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Adaptive Open-Loop Rear-Wheel-Steering

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Abstract — **Open-loop rear-wheel-steering (RWS) control can be enhanced by modifying the open-loop gain table that adapts to changes in vehicle dynamics parameters, such as front and rear cornering compliances. These changes can be caused by tire deflation/inflation, tire wear, loading change and suspension aging, etc. The vehicle parameters can be estimated by using a real-time vehicle parameter estimation technique. The open-loop gain table is modified based on three different strategies: yaw-rate gain matching, lateral-velocity gain matching, and lateral-velocity to yaw-rate ratio matching strategies. The three strategies are investigated using linear analysis. The yaw-rate gain matching strategy does recovers yaw-rate gain in the presence of vehicle cornering compliance changes. However it has undesired effect on lateral-velocity response. The other two strategies are both very effective in reducing lateral-velocity although the lateral-velocity to yaw-rate ratio matching strategy is preferred due to its simplicity. When the only change in vehicle dynamics parameters is the rear cornering compliance, the three proposed strategies are found to be equivalent. An adaptive open-loop RWS control structure is then proposed and analyzed with simulation studies. The simulation result shows the effectiveness of the adaptive open-loop control.**

I. INTRODUCTION

Rear-wheel-steering (RWS) control has been extensively investigated in the past decades as a new approach for enhancing vehicle handling and yaw-plane stability. Advantages for RWS systems include enhanced yaw-plane stability at high speeds, reduced sensitivity to external disturbances and improved maneuverability at low speeds. To date, a number of RWS systems have been proposed. These systems can be classified into two categories: open-loop control and closed-loop control. In an open-loop control RWS system [1-3], the steering-wheel angle and/or steering-wheel angle rate is used to command the rear wheel steering angle, while in a closed-loop control RWS system [4-6], the vehicle states are fed back to control the rear wheel steering angle. Open-loop control is simple and robust to some extent. Also with open-loop control the interference to the driver is minimal since driver still has full authority to control the vehicle. Recently, RWS has also been studied to enhance vehicle-trailer stability [7] and backing up control [8].

The first open-loop RWS application is found in Steer Angle Dependent 4WS system [3]. In 1987, the first production vehicle equipped with RWS was appeared in the market. In recent years this technique has again been introduced in some production trucks, such as GMC Sierra and Chevrolet Suburban. In general RWS angle is set proportional to the front steering angle by a scheduled gain at each speed. The open-loop gain schedule is tuned out-of-phase at low-speed range for better maneuverability and is tuned in-phase at high-speed range for better yaw-plane stability.

The open-loop RWS gain is scheduled based on the nominal vehicle conditions and is optimized for the out-of-factory status. However, the dynamics parameters of the vehicle such as front and rear cornering compliances can deviate from the nominal values due to aging, tire inflation/deflation, change of vehicle loading, tire wear, change of suspension characteristics, etc. When these changes are significant, the vehicle handling performance also deviates from the optimized one resulting in degraded handling and yaw-plane stability.

This paper presents a method of adaptive compensation of the open-loop RWS in response to changes in vehicle dynamics parameters. Section II describes the general concepts of open-loop RWS control and its expected nominal vehicle response. Section III discusses different strategies of achieving the adaptive compensation for different objectives. Section IV presents the pros and cons of each strategy and selects a best strategy. Section V shows simulation results of applying the selected compensation strategy to open-loop RWS. Finally, conclusions are shown in Section VI.

II. OPEN-LOOP REAR-WHEEL-STEERING

Figure 1 shows the schematics of typical open-loop control of a RWS vehicle. The hand-wheel angle δ_H is reduced by the lumped gear ratio Gm to yield the front

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wheel angle δ_F. The rear-wheel angle δ_R is proportional to the front-wheel angle δ_F with the open-loop gain $T(V_x)$, which is a function of vehicle speed V_x.

