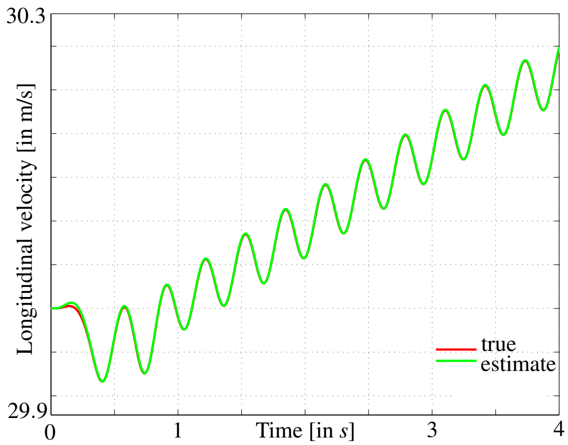

Fig. 2: Performance of the observer: the longitudinal velocity is particularly well estimated.

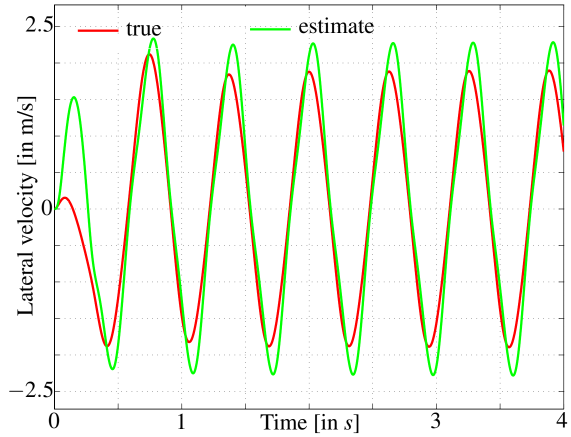

Fig. 3: Performance of the observer: as noticeable the observer tends to overestimate the lateral velocity. It also exibits some small delay.

IV. The integrated controller

A. The desired model

As clear, the lateral velocity has to be maintained as small as possible. In other words v_{y} should be steered to 0 by the control. At the same time the longitudinal velocity has to be maintained at a constant value. The yaw rate, instead of the reference model used in [3], is supposed here to follow the model below which is traced back to works by J. Ackermann:

$$v_{\psi d} = \frac{a v_{\mathrm{x}}}{1 + b v_{\mathrm{x}}^2}\delta \tag{11}$$

Here a and b are given by:

$$a = \frac{1}{L_{\mathrm{f}} + L_{\mathrm{r}}}, \quad b = \frac{m}{(L_{\mathrm{f}} + L_{\mathrm{r}})^2}\left(\frac{L_{\mathrm{r}}}{C_{\mathrm{f}}} - \frac{L_{\mathrm{f}}}{C_{\mathrm{r}}}\right).$$

B. Controller design

The feedback compensator, u, is designed to minimize the deviation of the state variables from their desired values. We

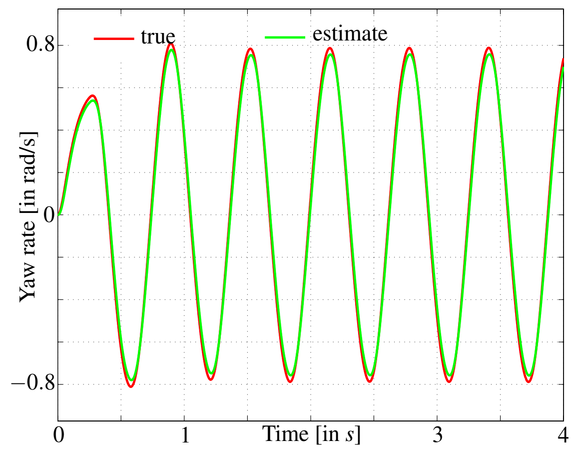

Fig. 4: Performance of the observer: the yaw rate is quite well estimated.

