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Influence of a slowdown warning system on a multi-vehicle stream

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***Abstract*— In this paper, we discuss the concept of a slowdown warning system in automobiles. If a driver on a highway decelerates suddenly or progresses abnormally slowly (thereby posing a hazard to the vehicles behind him), then, with such a system, all the cars behind him are provided information of this, near simultaneously. This advance information gives the drivers additional time to react, in anticipation of an impending slow-down, and this helps to alleviate collisions. Furthermore, it is seen that even if only a fraction of the cars in a platoon are equipped with such a system, this can still be sufficient to alleviate crashes even in the unequipped cars. This partial equipage also has the ability to considerably weaken the shock waves that would otherwise have occurred, if all cars were unequipped.**

I. INTRODUCTION

Car pile-up crashes have been a frequent occurrence on highways, especially in bad weather conditions [5], [6], [7], [8]. The cause for such crashes is that each driver gets warned of an impending slowdown ahead, only when the brake-lights of the car immediately in front of him, turn on. So, if we consider a platoon of cars travelling on a single lane, and the lead car executes an abrupt deceleration, this information is propagated from car to car in a staggered fashion (Fig 1a), as the brake-lights of each car come on, one after the other. There is an associated delay τ for each car as the information propagates through the line of cars, (τ comprises of the time it takes for each driver to realize that the front car's brake-lights are on, and to react with a corresponding deceleration that turns his own brake-lights on). Thus if car 1 (i.e. the lead car) poses a hazard by a sudden deceleration that turns on its brake-lights at time $t = 0$, then the kth car $(k > 2)$ gets warned of the slowdown ahead at $t = (k - 2)\tau$, and turns on its own brake-lights $(k - 1)\tau$ seconds after the first generation of the hazard. In this way, the driver's reaction time τ gets continously accumulated as the information propagates through the line of cars. As demonstrated in the next section, this can lead to a situation wherein cars with higher position number k in the platoon are more likely to crash. This illustrates that this mode of transmission of information of a slowdown (from car to car as in Fig 1a) is too slow, and does not allow the drivers that are far behind in the platoon, sufficient time to react. Car pile-up crashes are the result.

It is seen that these pile-ups often begin to occur several cars behind the car that first caused the hazard. One possible hypothesis that might explain their occurrence is the onset of asymptotic instability in a line of cars. We use the term asymptotic instability in the context defined in [1] : If a car in a platoon executes a velocity change, then as the resulting velocity fluctuation propagates down the platoon; if it grows in magnitude with increasing car index, then the string is asymptotically unstable. Furthermore, for such a string, the minimum value of the inter-car separation (and also the velocity) keeps decreasing with increasing car index. Thus if the string is sufficiently long, pile-ups will begin to occur several cars downstream in the string. The notion of asymptotic stability is closely related to the notion of string stability [4], in particular l_∞ string stability.

In this paper, we discuss the concept of a slowdown warning system that can significantly reduce the occurrence of pile-up crashes. For this, we discuss a slowdown warning concept, whereby cars are equipped with a slowdown warning system. A car equipped with a slowdown warning system has the ability to (a) Automatically transmit a warning signal when it decelerates abruptly, or its speed becomes dangerously low for highway driving conditions, and (b) Receive a slowdown warning signal, and alert the driver accordingly, if it deems the signal to be relevant. With this system, information of the lead car's deceleration is propagated to all the cars, near simultaneously, as in Figure 1(b). We show that if all cars are equipped with the slowdown warning system, it prevents the onset of asymptotic instability in the line of cars, and thus prevents the occurrence of pile-ups.

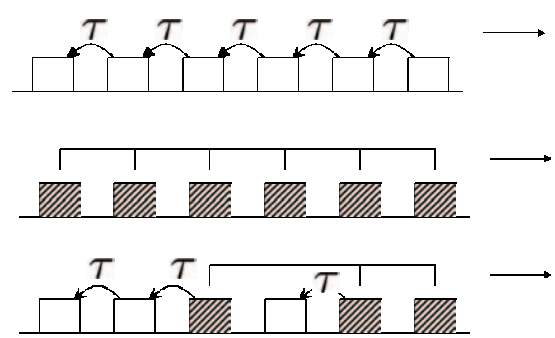

Fig. 1. Propagation of slowdown information when (a):No cars are equipped (b):All cars are equipped (c):Some cars are equipped. Equipped cars are shown as hatched rectangles.