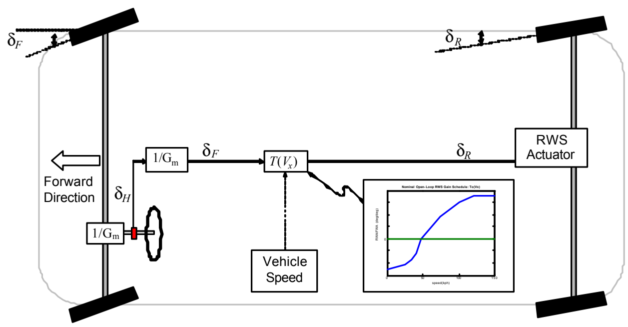

Figure 1. Open-Loop Rear Wheel Steering.

The open-loop gain is negative in the low-speed region and positive in the high-speed region. As a result, the open-loop RWS will command a steering angle opposite to the front steering (so-called out-of-phase steering) at low speeds to reduce the turning radius. At higher speeds, it will command a rear-wheel angle at the same direction as the front steering angle (so-called in-phase steering). The in-phase steering has an effect of stabilizing the vehicle during fast transient maneuvers. The rear wheel angle is expressed as:

$$\delta_R = T(V_x)\delta_F \tag{1}$$

Where $T(V_x)$ is the open-loop gain schedule which is function of vehicle speed. For notational simplicity, $T(V_x)$ is written as T, hereafter. The steady state response of the vehicle is then written as

$$V_{oy}(0) = \frac{[(gb/V_x - D_R V_x) + (ga/V_x + D_F V_x)T]}{(K_{us} + Lg/V_x^2)}\delta_F \tag{2}$$

$$\Omega(0) = \frac{g/V_x(1-T)}{(K_{us} + Lg/V_x^2)}\delta_F \tag{3}$$

The derivations of equations (2) and (3) are based on the bicycle model and the details are shown in Appendix. The parameters a and b determines the nominal c.g. location and are constants. D_F and D_R are the parameters that can be changed. For the nominal vehicle with nominal open-loop gain table, equations (2) and (3) are written as

$$V_{oy}^o(0) = \frac{[(gb/V_x - D_R^o V_x) + (ga/V_x + D_F^o V_x)T^o]}{(K_{us}^o + Lg/V_x^2)}\delta_F \tag{4}$$

$$\Omega^o(0) = \frac{g/V_x(1-T^o)}{(K_{us}^o + Lg/V_x^2)}\delta_F \tag{5}$$

III. Adaptation Strategies

When the vehicle is in nominal condition, equations (4) and (5) determine the steady state response. When the front and the rear cornering compliances D_F and D_R deviate from the nominal values, the steady-state response also deviates as shown in equations (2) and (3). The natural objective of the adaptive open-loop RWS control is to bring the deviated steady-state response back to the nominal response as close as possible, by changing open-loop gain table T.

A. *Limitations of Model Matching Strategy*

The ideal objective of the adaptive open-loop RWS is to match the steady-state response of the nominal vehicle and that of the deviated vehicle. This is called the steady-state model-matching strategy. The problem is to find T that satisfies both (2) = (4) and (3) = (5). Since there are two equations and one independent variable, the solution T does not always exist.

For the special case of $D_F = D_F^o$ and $D_R \neq D_R^o$., i.e., when the front cornering compliance remains the same and only the rear cornering compliance deviates from the nominal value, there is a solution for T that satisfies both (2) = (4) and (3) = (5). The existence of the solution is natural because the RWS effectively changes the rear cornering compliance and is capable of compensating for the deviation of rear cornering compliance. The solution is found to be

$$T = T^o + \frac{(1-T^o)V_x^2}{(K_{us}^o V_x^2 + Lg)}\Delta D_R \tag{6}$$

where $\Delta D_R = D_R - D_R^o$.

For the general cases of $D_F \neq D_F^o$ and $D_R \neq D_R^o$, there is no solution T for matching both lateral-velocity gain and yaw-rate gain at the same time. Therefore one should choose either yaw-rate gain matching, lateral-velocity gain matching or in between the two.

