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DECENTRALIZED CONTROL OF A LARGE PLATOON OF VEHICLES OPERATING ON A PLANE WITH STEERING DYNAMICS[1]

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Abstract: This paper studies the decentralized control of a large platoon of identical vehicles which are moving on a plane, where each vehicle is assumed to have steering dynamics; each vehicle in this case is assumed to be described by a 6^{th} order model and has two inputs: traction force and steering angle. A study of the string stability of the resulting platoon is made; in particular, it is shown that a decentralized controller consisting of non-identical controllers can be used to control the platoon such that it possesses string stability. An example of a 200-vehicle platoon is included to illustrate the results which can be obtained.

Keywords: Intelligent Transportation Systems, Decentralized Control, String Stability, Vehicle Control

1. INTRODUCTION

Due to the congestion of vehicles in highways, automated highway systems have recently become a research topic in transportation, and as a result there has been a good deal of attention paid recently to the problem of controlling a platoon of identical vehicles along a straight line e.g., see [1-10], [14-16]. In this case it is desired to find a controller for the platoon so as to bring about satisfactory tracking/regulation of the separation distance between a vehicle and its forward neighbour, independent of the velocity of the lead vehicle, where each vehicle has only limited communication with respect to its neighbours. One feature which arises in this problem is that it is necessary that the resultant closed loop system should be "string stable" [3] in order to prevent collisions occurring between vehicles, i.e. it is desired that the peak of the vehicles' separation distance should have the property that it is non-increasing along the platoon. In this problem, it is desired that the controller should have least complexity, i.e. ideally the controller would be decentralized, where each vehicle only measures the separation distance between itself and its neighbour, and such that a knowledge of the lead vehicle's velocity is not required. It is shown in [14], [15] that this can be accomplished using decentralized non-identical controllers.

This paper studies the same problem for a platoon of vehicles which move on a plane, and where steering of the lead vehicle now occurs in addition to velocity change. The lateral movement of such vehicles has been considered in [19] where the coordination between longitudinal and lateral parts has been highlighted. In [20] the impact of combined longitudinal/lateral/vertical control of the vehicles in a platoon has been considered; however the string stability of the platoon was introduced only for the longitudinal movement. In [17] it is shown how the maneuvers of the lead vehicle in a platoon can potentially effect the whole platoon, where only steering kinematics is included.

In this paper the string stability of a platoon, which has a closed loop controller applied to each vehicle, using a fully decentralized controller in both longitudinal and lateral directions is considered. In section 2 the nonlinear model of a vehicle with respect to the states relative to its immediate neighbour is described, and the model is then linearized about an equilibrium point. In section 3, the problem specification is described and in section 4 a controller design procedure is given. Finally simulation results for a platoon consisting of 200 vehicles with a decentralized controller applied is studied.

2. MODELING OF A VEHICLE WITH STEERING

In this paper we shall consider the case of a platoon of N identical vehicles, where the i^{th} vehicle and tire system is assumed to have the configuration given in Figures 1 and 2; in these Figures, the following notation is used to describe the vehicle (see Table 1 and Table 2):

Table 1 Description of parameters used to model vehicle

Par.	Description
x^a	Absolute x-position
y^a	Absolute y-position
θ^a	Absolute Orientation
v	Longitudinal velocity of the center of mass
ω	Angular velocity of the vehicle about its center of mass
β	Skidding angle

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F^f	Front wheel traction force (Input)
F^r	Rear wheel traction force (Input)
δ^f	Front wheel steering angle (Input)
δ^r	Rear wheel steering angle (Input)
f^f	Front wheel lateral force
f^r	Rear wheel lateral force
β^f	Front wheel skidding angle
β^r	Rear wheel skidding angle
v^f	Front wheel longitudinal velocity
v^r	Rear wheel longitudinal velocity
L^f	Distance between Front wheel and the center of mass
L^r	Distance between Rear wheel and the center of mass
μ^f	Front wheel Lateral friction coefficient
μ^r	Rear wheel Lateral friction coefficient
m	Mass of the vehicle
B	Linear Viscous Damping ratio (Air resistance)
J	Moment of inertia about the centre of Mass
b	Angular Viscous Damping ratio

Table 2 Description of relative parameters used to model vehicle

Par.	Description
R_i	Relative Distance from the previous agent
ϕ_i	Angle between the heading direction of a vehicle and vehicle-to-vehicle connection line (*following angle*)
θ_i	Relative Orientation (relative to the orientation angle of the previous vehicle)
v_i	Longitudinal velocity of the center of mass
ω_i	Angular velocity of the vehicle about its center of mass
β_i	Skidding angle