In a real-life scenario, however, it would be unrealistic to assume that all the cars in a platoon will indeed be equipped with a slow-down warning system. Indeed, the long lifetime of passenger cars and other road vehicles guarantees that, at least for a while, not all cars will be equipped with advanced computation and communication capabilities. It is pertinent therefore, to examine the influence of such a system, when

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only a (randomly selected) fraction of the total number of cars in the platoon are equipped with the system. In this case, information is propagated as in Fig 1(c). Thus if car 1 transmits a warning signal at time $t = 0$, then since cars 2 and 4 are equipped with the warning system, both of them receive the warning signal at $t = 0$. Furthermore, the unequipped cars 5 and 6 now receive (indirect) information of the slowdown at τ and 2τ, respectively, which contributes significantly to safety improvement, as compared to the case in Figure 1(a) when cars 5 and 6 receive the warning information only at $t = 3\tau$ and 4τ, respectively. The effects of this kind of partial deployment are also studied in this paper. We show that, in most cases, even if only a fraction of the cars are equipped, their influence on the traffic flow can be sufficient to alleviate the possibility of crashes even in the unequipped cars. This is because the equipped cars tend to break the trend of asymptotic instability as it propagates down the line of cars.

The use of inter-vehicle communication for enhancement of vehicle safety has been discussed in the past, for eg., in [15], [16], [17], though not in the context of alleviating car pile-up crashes with partial inter-vehicle communication. Besides this, while there has been recent research activity on mixed systems (comprising of semi-automated and manual vehicles), [9], [10], these systems do not have inter-vehicle communication. We believe there is value therefore in conducting research on the analysis of mixed systems with partial inter-vehicle communication, involving information propagation as in Fig. 1(c). This paper is a step in that direction, and addresses the specific issues of car pile-up crashes, shock waves and their alleviation. In [19], [20], we discussed results of some simulation studies using microscopic and cellular automaton models incorporating the slowdown warning system and details of road test results that were performed after equipping cars with the slowdown warning system.

This paper is organized as follows. Section 2 demonstrates the analogy between the occurrence of a pile-up crash and the occurrence of asymptotic instability along a line of cars and also demonstrates how equipping the cars with a slowdown warning system has a beneficial effect on the asymptotic stability. We also present some simulation results. In section 3, we model the effects of the slowdown warning system using partial differential equation (PDE) models. These models demonstrate the occurrence of a shock wave in the traffic flow in the case when information (of the lead car's slowdown) is propagated as in Fig 1(a), and the elimination or weakening of this shock wave in the instances when information is propagated as illustrated in Figs 1(b),(c). Finally, section 4 presents the conclusions.

II. EFFECT OF A SLOWDOWN WARNING SYSTEM ON ASYMPTOTIC STABILITY

A. Simulation Studies

Consider a string of cars driving on a single-lane highway. We assume that at $t = 0$, they are all driving with equal speeds and equal inter-car distances. The string of cars is modeled as an inter-connected system, with each car-driver system forming one element of the inter-connected system. The driver of the nth car is modeled by the following [2], [3]:

$$\frac{dV_n(t)}{dt} = K(x_{n-1}(t-\tau) - TV_n(t-\tau)) + \lambda(V_{n-1}(t-\tau) - V_n(t-\tau)) \quad (1)$$

where V_n indicates the velocity of the nth car and x_{n-1} represents the inter-car distance between the nth and the $(n-1)$th cars, with car 1 being the lead car. τ indicates the response delay of each car-driver system and T indicates the time headway (in seconds) maintained by each car to the car immediately ahead of it. λ represents the sensitivity of each driver to the velocity difference between his car and the one immediately ahead while K is the sensitivity to the difference between the desired inter-car distance and the true inter-car distance. The desired inter-car distance of each driver (to the car ahead) is proportional to his/her velocity.