B. *Yaw-Rate Matching Strategy*

The simplest strategy is to recover the yaw-rate gain without considering the lateral-velocity gain. With this strategy, the corrected gain can be obtained by equating Ω and Ω^o in equations (3) and (5), i.e.,

$$\frac{g/V_x(1-T)}{(K_{us} + Lg/V_x^2)} = \frac{g/V_x(1-T^o)}{(K_{us}^o + Lg/V_x^2)} \tag{7}$$

The solution T is then written as

$$T = T^o + \frac{(1-T^o)V_x^2}{(K_{us}^o V_x^2 + Lg)}(\Delta D_R - \Delta D_F) \tag{8}$$

C. Lateral-Velocity Matching Strategy

An alternate strategy is to recover the lateral-velocity gain without considering the yaw-rate gain. The exact solution can be obtained by equating lateral velocities in equations (2) and (4). However, the exact solution has a vehicle parameter D_F in its denominator. Since estimated parameter should be used, the denominator could be zero if the estimation is not correct. To prevent the use of estimation in the denominator, an approximation by linearization around the nominal status can be used. Let us define the lateral-velocity gain as G_{Vy}. From equation (2), G_{Vy} is

$$G_{Vy} = \frac{(gb/V_x - D_R V_x) + (ga/V_x + D_F V_x)T}{(K_{us} + Lg/V_x^2)} \tag{9}$$

Since the control objective is to maintain the lateral-velocity gain, the change of the gain from the nominal gain should be zero, i.e.,

$$\Delta G_{Vy} = \left.\frac{\partial G_{Vy}}{\partial D_F}\right|_o \Delta D_F + \left.\frac{\partial G_{Vy}}{\partial D_R}\right|_o \Delta D_R + \left.\frac{\partial G_{Vy}}{\partial T}\right|_o \Delta T = 0 \tag{10}$$

Equation (10) is evaluated as

$$T = T^o + \frac{(1-T^o)V_x^2}{(K_{us}^o V_x^2 + Lg)}\left\{\Delta D_R - \Delta D_F\left(\frac{D_R^o V_x^2 - bg}{D_F^o V_x^2 + ag}\right)\right\} \tag{11}$$

D. VY/YR Matching Strategy

The well-known design objective of the open-loop RWS is to make the steady-state lateral-velocity zero at the nominal c.g. location. Although this objective mathematically seems plausible, it is too aggressive especially for low-speed operations according to most drivers' subjective handling feel. In practical implementation, therefore, the open-loop schedule is tuned and modified based on the test driver's handling feel. This resultantly moves the zero lateral-velocity location back and forth from the nominal c.g. location for each speed. When the distance between the zero lateral-velocity location to the nominal c.g. location is ε, the lateral-velocity at the nominal c.g. location is represented as

$$V_{oy} = \varepsilon\Omega \tag{12}$$

In other words, the lateral-velocity at the nominal c.g. location is proportional to the yaw-rate with a specific proportional constant ε. Therefore, the design objective of a given table can be interpreted as driver's preferred lateral-velocity to yaw-rate ratio ε. This proportional constant is a function of vehicle speeds. For the nominal vehicle configuration, the nominal ratio relates the steady-state yaw rate and the steady-state lateral velocity, i.e.

$$V_{oy}^o(0) = \varepsilon^o(V_x)\Omega^0(0) \tag{13}$$

When vehicle parameters are changed, the expression of the new ratio ε is obtained from equations (2) and (3), i.e.,

$$\varepsilon = \frac{(gb - D_R V_x^2) + (ga + D_F V_x^2)T}{g(1-T)} \tag{14}$$

The linear approximation to maintain the ratio in the presence of parameter deviation can be calculated by solving the following equation.

$$0 = \Delta\varepsilon = \left.\frac{\partial\varepsilon}{\partial D_F}\right|_o \Delta D_F + \left.\frac{\partial\varepsilon}{\partial D_R}\right|_o \Delta D_R + \left.\frac{\partial\varepsilon}{\partial T}\right|_o \Delta T \tag{15}$$

The new open-loop gain is then

$$T = T^o + \frac{(1-T^o)V_x^2}{K_{us}^o V_x^2 + Lg}(\Delta D_R - \Delta D_F T^o) \tag{16}$$

IV. Comparison of Strategies

The three strategies in equations (8), (11) and (16) have very similar expressions. The only difference is the last term of each equation. Therefore the general expression is:

$$T = T^o + \frac{(1-T^o)V_x^2}{K_{us}^o V_x^2 + Lg}(\Delta D_R - \Delta D_F \Gamma) \tag{17}$$

where the factor Γ varies for different strategy.