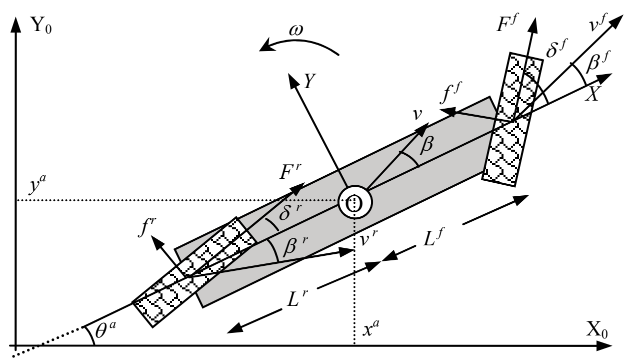

Figure 1 Geometrical parameters of a vehicle in the platoon

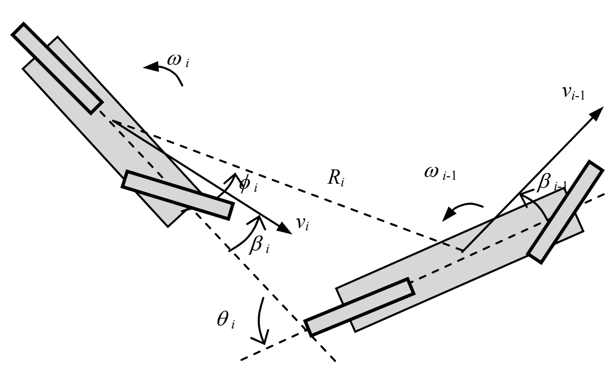

Figure 2 Relative parameters of a vehicle with respect to the vehicle in immediate neighbourhood

In this case, the "kinematics" and "dynamics" of the i^{th} vehicle can be described as follows [18]:

$$\begin{cases} \dot{x}_i^a = v_i \cos(\theta_i + \beta_i) \\ \dot{y}_i^a = v_i \sin(\theta_i + \beta_i) \\ \dot{\theta}_i^a = \omega_i \\ m\dot{v}_i = f_i^x \cos\beta_i + f_i^y \sin\beta_i \\ \dot{\beta}_i = -\dfrac{1}{mv_i} f_i^x \sin\beta_i + \dfrac{1}{mv_i} f_i^y \cos\beta_i - \omega_i \\ \dot{\omega}_i = \dfrac{1}{J}\tau_i^z \end{cases} \quad ,i=1, 2, \ldots, N \tag{1}$$

in which:

$$\begin{cases} f_i^x = -f_i^f \sin\delta_i^f + F_i^f \cos\delta_i^f - f_i^r \sin\delta_i^r + F_i^r \cos\delta_i^r - Bv_i \cos\beta_i \\ f_i^y = f_i^f \cos\delta_i^f + F_i^f \sin\delta_i^f + f_i^r \cos\delta_i^r + F_i^r \sin\delta_i^r - Bv_i \sin\beta_i \\ \tau_i^z = L^f(f_i^f \cos\delta_i^f + F_i^f \sin\delta_i^f) - L^r(f_i^r \cos\delta_i^r + F_i^r \sin\delta_i^r) - b\omega_i \end{cases} \tag{2}$$

where:

$$\begin{cases} f_i^f = f^f(\delta_i^f - \beta_i^f) \approx \mu^f(\delta_i^f - \beta_i^f) \\ f_i^r = f^r(\delta_i^r - \beta_i^r) \approx \mu^r(\delta_i^r - \beta_i^r) \end{cases} \tag{3}$$

and

$$\begin{cases} \tan(\beta_i^f) = \tan(\beta_i) + \dfrac{L^f\omega_i}{v_i\cos\beta_i} \Rightarrow \beta_i^f \approx \beta_i + \dfrac{L^f\omega_i}{v_i} \\ \tan(\beta_i^r) = \tan(\beta_i) + \dfrac{L^r\omega_i}{v_i\cos\beta_i} \Rightarrow \beta_i^r \approx \beta_i + \dfrac{L^r\omega_i}{v_i} \end{cases} \tag{4}$$

Here the control inputs to the vehicle are: F_i^f, F_i^r, δ_i^f, δ_i^r (see Table 1 for notation).

Since the absolute values of the position and orientation of the vehicles cannot be linearized about any equilibrium point, the relative measurements given in Figure 2 and Table 2 are used for the implementation of a nonlinear state-space model of the vehicle. By simple but massive geometrical calculations, the resulting state space equations of the i^{th} vehicle with respect to the relative parameters can then be obtained as:

$$\begin{cases} \dot{R}_i = -v_i\cos(\phi_i - \beta_i) + v_{i-1}\cos(\theta_i - \phi_i + \beta_{i-1}) \\ \dot{\phi}_i = \dfrac{1}{R_i} v_i \sin(\phi_i - \beta_i) - \omega_i + \dfrac{1}{R_i} v_{i-1}\sin(\theta_i - \phi_i + \beta_{i-1}) \\ \dot{\theta}_i = -\omega_i + \omega_{i-1} \\ m\dot{v}_i = f_i^x \cos\beta_i + f_i^y \sin\beta_i \\ \dot{\beta}_i = -\dfrac{1}{mv_i} f_i^x \sin\beta_i + \dfrac{1}{mv_i} f_i^y \cos\beta_i - \omega_i \\ \dot{\omega}_i = \dfrac{1}{J}\tau_i^z \end{cases} \tag{5}$$

