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A New Method for Adaptive Brake Control

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Abstract— **We consider the problem of minimizing the longitudinal braking distance for a single wheel rolling along a surface with unknown tyre-road characteristics. The friction coefficient is modelled by a nonlinear function of slip and tyre-road parameter which corresponds to the actual road conditions. Our method is based on recently proposed adaptive control technique that uses adaptation algorithms in integro-differential, or finite form. These algorithms are capable of dealing with nonlinear parametrizations, and they also ensure improved transient performance of the controlled system. We show that, for a class of practically relevant parameterizations of friction curves, it is possible to steer the system adaptively to the desired state without invoking sliding-mode or gain-scheduling control. At the same time we show that it is possible to estimate the optimal value of the tyre slip ensuring maximal braking force. These estimates, produced by essentially the standard PI algorithm, are used in the control loop to enhance efficiency of the brakes.**

I. INTRODUCTION

Effective wheel-slip control during braking/traction of a vehicle is one of the long-standing issues in the automotive industry. The problem dates back to the early 1947, when the first anti-lock braking systems where designed and implemented in B-47 bombers. Subsequent developments in automotive anti-lock braking systems promoted the understanding that not only anti-locking regimes are important but maintaining the optimal slip is highly desirable as well.

A fairly large number of publications is available, addressing the problems of road-dependent friction curve identification (see, for example, [7], [15], [10]) as well as robust slip control [14], [8], [9]. From a control theoretic point of view, the most advantageous strategy would be to combine identification of the tyre-road conditions with an adaptive/robust controller which calculates and ensures the optimal slip in the system [17]. The ideal controller should also be able to guarantee the desired dynamics of the wheel without the chattering in the brakes, and should prevent frequent large spikes of the braking torque.

As a candidate for a slip controller potentially replacing the existing robust control schemes [9], [3], one might consider the ideas of standard indirect adaptive control [12], [16], [6]. The problem with standard techniques, however, is that the uncertainty in the closed-loop system usually comes from nonlinearly parameterized functions [13], [3]. The issue of nonlinear parametrization provides severe theoretical challenges for conventional adaptive control methods, especially if non-dominating solutions are sought for. On the other hand, conventional adaptive control algorithms are often not robust, which makes their application technically challenging.

Recently, a new method for adaptive, non-dominating control was developed [18], [22], [21], [20]. The method is applicable to a large class of practically relevant, nonlinearly parameterized systems with nonlinearities monotonic in their parameters. It also guarantees improved transient performance and robustness under mild assumptions of sufficient excitation [19]. These features redder this method suitable for applications to the brake control problem.

In our present study we concentrate on model-based (indirect) adaptive control of the slipping wheel. For the experimentally validated static tyre-road model of the friction coefficient[1] [5], we propose a robust adaptive controller with on-line estimation of the tyre-road conditions. These conditions are described by a single parameter of the friction curve.

We show that our controller is able to maintain the desired slip and simultaneously provide asymptotic tracking of the tyre-road parameter. This property allows for on-line adjustment of the reference slip, which corresponds to the maximal value of the friction curve. The main advantage of our approach is that it does not require domination nor damping in the control. It also does not require linearization or overparametrization of the uncertainties. In addition it guarantees integrability of the square of the error derivatives and exponentially fast decay of the uncertainties with time. This, in principle, allows us to improve significantly the transient characteristics of the system, as compared with other adaptive control approaches.

The paper is organized as follows. In Section II we provide a brief summary of our theoretical results and introduce our notation. Section III contains the formulation of the problem and design of the controller. Section IV describes results of computer simulation of the model with our controller, followed by conclusions in Section V.

II. ADAPTIVE ALGORITHMS IN FINITE FORM

Let the following system be given:

$$\dot{\mathbf{x}} = \mathbf{f}(\mathbf{x}, \boldsymbol{\theta}) + \mathbf{g}(\mathbf{x})u, \tag{1}$$

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[1]During preparation of the manuscript the authors became aware of the work [2] where the same problem is approached by use of nonlinear observers. Although in [2] a more advanced, dynamic model of friction is considered [1], our approach pays off in much simpler and robust estimators. In particular, we show that our estimator can be realized by a simple linear PI controller.

where $\mathbf{x} \in \mathbb{R}^n$ is a state vector, $\boldsymbol{\theta} \in \Omega_\theta \in \mathbb{R}^d$ is a vector of unknown parameters, u is the control input (scalar), and functions $\mathbf{f} : \mathbb{R}^n \to \mathbb{R}^n$, $\mathbf{g} : \mathbb{R}^n \to \mathbb{R}^n$, are locally bounded[2]. We assume that Ω_θ is bounded, e.g., Ω_θ is a closed ball or hypercube in $\mathbb{R}^d$.