As discussed earlier, the three different strategies give the same solution if $\Delta D_F = 0$. This result is expected since the RWS is capable of compensating for rear cornering-compliance changes. In this case, both the lateral-velocity and the yaw-rate gains can be recovered to the nominal gains.

The comparisons of the case when D_F is increased by 45% from the nominal value and D_R remains the same as the nominal are shown in Figures 2-4. The yaw-rate matching strategy requires significant modification to the open-loop schedule. The curves for Vy matching and Vy/YR matching strategies are very close to each other especially for the high-speed range.

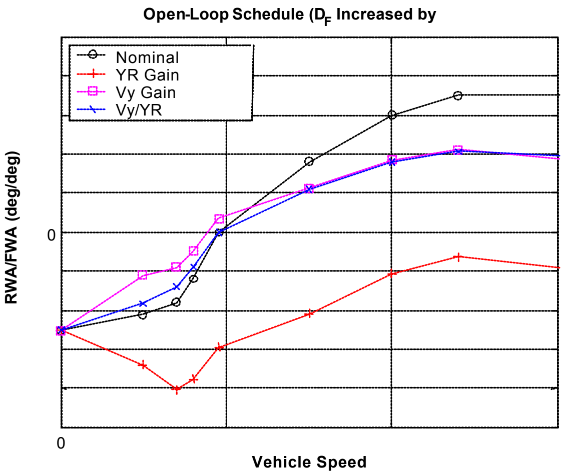

Figure 2. Open-Loop RWS Gear Ratio Schedule for Different Adaptation Strategies.

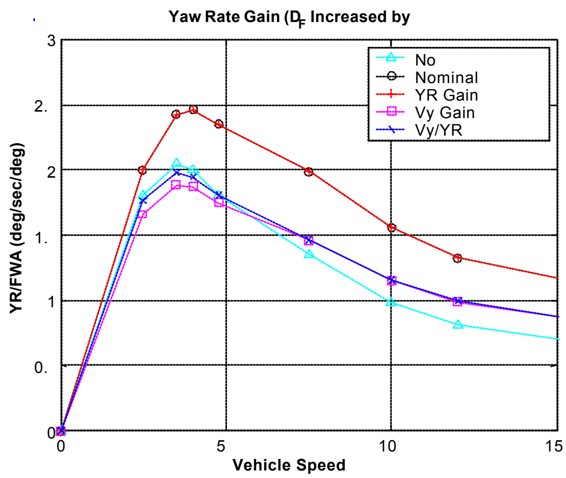

Figure 3. Steady-State Yaw-Rate Gains for Different Adaptation Strategies.

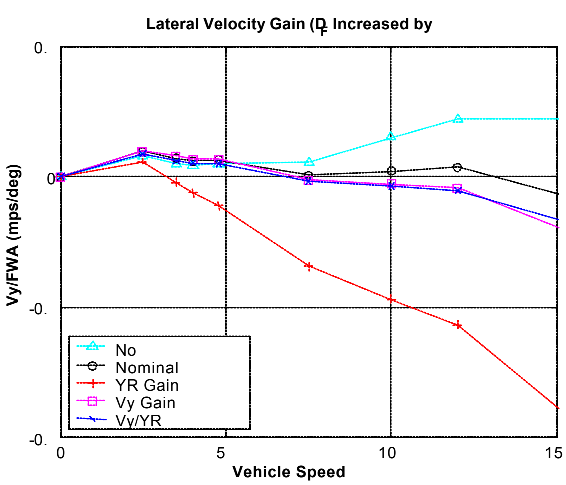

Figure 4. Steady-State Lateral Velocity Gains for Different Adaptation Strategies.

In Fig. 3, the yaw-rate gain matching strategy completely recovers the yaw-rate gain to the nominal gain. The other two strategies move the yaw-rate gain curve in between the curve of no-adaptation and that of the nominal case.

Figure 4 shows the lateral-velocity gains. When the yaw-rate gain matching strategy is applied, the lateral-velocity gain is even worse than the no-matching case. The other two cases, on the other hand, show similar performance in terms of lateral-velocity gain.

The plots for the case when D_F is decreased from the nominal value are omitted here since the effect is similar to the increased D_F case except the directions of changes for each case.