where the notation used is described in Table 2. In this paper, we shall assume that the rear wheel traction force F_i^r and rear wheel steering angle δ_i^r are not used, i.e. the vehicle uses "front-wheel-drive" and "front-wheel-steering", so that $F_i^r = 0$ and $\delta_i^r = 0$, i.e. each vehicle has 2 control input signals given by F_i^f and δ_i^f. We shall also assume that the desired separation distance is the same for all vehicles and that the desired following angle is also the same for all vehicles. In addition, there are external inputs to vehicle i arising from vehicle i-1 given by $(v_{i-1}, \beta_{i-1}, \omega_{i-1})'$. Let the equilibrium point of (5) with respect to the constant control inputs $F_i^f = F_{eq}^f$, $\delta_i^f = \delta_{eq}^f$, and external inputs from vehicle (i-1) $\beta_{i-1}=\beta_{eq}$, $v_{i-1}=v_{eq}$, $\omega_{i-1}=\omega_{eq}$, be denoted by: $[R_{eq} \;\; \phi_{eq} \;\; \theta_{eq} \;\; v_{eq} \;\; \beta_{eq} \;\; \omega_{eq}]$.

2.1 Linearization of the i^{th} vehicle model

Given the constant control inputs:

$$F_{eq}^f = F^{f0} \text{ and } \delta_{eq}^f = 0 \tag{6a}$$

then on solving for the equilibrium point from (5), the following result is obtained:

$$R_{eq}=R^0,\ \phi_{eq}=\phi^0,\ \theta_{eq}=0,\ v_{eq}=v^0,\ \beta_{eq}=0,\ \omega_{eq}=0 \tag{6b}$$

where R^0 and ϕ^0 are not unique (Here R^0 and ϕ^0 depend on the formation geometry of the platoon of vehicles), and the following linearized model of the vehicle (5) about this equilibrium point (6a), (6b) is obtained as:

$$\begin{bmatrix}\Delta\dot{R}_i\\ \Delta\dot{\phi}_i\\ \Delta\dot{\theta}_i\\ \Delta\dot{v}_i\\ \Delta\dot{\beta}_i\\ \Delta\dot{\omega}_i\end{bmatrix}=\begin{bmatrix}0 & 0 & v^0\sin\varphi^0 & -\cos\varphi^0 & -v^0\sin\varphi^0 & 0\\ 0 & 0 & \frac{v^0}{R^0}\cos\varphi^0 & \frac{1}{R^0}\sin\varphi^0 & -\frac{v^0}{R^0}\cos\varphi^0 & -1\\ 0 & 0 & 0 & 0 & 0 & -1\\ 0 & 0 & 0 & -B/m & 0 & 0\\ 0 & 0 & 0 & 0 & -\frac{1}{mv^0}[\mu^f+\mu^r+Bv_i^0] & -1+\frac{1}{mv^{0\,2}}[-\mu^f L^f+\mu^r L^r]\\ 0 & 0 & 0 & 0 & \frac{1}{J}[-\mu^f L^f+\mu^r L^r] & -\frac{1}{Jv^0}[\mu^f L^{f\,2}+\mu^r L^{r\,2}]-\frac{b}{J}\end{bmatrix}\begin{bmatrix}\Delta R_i\\ \Delta\phi_i\\ \Delta\theta_i\\ \Delta v_i\\ \Delta\beta_i\\ \Delta\omega_i\end{bmatrix}$$
$$+\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{1}{m} & 0\\ 0 & \frac{1}{mv^0}[\mu^f+F^{f0}]\\ 0 & \frac{L^f}{J}[\mu^f+F^{f0}]\end{bmatrix}\begin{bmatrix}\Delta F_i^f\\ \Delta\delta_i^f\end{bmatrix}+\begin{bmatrix}\cos\varphi^0 & v^0\sin\varphi^0 & 0\\ -\frac{1}{R^0}\sin\varphi^0 & \frac{v^0}{R^0}\cos\varphi^0 & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}\begin{bmatrix}\Delta v_{i-1}\\ \Delta\beta_{i-1}\\ \Delta\omega_{i-1}\end{bmatrix} \tag{7}$$