As a measure of closeness of the system trajectories to the desired solution, we introduce the smooth error function $\psi : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R},\ \psi \in C^1$. The function $\psi(\mathbf{x}, t)$ is bounded in t for every bounded $\mathbf{x}$. The target manifold, therefore, is given by

$$\psi(\mathbf{x}, t) = 0$$

Consider the transverse dynamics of system (1) with respect to $\psi(\mathbf{x}, t)$:

$$\dot{\psi} = L_{\mathbf{f}(\mathbf{x},\boldsymbol{\theta})}\psi(\mathbf{x}, t) + L_{\mathbf{g}(\mathbf{x})}\psi(\mathbf{x}, t)u + \frac{\partial \psi(\mathbf{x}, t)}{\partial t}, \qquad (2)$$

where $L_{\mathbf{f}(\mathbf{x},\boldsymbol{\theta})}$ is Lie derivative of function $\psi(\mathbf{x}, t)$ with respect to vector field $\mathbf{f}(\mathbf{x}, \boldsymbol{\theta})$. We assume that the inverse $L_{\mathbf{g}(\mathbf{x})}\psi(\mathbf{x}, t)^{-1}$ exists for any $\mathbf{x} \in \mathbb{R}^n$, $t \in \mathbb{R}_+$. Then there exists control input $u(\mathbf{x}, \hat{\boldsymbol{\theta}}, t)$, where $\hat{\boldsymbol{\theta}} \in \mathbb{R}^d$ is the vector of estimates of unknown parameters $\boldsymbol{\theta}$, which transforms equation (2) into the following form [20]:

$$\dot{\psi} = -\varphi(\psi) + z(\mathbf{x}, \boldsymbol{\theta}, t) - z(\mathbf{x}, \hat{\boldsymbol{\theta}}, t) \qquad (3)$$

where $z(\mathbf{x}, \boldsymbol{\theta}, t) = L_{\mathbf{f}(\mathbf{x},\boldsymbol{\theta})}\psi(\mathbf{x}, t)$ and

$$\varphi(\psi) \in C^0,\ \varphi(\psi)\psi > 0\ \forall \psi \neq 0,\ \lim_{\psi \to \infty} \int_0^\psi \varphi(\varsigma) d\varsigma = \infty$$

Let function $z(\mathbf{x}, \boldsymbol{\theta}, t)$ in (3) satisfy the following set of assumptions

Assumption 1: There exist function $\boldsymbol{\alpha}(\mathbf{x}, t) : R^n \times R \to R^d$ and constant $D_1 > 0$ such that for any $\mathbf{x}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\theta}}^*$, $t > 0$ the following inequalities hold:

$$(z(\mathbf{x}, \hat{\boldsymbol{\theta}}, t) - z(\mathbf{x}, \hat{\boldsymbol{\theta}}^*, t))(\boldsymbol{\alpha}(\mathbf{x}, t)^T(\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}^*)) > 0$$
$$|z(\mathbf{x}, \hat{\boldsymbol{\theta}}, t) - z(\mathbf{x}, \hat{\boldsymbol{\theta}}^*, t)| \leq D_1|\boldsymbol{\alpha}(\mathbf{x}, t)^T(\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}^*)|$$

Assumption 2: There exists a positive constant $D_2 > 0$ such that for any $\mathbf{x}$, $\hat{\boldsymbol{\theta}}$, $\hat{\boldsymbol{\theta}}^*$, $t > 0$ the following inequality holds:

$$|z(\mathbf{x}, \hat{\boldsymbol{\theta}}, t) - z(\mathbf{x}, \hat{\boldsymbol{\theta}}^*, t)| \geq D_2|\boldsymbol{\alpha}(\mathbf{x}, t)^T(\hat{\boldsymbol{\theta}} - \hat{\boldsymbol{\theta}}^*)|$$

Assumptions 1 and 2 state that the nonlinear function $z(\mathbf{x}, \boldsymbol{\theta}, t)$ is monotonic w.r.t. to a linear functional of parameters $\boldsymbol{\theta}$, and for every fixed $\mathbf{x}, t$ it satisfies a sort of sector condition (illustrated in Fig. 1).

Theorem 1: Let function $\psi(\mathbf{x}, t)$ be given, Assumptions 1–2 hold, and $L_{\mathbf{f}}\boldsymbol{\alpha}(\mathbf{x}, t)$ does not depend on $\boldsymbol{\theta}$ explicitly. Then for the system

$$\dot{\mathbf{x}} \ = \ \mathbf{f}(\mathbf{x}, \boldsymbol{\theta}) + \mathbf{g}(\mathbf{x})u \qquad (4)$$

[2]Function $\mathbf{f}(\mathbf{x}) : \mathbb{R}^n \to \mathbb{R}^m$ is said to be locally bounded if for any $\|\mathbf{x}\| < \delta$ there exists a constant $D(\delta) > 0$ such that the following holds: $\|\mathbf{f}(\mathbf{x})\| \leq D(\delta)$.