When only D_R is deviated from the nominal value, the three strategies are equivalent and both yaw-rate lateral-velocity gains are completely recovered. The plots of this case are also omitted because the result is trivial. For the general cases when both D_F and D_R deviate from the nominal values, the results are the linear combination of the two cases.

From the linear analysis discussed above, the yaw-rate matching strategy is not recommended. The lateral-velocity gain matching and the lateral-velocity to yaw-rate ratio matching strategies both have similar desirable effects on the yaw-rate gain and lateral-velocity gain. From equations (11) and (16), it is evident that equation (16) is simpler than equation (11). Since there is little difference in results using equations (11) and (16), equation (16), which is the VY/YR matching strategy, is recommended.

The lateral-velocity to yaw-rate ratio matching strategy has more plausible physical meaning. The driver himself or herself tries to compensate for the yaw-rate to follow the desired path of the vehicle. Since the ratio remains the same with adaptation, driver's yaw-rate compensation effort automatically compensates for the lateral-velocity. Or if the driver tries to compensate for the lateral-velocity, the same strategy automatically compensates for the yaw-rate.

V. Adaptive Open-Loop RWS Control

A. Control Structure

As discussed in the previous section, the open-loop adaptation strategy is expressed as

$$\Delta T = (T - T^o) = \frac{(1-T^o)V_x^2}{K_{us}^o V_x^2 + Lg}(\Delta D_R - \Delta D_F \Gamma) \tag{18}$$

The corresponding RWS angle is expressed as

$$\Delta\delta_R = \delta_R - \delta_R^o = (\Delta D_R - \Delta D_F \Gamma)\frac{V_x^2}{K_{us}^o V_x^2 + Lg}(\delta_F - \delta_R^o) \tag{19}$$

where $\Delta D_F = D_F - D_F^o$ and $\Delta D_R = D_R - D_R^o$

Since the exact parameters are not known, based on the certainty equivalence principle, we may use the estimated parameters,

$$\Delta\delta_R = \delta_R - \delta_R^o = (\Delta\hat{D}_R - \Delta\hat{D}_F\Gamma)\frac{V_x^2}{K_{us}^o V_x^2 + Lg}(\delta_F - \delta_R^o) \quad (20)$$

where $\Delta\hat{D}_F = \hat{D}_F - D_F^o$ and $\Delta\hat{D}_R = \hat{D}_R - D_R^o$

When combined with the parameter estimation algorithm, the overall control structure is shown in Fig. 5.

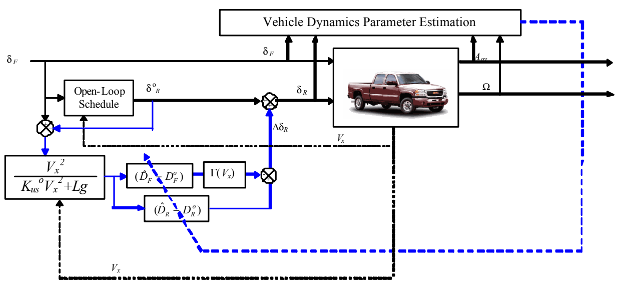

Figure 5. Adaptive Open-Loop RWS Control Structure

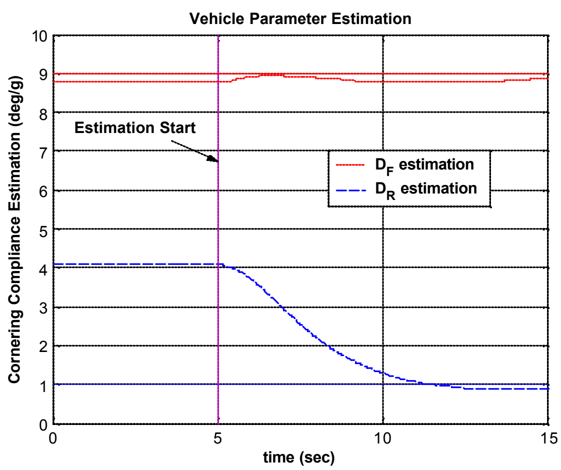

Figure 6. Vehicle Dynamics Parameter Estimation

The vehicle parameter estimation is developed based on the two D.O.F. model, and the details of the algorithm are shown in the separate paper [9]. The vehicle dynamics parameter estimation algorithm recursively updates the front and rear cornering compliances D_F and D_R. Figure 6 shows a simulation of vehicle parameter estimation. In this simulation scenario, the rear cornering compliance D_R is significantly decreased from about 4 (deg/g) to 1 (deg/g), and the vehicle parameter estimation algorithm detects this change. This estimation is performed during a series of double lane change maneuver. The estimated cornering compliances are used to calculate adaptive open-loop RWS correction at the same time.