show that we can do that by combining the two actuators and by using the only available measurements. Specifically, we impose new dynamics to the state deviation. In other words, even if we do not directly measure the lateral velocity, we may shape the dynamics of the lateral velocity and guarantee the asymptotic convergence to the desired value. We see that if the control law v have the form given by equations (9) then z converges to x and will be an observer. Conversely if u satisfies the following condition:

$$u = \left(I - B_2^{-1} K\, D_2\right)^{-1}\left[v - B_2^{-1} K\left(y - D_1 \delta_{\mathrm{d}} - C(x) z\right)\right] \tag{12}$$

where K is given by equation (10), then x converges to z. In other words we compute v to make z converge towards x_{ref} and from equation (12) we compute u to guarantee the convergence of x to z and consequently to x_{ref}. We note that in the expression of u, we use only the available measurements and the states of the observer. We have:

$$\begin{cases} \dot{e} = (A(x) - K C(x)) e \\ \dot{z} = A(x) z + B_2 v \end{cases} \tag{13}$$

Taking v as

$$v = B_2^{-1}(-A(x) z + \overline{A}_{\mathrm{d}}(z - x_{\mathrm{ref}}))$$

where $\overline{A}_{\mathrm{d}}$ can be any diagonal matrix with negative elements we obtain decoupled dynamic for z, with

$$\overline{A}_{\mathrm{d}} = \begin{pmatrix} -\sigma_1 & 0 & 0 \\ 0 & -\sigma_2 & 0 \\ 0 & 0 & -\sigma_3 \end{pmatrix}$$

C. Proof of stability

We consider the following Lyapunov function:

$$V = \frac{1}{2}\widetilde{e}'\widetilde{e}$$

where $\widetilde{e} = x - x_{\mathrm{ref}}$, we know that:

$$\widetilde{e} = x - z + z - x_{\mathrm{ref}} = e + \overline{e}$$

where $\overline{e} = z - x_{\mathrm{ref}}$. We have

$$V = \frac{1}{2}\sum_{1}^{3}(e_i + \overline{e}_i)^2$$

The derivative of this function is:

$$\dot{V} = \sum_{1}^{3} \dot{e}_i e_i + \dot{\overline{e}}_i \overline{e}_i + \dot{e}_i \overline{e}_i + e_i \dot{\overline{e}}_i$$

where the two inputs u and v are given by equations (12) and (13).

$$\dot{V} = -\sum_{1}^{3} \lambda_i e_i^2 + \sigma_i \overline{e}_i^2 + (\lambda_i + \sigma_i)\, e_i \overline{e}_i$$

A sufficient condition of stability is

$$2\sqrt{\lambda_i \sigma_i} = \lambda_i + \sigma_i$$

that is, $\lambda_i = \sigma_i$. For this choice of the controller parameters we have

$$\dot{V} = \sum_{1}^{3} -\lambda_i \left(e_i + \overline{e}_i\right)^2$$

which proves the stability.

D. Simulations

We use the same model as in the previous simulations. Here we illustrate the performance of the observer/controller for the standard specifications introduced previously. See Fig. 5, 6, 7, and 8. The driver steering angle is the same as one previously used in [7].

The next 3 figures (Fig. 9, 10, and 11) show how the controller manages to keep the vehicle's trajectory close to the specified one when, in addition to the driver steering angle of Fig. 5, the driver steps in the brake pedal. The brake action starts at $t = 1.5$ s and a constant acceleration of $a_{\mathrm{x}} = -5\ \mathrm{m/s}^2$ is assumed. In this case the desired value of the longitudinal velocity takes the following value