Let the cars be initially travelling at typical highway speeds of about 30 meters/sec, (i.e. 67.5 mph), with the inter-car distance being 36 meters (i.e. $T = 1.2sec$). At $t = 5sec$, the lead car begins to execute an abrupt deceleration, and decelerates continously for 5 seconds. We now present simulations showing the effect of the lead car's deceleration (on the cars behind), when information of this deceleration is transmitted in each of the modes demonstrated in Figure 1.

Refer Figure 2, which shows the velocity and inter-car distance profiles of 10 cars, when the information of the lead car's deceleration is transmitted from car to car, as in Figure 1(a). In these simulations, we use $\tau = 0.6sec$ for a driver normally, this reduces to $\tau = 0.4sec$ when the brake-lights of the car immediately ahead of him come on. (This moderately reduced value of τ is indicative of the increased driver alertness). It can be seen that the values of the minimum car velocity and minimum inter-car distance keep decreasing with increasing car index (i.e. the string is asymptotically unstable), until car 6 is rear-ended by car 7, and crashes occur for all the cars behind. It can be seen from the figure that if there were more cars behind car 10, they too would all collide, thus leading to a pile-up.

The onset of asymptotic instability leading to the occurrence of such a pile-up can be attributed to the following reason : information of the deceleration of the lead car is transmitted from car to car in a staggered fashion, viz., by the brake-lights of the successive cars coming on one after the other, and this rate of information travel is too slow to give the driver sufficient time to react to avoid the imminent collision.

Now, consider a scenario when all the 10 cars are equipped with the slowdown warning system, and the lead car executes an identical deceleration profile. In this case, all the cars get informed of the slowdown ahead, near simultaneously, from the instant the lead car begins to

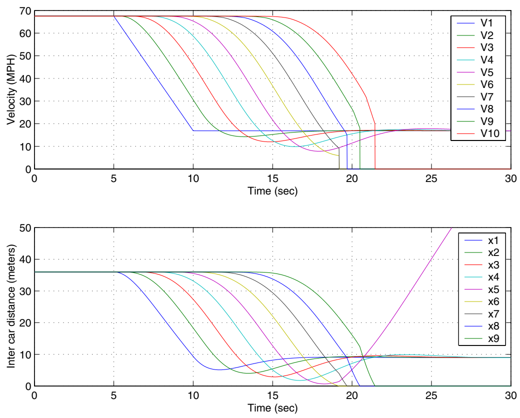

Fig. 2. All cars unequipped

decelerate (i.e. information of the lead car's deceleration is propagated as in Figure 1(b)). They are therefore able to react much earlier (car 10, for example, is able to react τ seconds after the lead car begins to decelerate, as opposed to 9τ seconds that it would otherwise have taken, if all cars were unequipped). We make the reasonable assumption that on receipt of the slowdown warning signal, each driver attempts to increase his time headway to the car in front of him, in anticipation of the imminent slowdown. Additionally, on receipt of the warning signal, the response delay τ of each driver reduces from its initial value of $\tau = 0.6sec$ to $\tau = 0.4sec$. The result is shown in Figure 3, where, on receipt of the warning signal, each driver tries to increase the time headway to the car immediately ahead (from the original $T = 1.2sec$ to $T = 1.65sec$). The trend of decreasing values of the minimum car velocity and minimum inter-car distance (with increasing car index) is no longer seen. In other words, the string of cars is asymptotically stable.