B. Simulations

Several simulation results are discussed in this section. The simulations are performed for a RWS truck model. The maneuver selected is a double-lane-change maneuver at 100 kph. Lateral-velocity to yaw-rate ratio matching strategy is applied for the reason discussed in the previous sections. Although the double-lane-change maneuver is not a steady-state maneuver, the correction of open-loop table is very effective for handling and yaw-plane stability.

Figures 7-9 shows the front wheel angle, yaw-rate and lateral-velocity responses when D_F is increased by 45%. Since the driver is in the loop, the drivertries to compensate for the yaw rate, as shown in Fig. 8, resulting in almost the same yaw-rate responses for all three cases, namely the nominal case, before adaptation case, and after adaptation case.

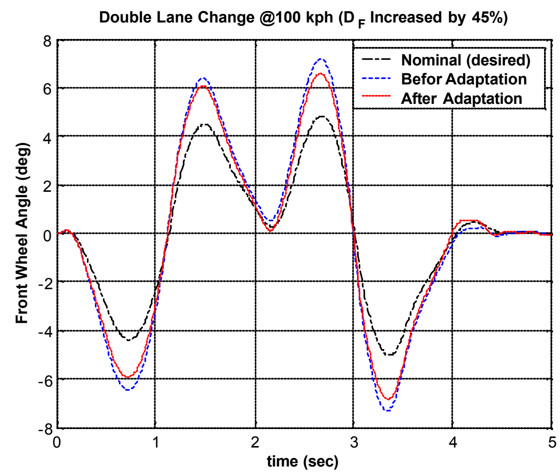

Figure 7. Front Wheel Angle with Adaptive RWS Control

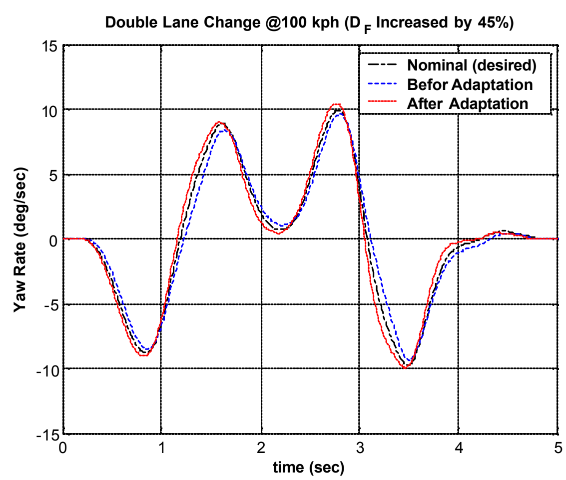

Figure 8. Yaw-Rate with Adaptive RWS Control

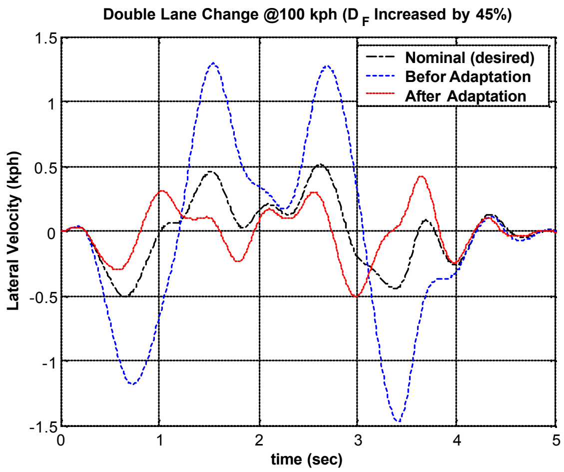

Figure 9. Lateral-Velocity with Adaptive RWS Control

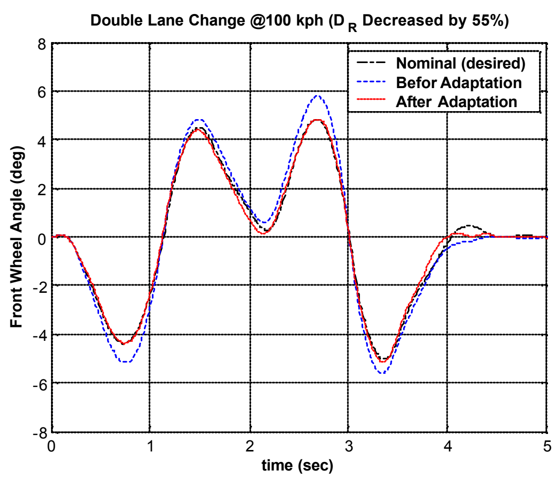

Figure 10. Front Wheel Angle with Adaptive RWS Control

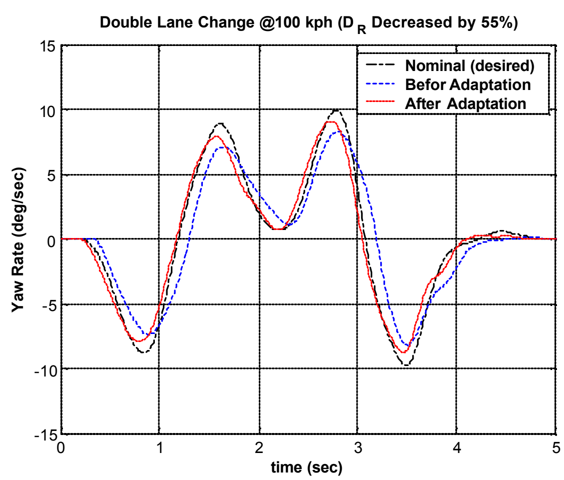