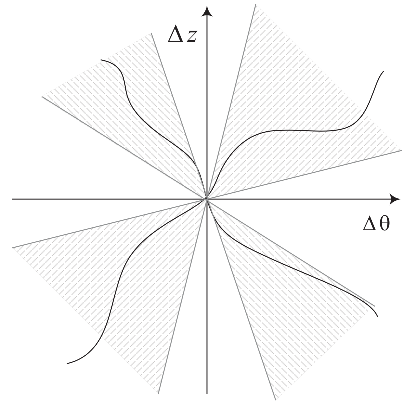

Fig. 1. Admissible parametrizations of function $z(\mathbf{x}, \boldsymbol{\theta}, t)$

there exists a control function $u(\mathbf{x}, \hat{\boldsymbol{\theta}}, t)$

$$u(\mathbf{x}, \hat{\boldsymbol{\theta}}, t) = (L_{\mathbf{g}(\mathbf{x})}\psi(\mathbf{x}, t))^{-1}(\varphi(\psi) - L_{\mathbf{f}(\mathbf{x},\hat{\boldsymbol{\theta}})}\psi(\mathbf{x}, t) - \partial\psi(\mathbf{x}, t)/\partial t) \qquad (5)$$

and adaptation algorithms:

$$\hat{\boldsymbol{\theta}}(\mathbf{x}, t) = \Gamma(\hat{\boldsymbol{\theta}}_P(\mathbf{x}, t) + \hat{\boldsymbol{\theta}}_I(t)),\ \Gamma > 0, \qquad (6)$$
$$\hat{\boldsymbol{\theta}}_P(\mathbf{x}, t) = \psi(\mathbf{x}, t)\boldsymbol{\alpha}(\mathbf{x}, t)$$
$$\dot{\hat{\boldsymbol{\theta}}}_I = \varphi(\psi(\mathbf{x}, t))\boldsymbol{\alpha}(\mathbf{x}, t) - \psi(\mathbf{x}, t) \times \partial\boldsymbol{\alpha}(\mathbf{x}, t)/\partial t - \psi(\mathbf{x}, t)L_{\mathbf{f}}\boldsymbol{\alpha}(\mathbf{x}, t) - \psi(\mathbf{x}, t)L_{\mathbf{g}}\boldsymbol{\alpha}(\mathbf{x}, t)u(\mathbf{x}, \hat{\boldsymbol{\theta}}, t)$$

such that the following statements hold:

P1) $\psi(\mathbf{x}, t) \in L_2 \cap L_\infty, \dot{\psi} \in L_2,\ \ z(\mathbf{x}, \boldsymbol{\theta}, t) - z(\mathbf{x}, \hat{\boldsymbol{\theta}}, t) \in L_2,\ \ \mathbf{x} \in L_\infty$;

P2) in addition, if derivatives $\partial\psi(\mathbf{x}, t)/\partial\mathbf{x}$, $\partial\psi(\mathbf{x}, t)/\partial t$ are uniformly bounded in t, then $\dot{\psi} \in L_\infty$, $\ z(\mathbf{x}, \boldsymbol{\theta}, t) - z(\mathbf{x}, \hat{\boldsymbol{\theta}}, t) \in L_\infty$, $\ \lim_{t\to\infty} \psi(\mathbf{x}(t), t) = 0$.

P3) Let $\alpha(\mathbf{x}, t)$ be persistently exciting [11], and $\varphi(\psi) = K\psi$. Then both $\psi(\mathbf{x}, t)$ and $\|\boldsymbol{\theta} - \hat{\boldsymbol{\theta}}\|$ converge exponentially fast to the origin.

This theorem is a special case of the *embedding* theorem in [21], [20], [19] and [18]. In the original paper the requirement that $L_{\mathbf{f}}\boldsymbol{\alpha}(\mathbf{x}, t)$ does not depend on $\boldsymbol{\theta}$ explicitly is replaced with that of existence of an auxiliary system with special properties. Although the present formulation is somewhat more restrictive than the original one, it is wide enough to be relevant in our current study. Proof of Theorem 1 trivially follows from the proof of Theorem 3 (with Proposition 1) in [19].