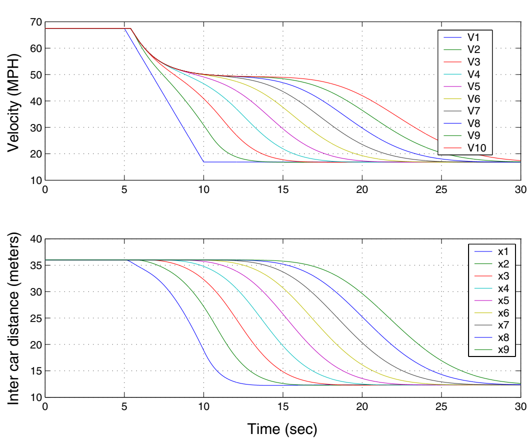

Fig. 3. All cars equipped

As mentioned earlier, it is unreasonable to assume that in a real-life scenario, all the cars would indeed be equipped with a slowdown warning system. It is pertinent, therefore, to examine the influence of the system when only a fraction of the cars are equipped. In other words, only some cars possess long distance sensing capabilities, while other cars possess only local (i.e. near neighbor) sensing capabilities. It turns out however, that in many cases, even if a fraction of the cars are equipped, this can still be sufficient to break the trend of asymptotic instability as it propagates down the line of cars, and this can prevent pile-up crashes. This is illustrated in Figure 4, where only cars 7 and 9 are equipped. It is seen that after the lead car decelerates, there is an onset of asymptotic instability in cars 2 to 6 - evident from the decreasing value of the minimum car velocity with increasing car index. However, the fact that car 7 is equipped breaks this trend, and in fact, the minimum value of V_7 (as also x_6) is higher than V_6 (respectively, x_5). Furthermore, since car 8 is unequipped, it re-initiates the trend of asymptotic instability and therefore the minimum value of V_8 (as also x_7) is indeed lower than that of V_7 (respectively, x_6); yet it is higher than that in the case when car 7 was unequipped (see Figure 2). Similarly, since car 9 is equipped, not only is the minimum value of x_8 high enough, but also that of x_9 is higher than what it was when car 9 was unequipped. Consequently, no crashes occur. This shows that it is possible that even if a fraction of the cars are equipped, they are able to ensure the safety of not only themselves, but the unequipped cars as well.

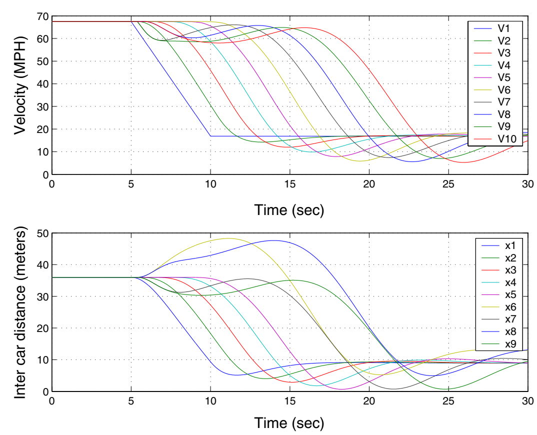

Fig. 4. Only cars 7 and 9 equipped

B. Analytic Studies

An analytic reasoning of the simulation results demonstrated above proceeds as follows : Let us rewrite eqn. (1) as

$$\frac{dV_n(t)}{dt} = K(d_n(t-\tau) - d_{n-1}(t-\tau) - TV_n(t-\tau)) + \lambda(V_{n-1}(t-\tau) - V_n(t-\tau)) \quad (2)$$

where d_n represents the absolute distance of the nth car with respect to some inertial frame. Let the lead car execute a displacement profile given by $d_1 = f_1 e^{i\omega t}$. Since eqn. (2) is linear, the steady state response of the nth car would be a displacement profile given by $d_n = f_n e^{i\omega t}$, where f_n can be found by substituting d_n and d_{n-1} in eqn. (2). Doing so, we get :

$$f_n = \frac{K + \lambda i\omega}{-w^2 e^{iw\tau} + K + i\omega KT + i\omega\lambda} f_{n-1} \quad (3)$$

from which it can be inferred that

$$f_{n+1} = \left(\frac{K + \lambda i\omega}{-w^2 e^{iw\tau} + K + i\omega KT + i\omega\lambda}\right)^n f_1 \quad (4)$$

From eqn. (3), we obtain that $|f_n| < |f_{n-1}|$ if

$$2K\cos(\omega\tau) + 2\omega KT\sin(\omega\tau) + 2\omega\lambda\sin\omega\tau < \omega^2 + K^2T^2 + 2\lambda KT \quad (5)$$