In the case of a flock of birds, ϕ^0 often would be non-zero (of course, the model (5) would now be different), but for the case of a platoon of vehicles, ϕ^0 will normally be equal to zero. In this case, the following state-space model for vehicle i is obtained:

$$\begin{cases}\dot{x}_i = Ax_i + Bu_i + Ew_{i-1}\\ y_i = Cx_i\\ w_i = Lx_i\end{cases},\ i = 2, 3, \ldots, N \tag{8}$$

where:

$$x_i = [\Delta R_i \quad \Delta\phi_i \quad \Delta\theta_i \quad \Delta v_i \quad \Delta\beta_i \quad \Delta\omega_i]' \tag{9a}$$

$$u_i = [\Delta F_i^f \quad \Delta\delta_i^f]' \tag{9b}$$

$$w_{i-1} = [\Delta v_{i-1} \quad \Delta\beta_{i-1} \quad \Delta\omega_{i-1}]' \tag{9c}$$

$$y_i = [\Delta R_i \quad \Delta\phi_i \quad \Delta\theta_i]' \tag{9d}$$

$$w_i = [\Delta v_i \quad \Delta\beta_i \quad \Delta\omega_i]' \tag{9e}$$

and A, B, E, C, and L are as follows:

$$A=\begin{bmatrix}0 & 0 & 0 & -1 & 0 & 0\\ 0 & 0 & \frac{v^0}{R^0} & 0 & -\frac{v^0}{R^0} & -1\\ 0 & 0 & 0 & 0 & 0 & -1\\ 0 & 0 & 0 & -B/m & 0 & 0\\ 0 & 0 & 0 & 0 & -\frac{1}{mv^0}[\mu^f+\mu^r+Bv_i^0] & -1+\frac{1}{mv^{0\,2}}[-\mu^f L^f+\mu^r L^r]\\ 0 & 0 & 0 & 0 & \frac{1}{J}[-\mu^f L^f+\mu^r L^r] & -\frac{1}{Jv^0}[\mu^f L^{f\,2}+\mu^r L^{r\,2}]-\frac{b}{J}\end{bmatrix} \tag{10a}$$

$$B=\begin{bmatrix}0 & 0\\ 0 & 0\\ 0 & 0\\ \frac{1}{m} & 0\\ 0 & \frac{1}{mv^0}[\mu^f+F^{f0}]\\ 0 & \frac{L^f}{J}[\mu^f+F^{f0}]\end{bmatrix} \tag{10b}$$

$$E=\begin{bmatrix}1 & 0 & 0\\ 0 & \frac{v^0}{R^0} & 0\\ 0 & 0 & 1\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix} \tag{10c}$$

$$C=\begin{bmatrix}1 & 0 & 0 & 0 & 0 & 0\\ 0 & 1 & 0 & 0 & 0 & 0\\ 0 & 0 & 1 & 0 & 0 & 0\end{bmatrix} \tag{10d}$$

$$L=\begin{bmatrix}0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\end{bmatrix} \tag{10e}$$

2.2 Decomposition of the linearized model

It can be directly observed that the model (8) can be decomposed into two decoupled models (i) a "longitudinal model" and (ii) a "rotational model" and are given as follows:

Longitudinal Model:

$$\begin{bmatrix}\Delta\dot{R}_i\\ \Delta\dot{v}_i\end{bmatrix}=\begin{bmatrix}0 & -1\\ 0 & -B/m\end{bmatrix}\begin{bmatrix}\Delta R_i\\ \Delta v_i\end{bmatrix}+\begin{bmatrix}0\\ 1/m\end{bmatrix}\Delta F_i^f+\begin{bmatrix}1\\ 0\end{bmatrix}\Delta v_{i-1}$$
$$\Delta R_i = \begin{bmatrix}1 & 0\end{bmatrix}\begin{bmatrix}\Delta R_i\\ \Delta v_i\end{bmatrix},\quad \Delta v_i = \begin{bmatrix}0 & 1\end{bmatrix}\begin{bmatrix}\Delta R_i\\ \Delta v_i\end{bmatrix} \tag{11}$$

Rotational Model:

$$\dot{z}_i = \left[\begin{array}{c:c}A_{11} & A_{12}\\ \hdashline 0 & A_{22}\end{array}\right]z_i+\left[\begin{array}{c}0\\ \hdashline B_2\end{array}\right]\delta_i^f+\left[\begin{array}{c}E_1\\ \hdashline 0\end{array}\right]\Delta w_{i-1} \tag{12a}$$

$$\begin{bmatrix}\Delta\phi_i\\ \Delta\theta_i\end{bmatrix}=C^o z_i=\left[\begin{array}{cc:cc}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\end{array}\right]z_i \tag{12b}$$

$$\begin{bmatrix}\Delta\beta_i\\ \Delta\omega_i\end{bmatrix} = L^o z_i = \left[\begin{array}{cc|cc}0 & 0 & 1 & 0\\ 0 & 0 & 0 & 1\end{array}\right] z_i \qquad (12c)$$

$$\Delta\phi_i = C_m^o z_i = \begin{bmatrix}1 & 0 & 0 & 0\end{bmatrix} z_i \qquad (12d)$$

where:

$$\Delta w_{i-1} = \begin{bmatrix}\Delta\beta_{i-1} & \Delta\omega_{i-1}\end{bmatrix}',\ z_i = \begin{bmatrix}\Delta\phi_i & \Delta\theta_i & \Delta\beta_i & \Delta\omega_i\end{bmatrix}'$$