$$v_{\mathrm{xd}} = \int a_{\mathrm{x}} + v_{\mathrm{x0}} \tag{14}$$

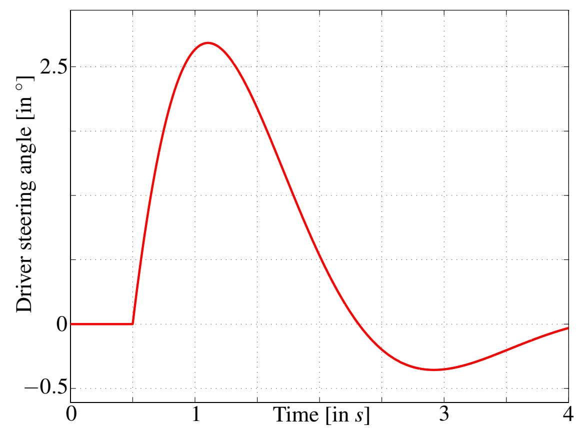

Fig. 5: The driver steering angle

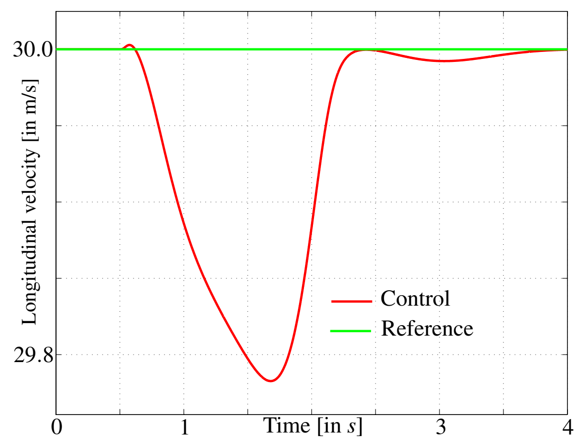

Fig. 6: Performance of the observer/controller: the longitudinal velocity is maintained at $30\,\mathrm{m/s}$ in about $3\,\mathrm{s}$ and its variation is under $0.3\,\mathrm{m/s}$ (about $1\,\mathrm{km/h}$).

V. Concluding remarks

The proposed integrated control of steering and braking/traction allows one to regulate the three variables of the chassis by using only the available measurements, which are the lateral acceleration and the yaw rate, and an approximation of the longitudinal velocity. This controller yields quite good performances by imposing desired dynamics to the state deviations. The design rests on a simple nonlinear control model of the vehicle with a quite reasonable mass distribution assumption. Such results would be hard to obtain in different strategies where, for instance, only braking/traction is used, or only steering is used. In other words, integrated steering and braking/traction control, here, eases the design contrary to what might have been thought at first place.

References

[1] J. Ackermann, "Robust car steering by yaw rate feedback," in *Proceedings of the Inetrnational Workshop on Robust Control, San Antonio, TX*, 1991.

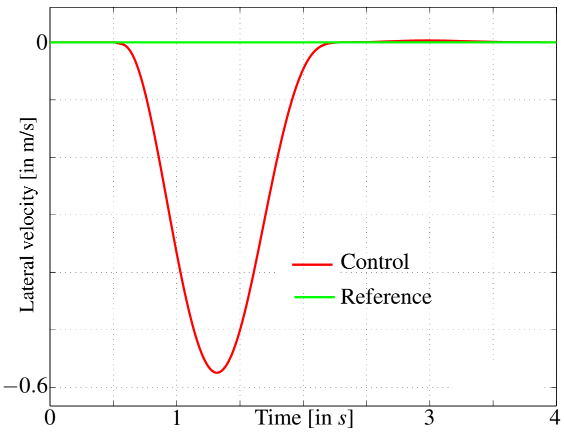

Fig. 7: Performance of the observer/controller: the lateral velocity is also stabilized in about 1 s, and its variation during the maneuver is less than 0.6 m/s.

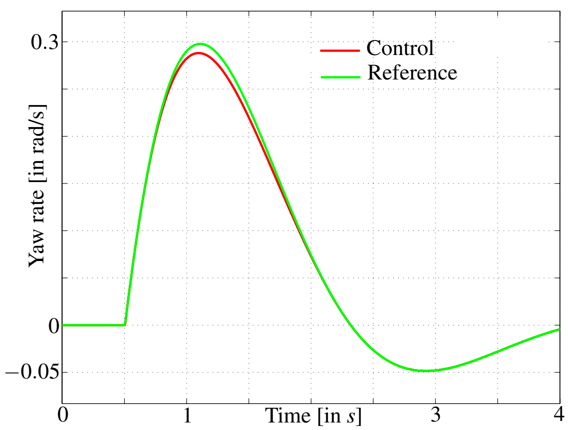

Fig. 8: Performance of the observer/controller: the yaw rate follows its prescribed reference very accurately.

[2] M. Lakehal-Ayat, S. Diop, E. Fenaux, F. Lamnabhi-Lagarrigue, and F. Zarka, "On global chassis control: combined braking and cornering and yaw rate control," in *Proceedings of the Intenational Symposium on Advanced Vehicle Control, AVEC'2000*, 2000, Paper 071.