In other words, if eqn. (5) is satisfied, then asymptotic stability [1] (recall the definition given in Section I) is guaranteed for all ω. Now, from eqn. (3), it can be inferred that for small, non-zero values of ω, as T increases, $\frac{|f_n|}{|f_{n-1}|}$ decreases. At the same time, for $\omega = 0$, we have that $\frac{|f_n|}{|f_{n-1}|} = 1$ for any T. Thus, we can infer that above some threshold value of T, the maximum value of $\frac{|f_n|}{|f_{n-1}|}$ is unity, and this value occurs at $\omega = 0$. To find this threshold value of T, we substitute $\omega = 0$ in eqn. (5), and doing so, we obtain the requirement for asymptotic stability as

$$K^2T^2 + 2\lambda KT > 2K \quad (6)$$

Eqn. (6) shows that for fixed values of K and λ (that ensure stable, non-oscillatory behavior of eqns. (1) and (2) for the given τ), we can see that increasing values of T lead to increasing levels of asymptotic stability. Thus, for the $((n-1), n)$th car pair, if T is small enough so that eqn. (6) is violated, it implies asymptotic instability for this pair. In other words, the $(n-1)$th car's velocity perturbation gets amplified at the nth car by a factor greater than unity; and this maximum amplification occurs at some non zero value of ω. On the other hand, if the $((n-1), n)$th car pair share a T large enough so that eqn. (6) is satisfied, then this implies asymptotic stability for this pair. Fig. 5 shows $\frac{|f_n|}{|f_{n-1}|}$ for two different values of T, for one of which the asymptotic stability condition is violated, and another for which it is satisfied.

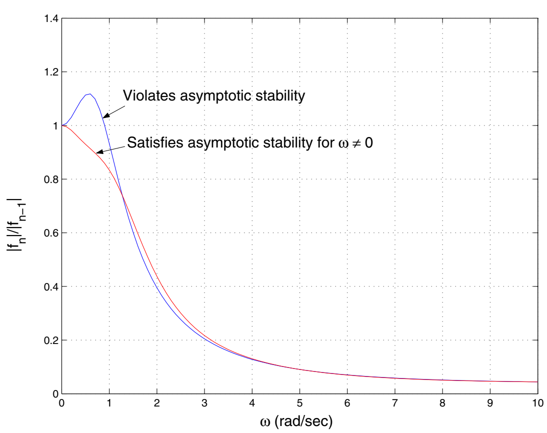

Fig. 5. $\frac{|f_n|}{|f_{n-1}|}$ for different values of T

Now, the inter-car distance is given by $x_n = d_{n-1} - d_n = (f_{n-1} - f_n)e^{i\omega t}$. Occurrence of a collision is indicated by the inter-car distance dropping to zero, thus if pile-ups begin to occur from the pth car onwards, then p is found from the smallest n that causes $|f_{n-1} - f_n|$ to become greater than the initial inter-car distance M, i.e. p is obtained from the expression

$$\frac{|K + \lambda i\omega|^{p-1}| - \omega^2 e^{i\omega\tau} + i\omega KT|}{| - \omega^2 e^{i\omega\tau} + K + i\omega KT + i\omega\lambda|^p} > M \quad (7)$$

We can use this p to find a quantity γ, that represents the maximum level of asymptotic instability that can be reached, before pile-ups begin to occur.

$$\gamma = \left|\frac{K + \lambda i\omega}{-\omega^2 e^{i\omega\tau} + KTi\omega + K + \lambda i\omega\lambda KT}\right|^p \quad (8)$$

Thus, for the nth car, as long as $\frac{|f_n|}{|f_1|} < \gamma$, it will not collide with the $(n-1)th$ car.

If all the N cars in the string are unequipped, and they are driving with a $T = T_1$ that is small enough such that eqn. (6) is violated, then pile-ups will begin to occur. These pile-ups will begin to occur from that car onwards, at whose location the level of asymptotic instability exceeds γ.