and:

$$A_{11} = \begin{bmatrix}0 & v^0/R^0\\ 0 & 0\end{bmatrix},\ A_{12} = \begin{bmatrix}-v^0/R^0 & -1\\ 0 & -1\end{bmatrix}$$

$$A_{22} = \begin{bmatrix}-\frac{1}{mv^0}[\mu^f+\mu^r+Bv^0] & -1+\frac{1}{mv^{0\,2}}[-\mu^f L^f+\mu^r L^r]\\ \frac{1}{J}[-\mu^f L^f+\mu^r L^r] & -\frac{1}{Jv^0}[\mu^f L^{f\,2}+\mu^r L^{r\,2}]-\frac{b}{J}\end{bmatrix}$$

$$B_2 = \begin{bmatrix}\frac{1}{mv^0}[\mu^f+F^{f0}]\\ \frac{L^f}{J}[\mu^f+F^{f0}]\end{bmatrix},\quad E_1 = \begin{bmatrix}\frac{v^0}{R^0} & 0\\ 0 & 1\end{bmatrix}$$

3. PROBLEM SPECIFICATION

Assumptions:

The controller to be used for the platoon is assumed to be fully decentralized without any communication existing between the vehicles, which implies that each vehicle can only measure the relative distance (R_i) and also the following angle (ϕ_i) between itself and its immediate neighbour; it is also assumed that the velocity (v_i) of the vehicle is measurable. It is further assumed that the velocity of the previous vehicle (v_{i-1}) is measurable; this can be obtained by taking the derivative of its relative distance from the immediate previous neighbour and subtracting the result from the velocity of the vehicle itself. The relative orientation (θ_i) is not easily measurable; however it is very useful in calculating the performance of the closed loop system, so it can be estimated, as well as the skidding angle (β_i) and angular velocity (ω_i) of the vehicle, using a full order or reduced order observer.

Although the vehicles of the platoon are assumed to be identical, it is not assumed necessarily that the decentralized controllers to be used in the platoon are identical. Figure 3 gives the structure of the closed loop system when decentralized control is used.

Problem Requirements:

It is desired to find a decentralized controller which has local inputs u_i and local outputs y_i for the system (8), to solve the servomechanism problem [11],[12] for (8) so that:

i- Asymptotic regulation occurs, i.e.,
$\lim_{t\to\infty} y_i(t) = (0,0,0)'$, i= 2, 3, …, N
for all constant linear and angular velocities $(\Delta v_0, \Delta\beta_0, \Delta\omega_0)'$ of the lead vehicle, i.e. "spacing distance control" and "following angle control" occurs,

ii- Asymptotic tracking occurs, i.e.,
$\lim_{t\to\infty} y_i(t) = \begin{bmatrix}\Delta R^{ref} & \Delta\phi^{ref} & 0\end{bmatrix}'$, i= 2, 3, …, N
for all constant reference signals $\Delta R^{ref} > 0$ and $-\pi/2 < \Delta\phi^{ref} < \pi/2$.

iii- The resultant closed loop system is asymptotically stable.

iv- Eventual String Stability [15], with respect to spacing distance, velocity, absolute orientation, angular velocity, following angle and skidding angle occurs.

It is to be noted that eventual string stability in a platoon is that property of the system, by which there exists an index N_0 for which the system is string stable [3] for vehicles with index $i > N_0$. It is also to be noted that for a platoon of identical vehicles, using decentralized control, it is impossible to achieve string stability if the decentralized controllers are identical [3], [14].

4. DEVELOPMENT OF DECENTRALIZED CONTROL FOR THE PLATOON

It is clear that the controller design for decentralized control of the platoon can be decoupled with respect to the longitudinal and rotational models (11), (12).

Existence conditions:

It can be directly verified from the structure of (11), (12), that the existence conditions for a solution to the decentralized robust servomechanism problem for (11) and (12), for constant tracking signals / disturbance signals always exists [11] for all parameters of (11), (12). Thus a decentralized controller which satisfies problem requirements (i), (ii), (iii) always exists. To determine if problem requirement (iv) is satisfied requires an examination of the actual controller which is used to control the system.