[3] M. Shino, P. Raksincharoensak, and M. Nagai, "Vehicle handling and stability control by integrated control of direct yaw moment and active steering," in *Proceedings of the Intenational Symposium on Advanced Vehicle Control, AVEC'2002*, 2002, Paper 052.

[4] H. Smakman, "Functional integration of slip control with active suspension for improved lateral vehicle dynamics," Ph. D. dissertation, Delft University of Technology, Delft, The Nethelands, 2000.

[5] H. Cherouat and S. Diop, "Vehicle velocity estimation and vehicle body side slip angle and yaw rate observer," to appear in *Proc. IEEE Internat. Symposium Industrial Electronics, ISIE 2005, Mini Track On Automotive*, 2005.

[6] U. Kiencke and L. Nielsen, *Automotive Control Systems For Engine, Driveline, and Vehicle*. New York: Springer-Verlag, 2000.

[7] H. Cherouat, M. Lakehal-Ayat, and S. Diop, "An integrated braking and steering control for a cornering vehicle," in *Proceedings of the Intenational Symposium on Advanced Vehicle Control, AVEC'2004*, 2004, Paper 056_Diop.

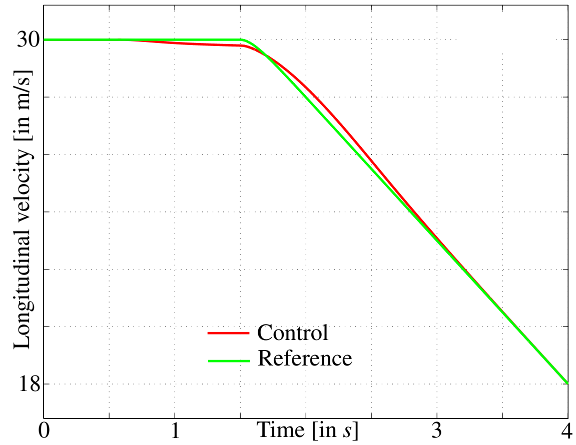

Fig. 9: Performance of the observer/controller: the longitudinal velocity converges to the new desired value given by equation (14).

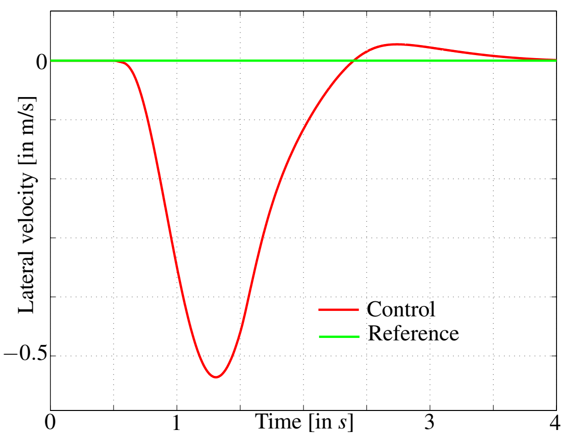

Fig. 10: Performance of the observer/controller: the influence of the brake action on the lateral velocity regulation is minimized, compare with Fig. 7.

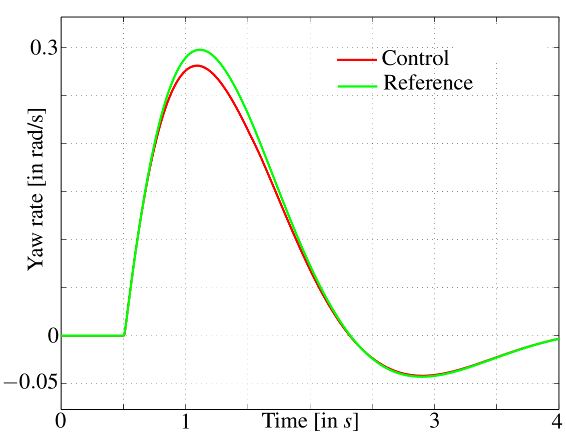

Fig. 11: Performance of the observer/controller: the influence of the brake action on the yaw rate regulation is also minimized. Compare with Fig. 8.