If all the N cars in the string are equipped, and we assume that on receipt of the slowdown warning signal, they increase their headway to a value $T = T_2$ such that eqn. (6) is satisfied, then there is no asymptotic instability whatsoever and no pile-ups will occur. If only a fraction of the N cars is equipped, then these equipped cars can serve to keep the level of asymptotic instability below the threshold that would lead to pile-ups. There would exist a critical fraction of these equipped cars (with an associated critical distribution of these equipped cars in the string) above which crashes can be averted, and below which crashes cannot be averted. Expressions for determining this critical fraction can be obtained as follows :

Considering now the nth car, if m cars in front of it are equipped and $n-m-1$ cars are unequipped, then we have

$$f_n = \left(\frac{K + \lambda i\omega}{-w^2 e^{iw\tau} + K + i\omega KT_1 + i\omega\lambda}\right)^{n-m-1} \times \left(\frac{K + \lambda i\omega}{-w^2 e^{iw\tau} + K + i\omega KT_2 + i\omega\lambda}\right)^m f_1 \quad (9)$$

where T_1 represents the time headway of the unequipped cars and T_2 is the time headway of the equipped cars.

Therefore, if T_1, T_2, m, n are such that

$$\left|\frac{K + \lambda i\omega}{-w^2 e^{iw\tau} + K + i\omega KT_1 + i\omega\lambda}\right|^{n-m-1} \times \left|\frac{K + \lambda i\omega}{-w^2 e^{iw\tau} + K + i\omega KT_2 + i\omega\lambda}\right|^m < \gamma \quad (10)$$

then the nth car will not be involved in the pile-up. Additionally, if all of $f_1, f_2, ..., f_n$ are less than γ, then all the cars $1, 2, ..., n$ will not be involved in a pile-up.

Fig. 6 illustrates the above discussion for the simulation scenario shown in Figs. 2-4 for the critical frequency (i.e. the frequency at which the unequipped cars show the maximum asymptotic instability). It shows the build up of asymptotic instability in the case of all cars being unequipped and the absence of asymptotic instability in the case of all cars being equipped. Finally, in the case of partial equipage, it shows the manner in which the equipped cars tend to keep the level of asymptotic instability below the threshold represented by γ. In the case of partial equipage, cars 7 and 9 are the equipped cars (which was the simulation scenario of Fig. 4).

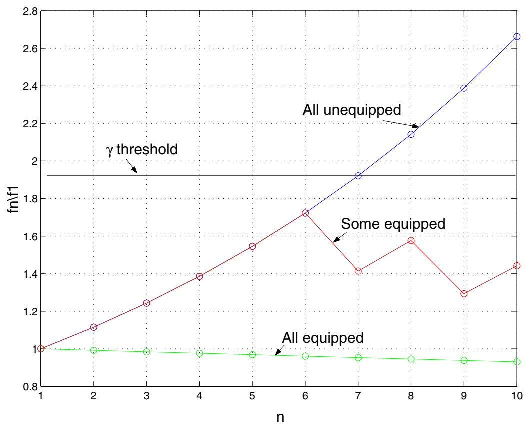

Fig. 6. Asymptotic instability levels at each car

Eqn. (10) thus gives an expression for the precise number of equipped cars required to prevent pile-up crashes in a mixed stream (comprising of equipped and unequipped cars). This section thus demonstrates how even partial equipage of the slowdown warning system can be sufficient to alleviate pile-up crashes.

III. MODELING VIA PDE'S

In this section, we discuss a continuum approach for modeling the effects of the slowdown warning system. For the purpose of this paper, we use only simple first order models based on the continuity equation [14]. The objective is to analyze the influence of the slowdown warning system on the kinematic and shock waves [12], [11] that arise in traffic flow.

The standard form of the continuity equation is :

$$\frac{\partial \rho}{\partial t} + \frac{\partial(\rho v)}{\partial x} = 0 \tag{11}$$

where $\rho(x)$ and $v(x)$ represent the normalized distributions of car density and car velocity respectively, as a function of x. x and t also represent normalized quantities, and the normalization is such that each of the quantities x, t, $\rho(x)$ and $v(x)$ can take values between 0 and 1. It is customary in traffic flow literature [14], [13] to represent velocity as a function of the car density, i.e. $v = v(\rho)$. Different forms of $v(\rho)$ have been proposed in the literature. In this paper, we choose one of the simplest forms, i.e. $v(\rho) = 1 - \rho^2$. This form has been discussed by researchers in the past [14], [13]; note that it satisfies the reasonable assumptions that when $\rho = 0$, then $v = 1$ (which represents the maximum possible value of car velocity), and $v = 0$ when $\rho = 1$ (which represents all the cars being lined up bumper-to-bumper).