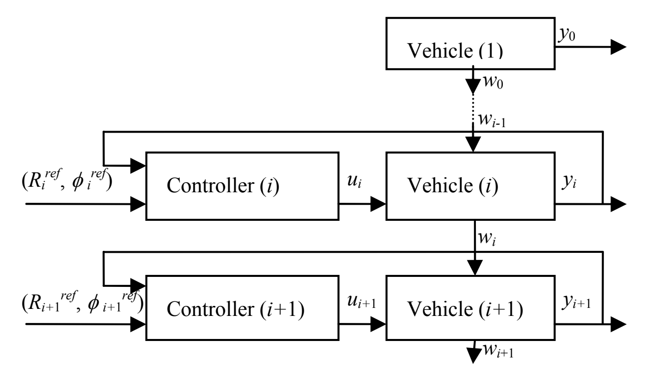

Figure 3 Flow of variables along the closed loop platoon, i= 2, 3, …, N where Controller (i), (i+1) denote decentralized controllers and Vehicle (1) is the lead vehicle of the platoon

4.1 Longitudinal Control:

Longitudinal control can be implemented using a decentralized *PID* term controller on each vehicle:

$$F_i^f = (P_i + \frac{I_i}{s} + D_i s)(R_i(s) - R^{ref}(s)) \ , i = 2 , 3,\ldots, N \qquad (13)$$

but to obtain string stability in the relative distance between vehicles using decentralized control, it is necessary to have non-identical controllers applied to the platoon of *N* identical vehicles [14]. In this case, the vehicles in the platoon are numbered so that controller *i* is applied to vehicle *i.* In particular, to obtain eventual string stability [15] in the relative distance, and bounded stability in the velocity of the vehicles, one can use the *PID* controller described in [15]; this controller is given by (13) where the *PID* gains are updated with respect to the vehicle index as follows:

$$\begin{cases} I_i = I_0 > 0 \\ P_i = P_0 + \alpha\, i \quad , \quad \alpha > 0 \\ D_i = D_0 + \beta\, i \quad , \quad \beta > 0 \end{cases} , i = 2 , 3,\ldots, N \qquad (14)$$

where $\beta \geq \max\left\{\sqrt{b^2/4 + m\alpha} - b/2 \ , \ m\alpha/b\right\}$ [15]; here $I_0>0$, $P_0>0$, and $D_0>0$ are chosen to stabilize the first vehicle (11) in the platoon so that it has a "reasonable non-oscillatory response". Because (11) is controllable and observable and of order n=2, this can always be achieved.

4.2 Rotational Control:

In finding a controller for the rotational model (12), it is assumed that only the following angle is measurable, and that the other bearing states: relative orientation, angular velocity, and skidding angle are estimated via an observer. This is always possible to do, since (12) is observable for the following angle and is controllable. It is to be noted that since the transfer function from the steering angle control input to any of the outputs is a type II system, and that the transfer function from the skidding angle and angular velocity of the previous vehicle to the output of the system is at least a type I system, then no integrator is required to regulate the outputs of the system (12). Thus the following type of controller can be applied:

$$\begin{cases} \dot{\eta}_i = (A^o - \Lambda^o C_m^o - B^o K^o)\eta_i + \Lambda^o y_i^o \\ u_i^o = -K^o \eta_i \end{cases} , i = 2, 3, \ldots, N \qquad (15)$$

where

$$A^o = \begin{bmatrix} A_{11} & A_{12} \\ 0 & A_{22} \end{bmatrix},\ B^o = \begin{bmatrix} 0 \\ B_2 \end{bmatrix},\ E^o = \begin{bmatrix} E_1 \\ 0 \end{bmatrix},\ y_i^o = \Delta\phi_i,\ u_i^o = \delta_i^f$$

are defined in (12), and Λ^o and K^o are the observer gain, and controller gain respectively which can be found using conventional linear design methods. Unlike the case of longitudinal control, the rotational control uses identical controllers given by (15) for all of the platoon vehicles.

4.3 Properties of Transfer Function of Resultant Closed loop System

The following closed loop transfer functions are obtained on applying the longitudinal controller (14) to (11) and the rotational controller (15) to (12):

Transfer functions associated with the longitudinal behaviour:

On applying the 3-term controller (14) to (11), the transfer functions associated with the relative distance and velocity between vehicles from one vehicle to another can be obtained as:

$$\frac{R_i(s)}{R_{i-1}(s)} = \frac{(D_{i-1}s^2 + P_{i-1}s + I_{i-1})}{s^2(ms+B) + (D_i s^2 + P_i s + I_i)} , i = 3 , 4,\ldots, N \quad (16)$$

$$\frac{v_i(s)}{v_{i-1}(s)} = \frac{(D_i s^2 + P_i s + I_i)}{s^2(ms+B) + (D_i s^2 + P_i s + I_i)} , i = 2 , 3,\ldots, N \quad (17)$$

Transfer functions associated with the rotational behaviour:

Consider the closed loop system obtained by applying the controller (15) to (12) and define the following transfer function matrices:

$$\begin{bmatrix} \begin{pmatrix} \phi_{i+1} \\ \theta_{i+1} \end{pmatrix} \\ \begin{pmatrix} \beta_{i+1} \\ \omega_{i+1} \end{pmatrix} \end{bmatrix} = \begin{bmatrix} G_{yy}(s) & G_{yv}(s) \\ G_{vy}(s) & G_{vv}(s) \end{bmatrix} \begin{bmatrix} \begin{pmatrix} \phi_i \\ \theta_i \end{pmatrix} \\ \begin{pmatrix} \beta_i \\ \omega_i \end{pmatrix} \end{bmatrix} \qquad (18a)$$

and

$$\left[\begin{pmatrix} \beta_i \\ \omega_i \end{pmatrix}\right] = H_{vy}\left[\begin{pmatrix} \phi_i \\ \theta_i \end{pmatrix}\right] \qquad (18b)$$

thus from (15): $u_i^o(s) = G_c(s) y_i^o(s)$

where

$$G_c(s) = \frac{u_i^o}{y_i^o} = -K^o\left(sI_4 - (A^o - \Lambda^o C_m^o - K^o B^o)\right)^{-1}\Lambda^o \qquad (19)$$

and so from (19) and (12), the following result is obtained:

$$G_{vv}(s) = L^o\left(sI_4 - A^o - B^o G_c(s) C_m^o\right)^{-1} E^o \qquad (20)$$

Also $G_{yv}(s)$ can be directly obtained from (19) and (12) as:

$$G_{yv}(s) = C^o\left(sI_4 - A^o - B^o G_c(s) C_m^o\right)^{-1} E^o \qquad (21)$$

and $H_{vy}(s)$ can be obtained from (12) as follows:

$$H_{vy}(s) = (sI_2 - A_{22})^{-1}(B_2 G_c(s)(1 \quad 0)) \qquad (22)$$

where A_{22} and B_2 are given in (12) and $G_c(s)$ is given in (19), and finally $G_{yy}(s)$ can be obtained from (21) and (22) on noting that:

$$G_{yy}(s) = G_{yv}(s)\, H_{vy}(s) \tag{23}$$

A sufficient condition for the rotational closed loop system to be string stable with respect to the rotational states is that:

$$\bar{\sigma}\left|G_{vv}(j\omega)\right| \le 1 \tag{24a}$$

$$\bar{\sigma}\left|G_{yy}(j\omega)\right| \le 1 \tag{24b}$$

where $\bar{\sigma}$ represents the maximum singular value, and so in this case, one would design the controller (15) to satisfy (24) if possible. In the examples, it is shown that the resultant closed loop system has eventual string stability, with respect to the bearing of the vehicles.

5. EXAMPLE

In the following example, a platoon of N=200 identical vehicles with the nonlinear model (5) and model parameters given in Table 3, is considered. The reference for the *relative distance* is set to R_i^{ref}=15 m and for the *following angle* to ϕ_i^{ref}=0.

Table 3 Numerical values for the parameters of the vehicle

Par.	Value	Par.	Value
L^f	2 *m*	*m*	1 *Kg*
L^r	2 *m*	*B*	1 N/ms^{-1}
μ^f	5 *N/rad*	*J*	1 $Nm/rad.s^{-2}$
μ^r	5 *N/rad*	*b*	1 $Nm/rad.s^{-1}$

The longitudinal controller parameters of (14) are obtained using the method of [15] and are given as follows:

$P_0 = 5$, $I_0 = 1$, $D_0 = 5$, $\alpha = 0.1$, $\beta = 0.2$

The rotational controller of equation (15) has been obtained by placing the closed loop poles at [-1.0, -1.5, -2.0, -2.5] and the observer poles at [-10, -15, -20, -25] which results in:

$$K^o=[-0.1125 \quad -0.1680 \quad 0.0672 \quad 0.0394]' \tag{25}$$

and

$$\Lambda^o=[65 \quad 11515 \quad 12870 \quad -2779]. \tag{26}$$

In the example, an approximately 100-degree left-turn maneuver is applied to the leader and the response of the resulting nonlinear platoon with a closed loop fully decentralized controller is considered. The resulting historical footsteps of the vehicles obtained, using the resultant decentralized controller are shown in Figure 4, and Figure 5 shows the time response of the platoon. Figures 6 and 7 give a Bode plot of the magnitude of the resultant closed loop transfer function matrices for the obtained rotational states.

Discussion:

It is seen that the platoon under closed loop decentralized controller closely follows the leader in tracking a 100-degree turn, and that the resulting system has excellent eventual string stability. Figure 5 shows that we have: (i) eventual string stability for relative distance and velocity and (ii) desired behaviour for all the remaining responses. Figures 6 and 7 show that the sufficient conditions (24a), (24b) do not hold in spite of the fact that the responses in Figure 5 are completely satisfactory.

6. CONCLUSIONS

In this paper, a study of a large platoon of identical vehicles moving on a plane is made, in which a decentralized controller, consisting of non-identical controllers for each vehicle, is used. The model for the vehicle contains both dynamics and kinematics, with reasonably complete steering dynamics. In this case, it is shown that the longitudinal and rotational movements of the linearized model of the vehicle are decoupled, and thus that the decentralized controller design problem for the platoon, can be decomposed into two classes of problems: (i) the longitudinal controller design problem, and (ii) the rotational controller design problem. A method for constructing a decentralized controller which possesses eventual string stability, and which tracks constant reference separation distances and constant steering angles is then described, and an example of this decentralized controller design is made for a platoon consisting of 200 vehicles, in which it is shown that the resulting nonlinear vehicle model closed loop system has excellent eventual string stability.