III. INDIRECT ADAPTIVE BRAKE CONTROL

In this section we consider the problem of minimizing the braking distance for a single wheel rolling along a surface. The surface properties are assumed to vary depending on the current position of the wheel. Wheel dynamics can be described by the following system of differential equations

[14]:

$$\begin{aligned}\dot{x}_1 &= -\frac{1}{m}F_s(F_n,\mathbf{x},\theta)\\ \dot{x}_2 &= \frac{1}{J}(F_s(F_n,\mathbf{x},\theta)r-u) \qquad (7)\\ \dot{x}_3 &= -\frac{1}{x_1}((\frac{1}{m}(1-x_3)+\frac{r^2}{J})F_s(F_n,\mathbf{x},\theta)-\frac{r}{J}u),\end{aligned}$$

where x_1 is longitudinal velocity, x_2 is angular velocity,

$$x_3 = (x_1 - rx_2)/x_1$$

is wheel slip, m is the mass of the wheel, J is the moment of inertia, r is the radius of the wheel, u is control input (brake torque), $F_s(F_n, \mathbf{x}, \theta)$ is a function specifying the tyre-road friction force depending on the surface-dependent parameter θ and bounded load force F_n. This function, for example, can be derived from steady-state behavior of the LuGre tyre-road friction model [3],[5]:

$$F_s(F_n,\mathbf{x},\theta) = F_n \mathrm{sign}(x_2)\frac{\frac{\sigma_0}{L}g(x_2,x_3,\theta)\frac{x_3}{1-x_3}}{\frac{\sigma_0}{L}\frac{x_3}{1-x_3}+g(x_2,x_3,\theta)}, \qquad (8)$$

$$g(x_2,x_3,\theta) = \theta(\mu_C + (\mu_S-\mu_C)e^{-\frac{|rx_2x_3|}{|1-x_3|v_s}}),$$

where μ_C, μ_S are Coulomb and static friction coefficients, v_s is the Stribeck velocity, σ_0 is the normalized rubber longitudinal stiffness, L is the length of the road contact patch. In order to avoid singularities in the solutions of model (7) we assume, as suggested in [14], that the system is turned off when velocity x_1 reaches a small neighborhood of zero (i.e when $x_1 < \delta_{x_1}$, $\delta_{x_1} \in \mathbb{R}_{>0}$). As soon as the system is turned off when $x_1 < \delta_{x_1}$, we can safely assume that the slip is always nonzero[3]. Practical considerations also suggest that the relevant slip values should not reach the point $x_3 = 1$ as explicit implementation of the friction model (8) would require the values of $x_3/(1-x_3)$. Therefore we shall assume that there exist $\delta \in \mathbb{R}_{>0}$ such that $0 < \delta < x_3 < 1-\delta$[4].

The typical shape of function $F_s(F_n, \mathbf{x}, \theta)$ is illustrated in Fig. 2. The value of slip x_3^* corresponding to the maximal value of the friction coefficient fluctuates broadly. As a rule of thumb, the value of slip to be maintained during braking is set around $x_3 = 0.2$. This choice significantly simplifies the design procedure of the slip controller. Yet, the choice is not always optimal due to unpredictable changes of the road surface. Hence in order to improve performance of the brakes, effective on-line estimation of the optimal slip is needed. One possible way to realize this is to estimate

[3]In fact model (8) assumes that the friction is zero for the zero values of slip x_3. On the other hand, it is the friction force which allows the wheel to move.

[4]Given that the variables x_2 and x_1 are, in principle, available it is always possible to monitor if the term $x_3 = (x_1 - rx_2)/x_1$ reaches a neighborhood of the point $x_3 = 1$. If the critical value of x_3 is reached, we can switch to the conventional controller, which steers the system back to the relevant domain.