Substituting this form of $v(\rho)$ in eqn. (11), we get

$$\frac{\partial \rho}{\partial t} + (1 - 3\rho^2)\frac{\partial \rho}{\partial x} = 0 \tag{12}$$

Eqn. (12) represents the PDE model that we shall use for the case when all cars are unequipped.

Now, if we consider a situation wherein $\rho(x)$ is initially uniform over all $x \in [0, 1]$, and a car in this control volume suddenly declerates abruptly, its effect can be modeled as one that creates a density profile represented by the red curve in Fig. 7(a). If we now solve eqn. (12) with the initial condition on ρ as given by the red curve in Fig. 7(a) and the boundary condition given by $\rho(1, t) = \rho(1, 0)$ then we see that the initial condition propagates in a manner to form a shock. Fig. 8 then shows the individual car trajectories on the (x, t) plane, and the effect of the shock can be seen on this plane too.

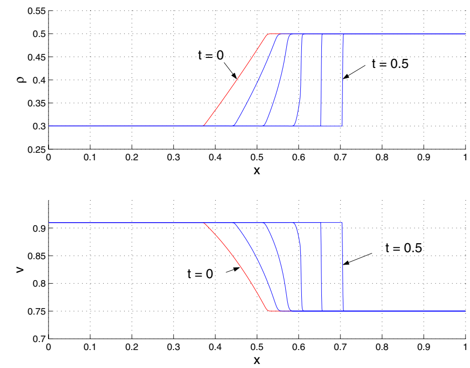

Fig. 7. Density and velocity distributions when no cars are equipped

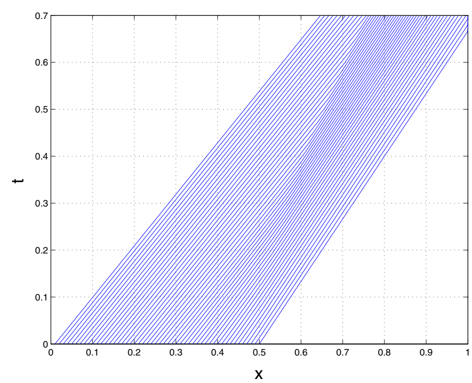

Fig. 8. Car Trajectories when none are equipped

We now model the case when cars are equipped with the slowdown warning system. In this model, we assume that along with the warning signal, information of the maximum density ahead (which includes the value of the maximum density ahead, as well as its location) is also broadcast to the equipped cars, with infinite speed. We also assume that when an equipped car receives the slowdown warning signal, it adjusts its velocity to follow the rule -

$$v(\rho) = \alpha(1 - \rho^2) + (1 - \alpha)(1 - max_{[x,1]}\rho^2) \tag{13}$$

i.e. an equipped car adjusts its velocity in correspondence to the maximum density ahead of it (as opposed to just the local density, which is true in the case of an un-equipped car). In the above equation, $\alpha = \frac{d(t)}{d(0)} \in [0, 1]$ is a Lagrangian quantity representing a weighting factor. It represents the distance of each equipped car to the location of the maximum density ahead, normalized with respect to the distance at time $t = 0$. In other words, we assume that each equipped car, on receiving a slowdown warning signal, transitions from its current velocity (that corresponds to the local density) to a new value (that corresponds to the maximum density ahead), at a rate that ensures that it attains this desired velocity by the time it reaches the x location of the maximum density.

Substituting (13) in (11), we get the PDE for the case when all cars are equipped, to be

$$\frac{\partial \rho}{\partial t} + (\alpha(1-3\rho^2) + \alpha(1-max_{[x,1]}\rho^2))\frac{\partial \rho}{\partial x} = 0 \qquad (14)$$

Solving the above PDE for the same initial conditions as in the all unequipped case, we see from Fig. 9 that the resulting trajectories on the (x, t) plane show no shock wave.