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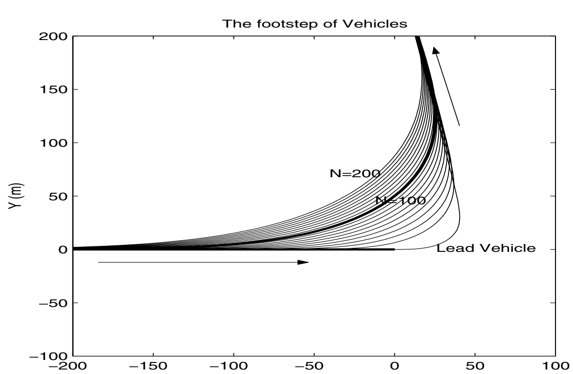

Figure 4 Footstep of the vehicles near the ramp.

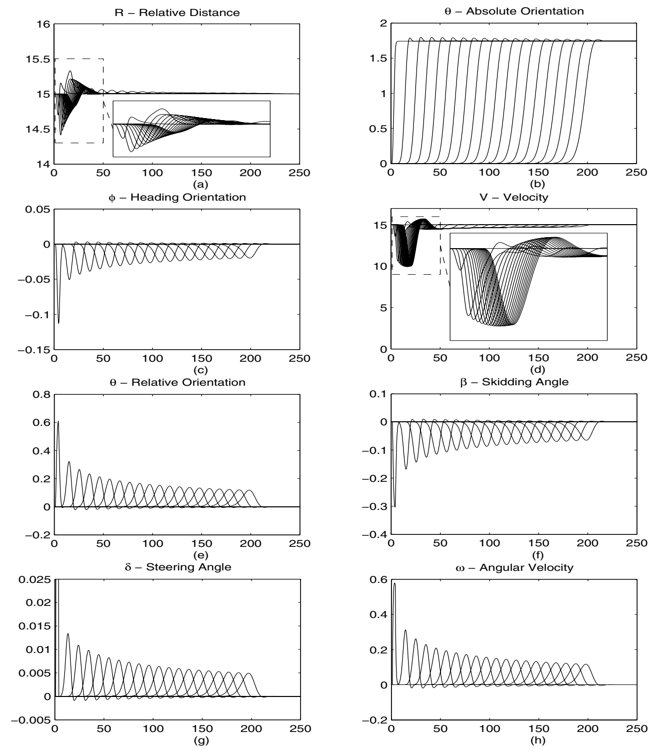

Figure 5 Time responses of the platoon of N=200 vehicles
Note: responses plotted only for vehicles i=1, 10, 20, 30,…,200

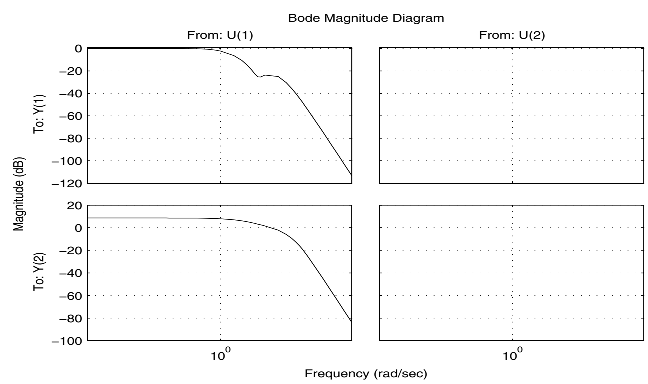

Figure 6 Bode diagram for the transfer function $[\beta_{i+1} \quad \omega_{i+1}]' = G_{vv}(s)[\beta_i \quad \omega_i]'$

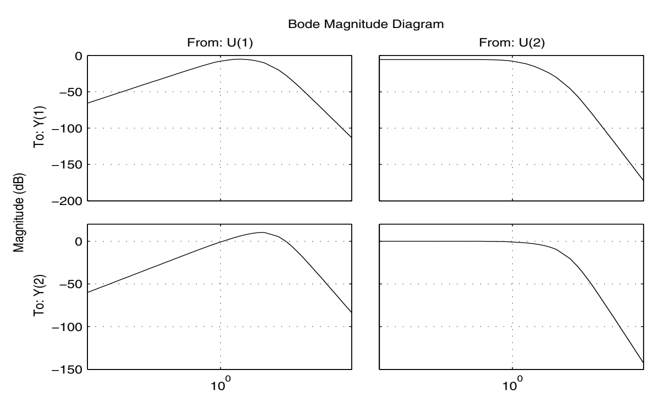

Figure 7 Bode diagram for the transfer function $[\phi_{i+1} \quad \theta_{i+1}]' = G_{yy}(s)[\phi_i \quad \theta_i]'$