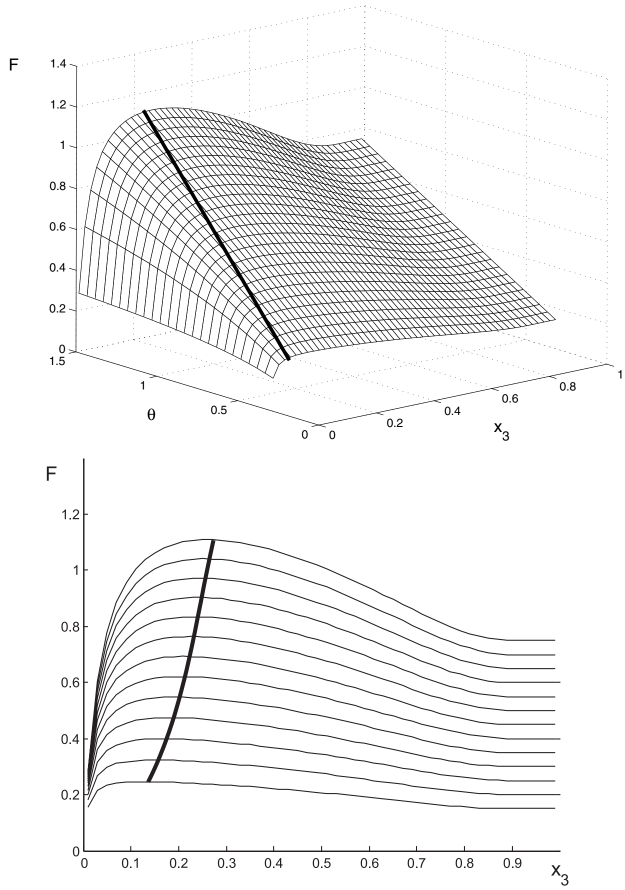

Fig. 2. Tire-road friction coefficient $F_s(1, \mathbf{x}, \theta)$ as a function of parameter θ) and slip x_3 for the fixed values of longitudinal velocity $x_1 = 30\frac{\mathrm{m}}{\mathrm{sec}}$ (upper plot). Projection of function $g(x_2, x_3, \theta)$ to axis x_3 (bottom plot). The thick black line depicts the set of points $(x_3, F_s(1, \mathbf{x}, \theta))$ which corresponds to the maximum value of $F_s(1, \mathbf{x}, \theta))$ for each θ

the actual tyre-road friction parameter θ as a function of slip and then calculate

$$x_3^* = \arg\max_{x_3} F_s(F_n, \mathbf{x}, \theta) \qquad (9)$$

The value of x_3^* is to be used in the main loop controller which would steer the system state to x_3^* ensuring the maximum deceleration force and the shortest braking distance.

In order to design an estimator of the friction coefficient, we employ the model (8). Whereas the majority of parameters in (8) can be set or estimated a priori, the tyre-road parameter θ depends explicitly on the actual conditions of the road surface. We assume quasi-stationarity of the road conditions, i.e., the road conditions (and the corresponding parameter θ) can be thought of as a piecewise constant function. According to (9), identification of parameter θ automatically results in successful estimation of the optimal slip x_3^*.

The main loop controller is derived in accordance with the standard certainty-equivalence principle, yielding:

$$u(\mathbf{x},\hat{\theta},x_3^*) = \frac{J}{r}((\frac{1}{m}(1-x_3)+\frac{r^2}{J})F_s(F_n,\mathbf{x},\hat{\theta}) - K_s x_1(x_3-x_3^*)),\ K_s>0 \quad (10)$$

In order to estimate parameter θ by measuring the values of variables x_1, x_2 and x_3, we construct the following subsystem:

$$\dot{\hat{x}}_3 = -\frac{1}{x_1}((\frac{1}{m}(1-x_3)+\frac{r^2}{J})F_s(F_n,\mathbf{x},\hat{\theta}) - \frac{r}{J}u) + (x_3-\hat{x}_3)$$

and consider the dynamics of the error function $\psi(\mathbf{x},t) = \psi(x_3,\hat{x}_3) = x_3-\hat{x}_3$:

$$\dot{\psi} = -\psi - \frac{1}{x_1}((\frac{1}{m}(1-x_3)+\frac{r^2}{J})(F_s(F_n,\mathbf{x},\theta) - F_s(F_n,\mathbf{x},\hat{\theta})) \quad (11)$$

Function $\kappa = \frac{1}{x_1}(\frac{1}{m}(1-x_3)+\frac{r^2}{J})F_s(F_n,\mathbf{x},\theta)$ is monotonic in θ and satisfies Assumptions 1, 2 with

$$\alpha(\mathbf{x},t) = \alpha_c,\ \alpha_c \in \mathbb{R}_+ \quad (12)$$

In order to verify that κ is monotonic, note that function (8)

$$F_s(F_n,\mathbf{x},\theta) = F_n \mathrm{sign}(x_2)\frac{\frac{\sigma_0}{L}g(x_2,x_3,\theta)\frac{x_3}{1-x_3}}{\frac{\sigma_0}{L}\frac{x_3}{1-x_3}+g(x_2,x_3,\theta)}$$

is monotonic in $g(x_2,x_3,\theta)$ and grows as $g(x_2,x_3,\theta)$ increases ($x_3/(1-x_3)$ is positive). Furthermore, function $g(x_2,x_3,\theta)$ is monotonic in θ (in fact, it is linear), and the sequence $g(x_2,x_3,\theta_i)$ is nondecreasing for every nondecreasing sequence θ_i. Hence we can conclude that $\kappa = \frac{1}{x_1}(\frac{1}{m}(1-x_3)+\frac{r^2}{J})F_s(F_n,\mathbf{x},\theta)$ is monotonic in both θ and $g(x_2,x_3,\theta)$. State $\mathbf{x}$ of system (7) is bounded by virtue of the physical laws governing the motion of the system. Therefore applying the arguments of continuity and monotonicity of $F_s(F_n,\mathbf{x},\theta)$ w.r.t. $g(x_2,x_3,\theta)$, we can bound the function $F_s(F_n,\mathbf{x},\theta)-F_s(F_n,\mathbf{x},\theta')$ as follows:

$$\begin{aligned}
&|F_s(F_n,\mathbf{x},\theta)-F_s(F_n,\mathbf{x},\theta')| \leq D_{g,1}|g(x_2,x_3,\theta) - g(x_2,x_3,\theta')\frac{x_3}{1-x_3}| = \\
&D_{g,1}|g(x_2,x_3,1)\frac{x_3}{1-x_3}||\theta-\theta'|, \\
&|F_s(F_n,\mathbf{x},\theta)-F_s(F_n,\mathbf{x},\theta')| \geq D_{g,2}|g(x_2,x_3,\theta) - g(x_2,x_3,\theta')\frac{x_3}{1-x_3}| = \\
&D_{g,2}|g(x_2,x_3,1)\frac{x_3}{1-x_3}||\theta-\theta'| \\
&D_{g,1},\ D_{g,2} > 0
\end{aligned} \quad (13)$$

Though we do not specifically address the issue of keeping the slip within desired bounds, one can easily derive a controller with such properties by choosing parameter K_s in (10) sufficiently large and simultaneously keeping $\hat{\theta}$ within the bounded domain by employing a projection technique. The use of large gains K_s is only to ensure that $\delta \leq x_3 \leq 1-\delta$. It does not necessarily mean that K_s is kept large for the whole braking period. In fact, it would be enough to apply this high-gain control only if the slip reaches a certain critical value, which is close to the bounds of interval $[\delta, 1-\delta]$. The gain can be set to its normal (desired) value as soon as the slip returns back to normal values.

Taking into account $0<\delta<x_3<1-\delta$, boundedness of $\mathbf{x}$ and continuity of $g(x_2,x_3,1)$, we can rewrite (13):

$$\begin{aligned}
&|F_s(F_n,\mathbf{x},\theta)-F_s(F_n,\mathbf{x},\theta')| \leq \bar{D}_{g,1}\frac{1-\delta}{\delta}|\theta-\theta'| \\
&|F_s(F_n,\mathbf{x},\theta)-F_s(F_n,\mathbf{x},\theta')| \geq \bar{D}_{g,2}\frac{\delta}{1-\delta}|\theta-\theta'| \\
&\bar{D}_{g,1} = D_{g,1}\max_{x_2,x_3}\{g(x_2,x_3,1)\} \\
&\bar{D}_{g,2} = D_{g,2}\max_{x_2,x_3}\{g(x_2,x_3,1)\}
\end{aligned} \quad (14)$$

Therefore, Assumptions 1, 2 are satisfied with $\alpha(\mathbf{x},t)=\alpha_c$ as specified in (12).

So far we have shown that Assumptions 1–2 hold, hence we can apply Theorem 1. Taking into account that $\alpha = \mathrm{const} > 0$, and $\varphi(\psi)=\psi$ - this follows from (11) - we can derive from (6) the following adaptation algorithm:

$$\hat{\theta} = -\gamma((x_3-\hat{x}_3)+\hat{\theta}_I),\quad \gamma>0 \quad (15)$$

$$\dot{\hat{\theta}}_I = x_3-\hat{x}_3$$

where $\gamma=\Gamma\alpha$, and $\alpha=\alpha_c$ is persistently exciting (i.e., $\int_t^{t+T}\alpha^T(\mathbf{x},\tau)\alpha(\mathbf{x},\tau)d\tau = \alpha_c^2 T$). Based on Theorem 1 (P3), we conclude that adaptation algorithm (15) ensures exponentially fast convergence of $\theta-\hat{\theta}$, $x_3-\hat{x}_3$ to the origin. Taking into account the smoothness of function $F_s(F_n,\mathbf{x},\theta)$ for $x_1>0$, we also conclude that control function (10) guarantees exponentially fast convergence of x_3 to the desired x_3^*. The rate of convergence is determined by constants K_s, $\gamma>0$. This result can be summarized as follows:

Corollary 1: Let system (7) be given and control function satisfies the following equations

$$\begin{aligned}
u(\mathbf{x},\hat{\theta},x_3^*) &= \frac{J}{r}((\frac{1}{m}(1-x_3)+\frac{r^2}{J})F_s(F_n,\mathbf{x},\hat{\theta}) - K_s x_1(x_3-x_3^*)),\ K_s>0 \\
\hat{\theta} &= -\gamma((x_3-\hat{x}_3)+\hat{\theta}_I),\quad \gamma>0 \\
\dot{\hat{\theta}}_I &= x_3-\hat{x}_3
\end{aligned}$$