Figure 11. Yaw-Rate with Adaptive RWS Control

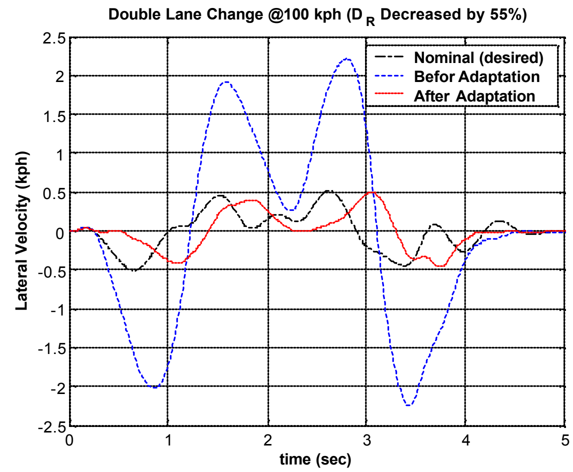

Figure 12. Lateral-Velocity with Adaptive RWS Control

Therefore, the front wheel angles in Fig. 7 are different for different cases. However, for the lateral-velocity response, the response before adaptation produces large lateral velocities while the after adaptation maintains the magnitude of the response as small as the nominal case.

Figures 10-12 shows the front wheel angle, yaw-rate and lateral-velocity responses when D_R is decreased by 55%. As discussed above, the adaptive control can recover both yaw-rate gain and lateral-velocity gain at the same time when the deviation is from D_R only. Therefore, the front wheel angle, yaw rate and lateral velocity are all close to the nominal response when the adaptation is applied.

VI. Conclusion

The lateral-velocity to yaw-rate ratio matching strategy is found to be most effective among the three proposed strategies. By combining with real-time vehicle parameter estimation algorithm, the adaptive open-loop RWS control effectively maintains the nominal handling and yaw-plane stability enhancement capability as close as possible in the presence of vehicle dynamics parameter deviations.