Then for any bounded $F_n>0$ and arbitrary small $\delta>0$ there exists $K_s>0$ such that for every $x_3^*,\ x_3(0)\in[2\delta,1-2\delta]$, $\theta\in\mathbb{R}_+$, and $x_1(t)>\delta_0\in\mathbb{R}_+$, the estimate $\hat{\theta}(t)$ is bounded, and $x_3(t)-x_3^*$, and $\hat{\theta}-\theta$ converge exponentially fast to the origin as long as $x_1(t)>\delta_0\in\mathbb{R}_+$.

Proof. We must show that $x_3\in[\delta,1-\delta]$ and $\hat{\theta}$ is bounded.

First, notice that $\hat{\theta}$ is bounded. Indeed, differentiation of $\hat{\theta}$ with respect to time results in the following equation

$$\dot{\hat{\theta}} = -\gamma \frac{1}{x_1}((\frac{1}{m}(1-x_3) + \frac{r^2}{J})F_n \text{sign}(x_2) \times \frac{\frac{\sigma_0}{L} g(x_2, x_3, 1) \frac{x_3}{1-x_3}}{\frac{\sigma_0}{L}\frac{x_3}{1-x_3} + g(x_2, x_3, 1)}(\theta - \hat{\theta}), \quad (16)$$

where $x_3 \in [0,1]$ by definition and

$$\frac{\frac{\sigma_0}{L} g(x_2, x_3, 1) \frac{x_3}{1-x_3}}{\frac{\sigma_0}{L}\frac{x_3}{1-x_3} + g(x_2, x_3, 1)}$$

is nonnegative (positive for every $1 \geq x_3 \geq \delta$). For any initial conditions $\hat{\theta}(0)$ and bounded θ, solutions $\hat{\theta}(t)$ of system (16) remain, therefore, bounded.

Let us prove that there exists K_s which keeps x_3 in $[\delta, 1-\delta]$. This follows explicitly from the boundedness of function F_n and properties of function F_s given by (8); the values of $(g(x_2, x_3, \hat{\theta})$, $g(x_2, x_3, \theta)$ are bounded for bounded θ, and F_s is bounded for every $x_3 \in [0,1]$ and bounded g). Therefore, difference

$$\frac{1}{x_1}(\frac{1}{m}(1-x_3) + \frac{r^2}{J})(F_s(F_n, \mathbf{x}, \theta) - F_s(F_n, \mathbf{x}, \hat{\theta}))$$

is always bounded by constant M. This implies existence of $K_s > 0$ such that for any x_3^*, $x_3(0) \in [2\delta, 1-2\delta]$ solutions of the controlled system (slip part)

$$\begin{aligned} \dot{x}_3 &= \frac{1}{x_1}(\frac{1}{m}(1-x_3) + \frac{r^2}{J})(F_s(F_n, \mathbf{x}, \theta) - \\ &\quad F_s(F_n, \mathbf{x}, \hat{\theta})) - K_s(x_3^* - x_3) \end{aligned}$$

belong to $[\delta, 1-\delta]$. To show this, one can take the following quadratic function $V = 0.5(x_3 - x_3^*)^2$ and estimate its derivative

$$\begin{aligned} \dot{V} &= -2K_s(x_3 - x_3^*)^2 + |(x_3 - x_3^*)|M \leq 0, \\ &\quad \forall |x_3 - x_3^*| \geq \delta, \; K_s \geq \frac{M}{\delta} \end{aligned} \quad (17)$$

Inequality (17) implies that trajectories of system (17) converge uniformly into $|x_3 - x_3^*| \leq \delta$. Under assumption that x_3^*, $x_3(0) \in [2\delta, 1-2\delta]$ this proves that $x_3(t) \in [\delta, 1-\delta]$ for any $x_1 > \delta_0$. Q.E.D.

IV. SIMULATIONS

We illustrate our theoretical results with a numerical simulation. We consider system (7) – (15) with the following setup of parameters: $\sigma_0 = 200$, $L = 0.25$, $\mu_C = 0.5$, $\mu_S = 0.9$, $v_s = 12.5$, $r = 0.3$, $m = 200$, $J = 0.23$, $F_n = 3000$, $K_s = 30$, $\gamma = 100$. The effectiveness of algorithm (15) is illustrated in Figures 3 and 4 (the solid thick lines correspond to our adaptive controller with on-line estimation of the optimal values of slip, and the dashed lines correspond to the controller with preset constant x_3^* in the rage $[0.1, 0.2]$). Figure 3 shows trajectories of the system for the road conditions given by the piece-wise constant function:

$$\theta(s) = \begin{cases} 0.3, & s \in [0, 10] \\ 1.3, & s \in (10, 20] \\ 0.7, & s \in (20, 30] \\ 0.4, & s \in (30, 40] \\ 1.5, & s \in (40, 50] \\ 0.6, & s \in (50, \infty) \end{cases}, s = \int_0^t x_1(\tau)d\tau \quad (18)$$

Both our adaptive controllers (with constant x_3^* and $x_3^*(t)$ calculated according to (9)) show acceptable performance. Estimates $\hat{\theta}$ approach the actual values of parameter θ sufficiently fast (see Fig. 4) for the controller to calculate the optimal slip value x_3^* and steer the system toward this point. The control torque remains within realistic bounds (see also [14] where the plots containing the actual values of the braking torque generated in the experimental ABS are provided).

The effectiveness of the identification-based control can be confirmed by comparing the braking distance in the system with on-line estimation of x_3^* with $\theta = \hat{\theta}$ according to (9) with the one in which the values of x_3^* were kept constant. For the specified model parameters and the road conditions 18, the simulated braking distance obtained with our on-line estimation procedure of x_3^* is 49.7 meters. This result compares favorably with the values obtained for preset values of x_3^*, which range between 53.2 and 49.9 (for $x_3^* = 0.1$ and $x_3^* = 0.2$, respectively). Similar advantages of our method are observed for other parameter settings and initial conditions.

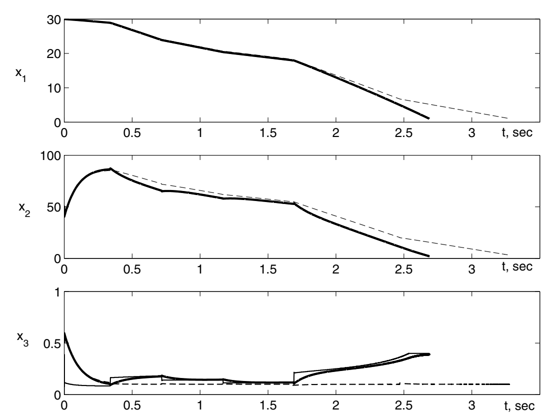

Fig. 3. Trajectory plots of system (7). The top panel is longitudinal velocity, the middle panel is angular speed, the bottom panel is the slip. Estimates of the optimal slip values obtained from (9) with $\hat{\theta}(s)$ are shown by the solid thin line; the trajectory of the slip $\hat{x}_3$ in the system with controller estimating x_3^* on-line is shown by the solid thick line; dynamics of the slip with in the system with pre-set x_3^* in adaptive controller is depicted by a dashed line.

V. CONCLUSIONS AND FUTURE WORK

We provided a non-dominating adaptive controller for the problem of effective adaptive brake control. Despite nonlinear parametrization of the friction coefficient in the

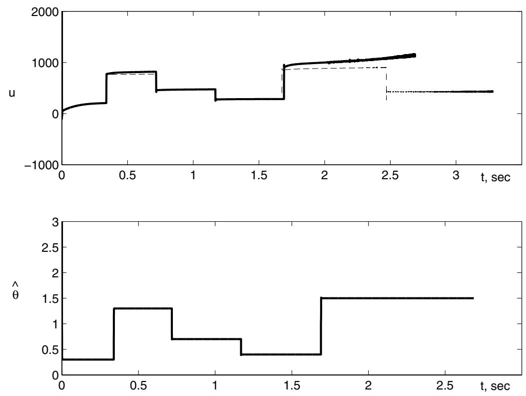

Fig. 4. Plots of the braking torque u and estimate $\hat{\theta}$ of the tyre-road condition given by (18). The top panel is the plots of the braking torques. Solid thick line shows the braking torque in the system with on-line calculation of the optimal slip x_3^*, dashed line stands for the braking torque in the system with pre-set values of the desired slip $x_3^* = 0.1$. The second panel shows estimate $\hat{\theta}$ as a function of time. It can be seen from the pictures that $\hat{\theta}(t)$ virtually coincides with the parameter θ of the actual tyre-road conditions

model, it is possible to design an estimator of the tyre-road parameter which converges exponentially fast to the values corresponding to the actual road conditions. The estimation algorithm given is robust (defined by exponential stability of the estimator) and hardware implementable. In fact, the proposed estimation algorithm can be realized via standard PI controllers widely used in industry.

Our current algorithm for effective brake control relies on explicit measurement of the longitudinal velocity. The most promising improvement would be to eliminate the needs for velocity sensors in the system. Combination of the nonlinear velocity observers (for instance, those proposed in [4]) may be a theme of our future study.

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