Appendix

Nomenclature

- m: vehicle mass.
- I_c: vehicle yaw moment of inertia about true c.g.
- A_{cy}: vehicle lateral acceleration at true c.g.
- A_{oy}: vehicle lateral acceleration at the nominal c.g.
- α_F: front wheel side-slip angle.
- α_R: rear wheel side-slip angle.
- a: distance between the front axle to the nominal c.g.
- b: distance between the rear axle to the nominal c.g.
- δ_F: front steering angle.
- δ_R: rear steering angle.
- e: distance between the true c.g. and the lateral

accelerometer.

l_F: distance between the front axle to the true c.g.
l_R: distance between the rear axle to the true c.g.
L: wheel base ($L = a + b$).
g: gravitational acceleration.
F_F: lateral force of the front axle.
F_R: lateral force of the rear axle.
C_F: front axle cornering stiffness.
C_R: rear axle cornering stiffness.
D_F: front axle cornering compliance.
D_R: rear axle cornering compliance.
K_{us}: understeer coefficient (D_F - D_R).

Figure A.1 shows the description of variables of the bicycle model with rear wheel steer. The equations of motion are obtained simply by Newton's second law.

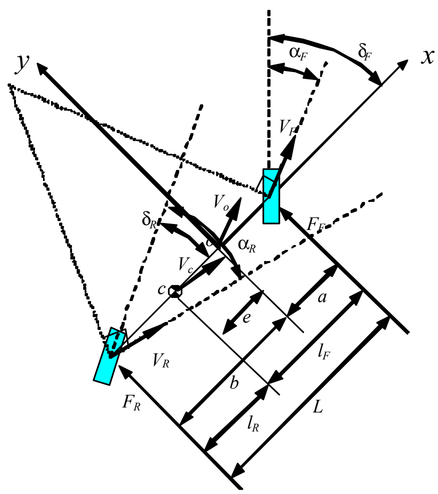

Figure A.1. Description of Bicycle Model with RWS

$$\begin{aligned} mA_{cy} &= F_F + F_F \\ I_c\dot{\Omega} &= l_F F_F - l_F F_F \end{aligned} \tag{A.1}$$

Equation (A.1) can be expressed with respect to the lateral accelerometer location 'o', which may or may not be the same as c.g.

$$\begin{bmatrix} m & -me \\ 0 & I_c \end{bmatrix} \begin{Bmatrix} A_{oy} \\ \dot{\Omega} \end{Bmatrix} = \begin{bmatrix} 1 & 1 \\ l_F & -l_R \end{bmatrix} \begin{Bmatrix} F_F \\ F_R \end{Bmatrix} \tag{A.2}$$

Let us define several vehicle parameters as

$$D_F = \frac{W_F}{C_F} = \frac{mgl_R}{LC_F}, \quad D_R = \frac{W_R}{C_R} = \frac{mgl_F}{LC_R},$$

$$\lambda = \frac{I_c}{ml_R l_F}, \quad E_F = (\lambda l_F - e)D_F \text{ and } E_R = (\lambda l_R + e)D_R .$$

Now the equation of motion (A.2) can be expressed as

$$\begin{bmatrix} D_F & E_F \\ D_R & -E_R \end{bmatrix} \begin{Bmatrix} A_{oy} \\ \dot{\Omega} \end{Bmatrix} = g \begin{Bmatrix} F_F / C_F \\ F_R / C_R \end{Bmatrix} = g \begin{Bmatrix} \alpha_F \\ \alpha_R \end{Bmatrix} . \tag{A.3}$$

From the kinematics, the side-slip angles are expressed as

$$\begin{Bmatrix} \alpha_F \\ \alpha_R \end{Bmatrix} = g \begin{bmatrix} -1/V_x & -a/V_x \\ -1/V_x & b/V_x \end{bmatrix} \begin{Bmatrix} V_{oy} \\ \Omega \end{Bmatrix} + g \begin{Bmatrix} \delta_F \\ \delta_R \end{Bmatrix} . \tag{A.4}$$

By combining equations (A.3) and (A.4), the steady state response can be easily obtained as:

$$\begin{Bmatrix} V_{oy}(0) \\ \Omega(0) \end{Bmatrix} = \frac{1}{(K_{us} + Lg/V_x^2)} \times \begin{Bmatrix} [(gb/V_x - D_R V_x)\delta_F + (ga/V_x + D_F V_x)\delta_R] \\ g/V_x(\delta_F - \delta_R) \end{Bmatrix} \tag{A.5}$$

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