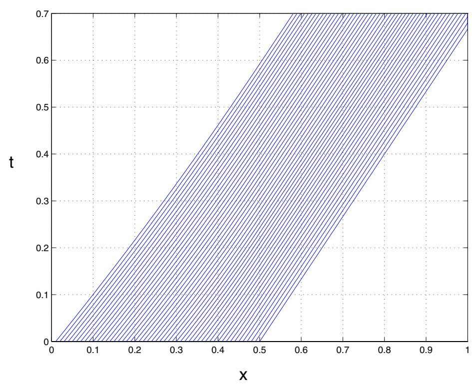

Fig. 9. Car Trajectories when all are equipped

We now consider the case when only some cars are equipped. At each time instant t, let $\beta(x)$ represent the distribution of the equipped cars, i.e. at each t, $\beta(x) = 1$ for those x that have equipped cars, and $\beta(x) = 0$ for those x that have unequipped cars. We then have

$$v(\rho, x) = \beta(x,t)(\alpha(1-\rho^2)+(1-\alpha)(1-max_{[x,1]}\rho^2)) + (1-\beta(x,t))(1-\rho^2) \qquad (15)$$

To model the effects of mixed distribution, it is necessary to use a smooth function of β to ensure that it is possible to obtain a PDE by substituting eqn. (15) in (11). We do this by approximating the points of discontinuity in the initial β profile by cosine functions. Also, note that since the equipped cars are themselves moving through the control volume, therefore the β distribution changes with time, and consequently $\beta = \beta(x, t)$.

Results obtained in the case of a mixed distribution are shown (for the same initial conditions as in the previous two cases) in Figs. 10 and 11. Fig. 10 shows the car trajectories on the (x, t) plane, with the trajectories of the equipped cars shown in red, and the unequipped ones in blue. It is seen that the unequipped cars in front do experience a shock (this is also evident from the individual car velocities shown in Fig. 11(a)). The equipped cars behind do not experience a shock however, and furthermore, even the unequipped cars behind them are forced to slow down earlier than they otherwise would have, and consequently they too do not experience a shock. This is seen in Figs. 10 and 11(b). Fig. 11(b) shows the individual car velocities of the equipped cars (in red), and the unequipped cars behind them (in blue).

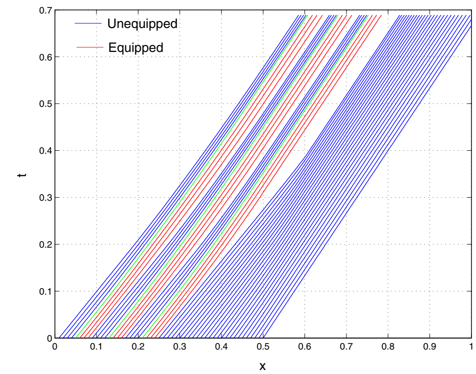

Fig. 10. Car Trajectories when some are equipped

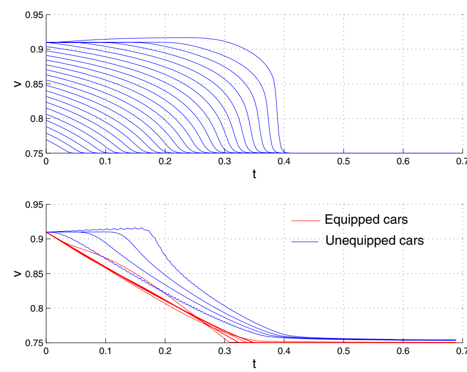

Fig. 11. Car Velocities when some are equipped

The PDE model used in the above analysis still does not give a complete representation of the macroscopic effects of the slowdown warning system. For a more complete representation, more comprehensive models that use separate continuity equations for the equipped and unequipped cars need to be used. This will enable a more accurate representation of the effects of a mixed sensing environement. Furthermore, the momentum equation [18] too should be used. Work is currently in progress on the analysis of the slowdown warning system using these more sophisticated models and these results will be communicated soon.

IV. CONCLUSIONS AND FUTURE WORKS

A. Conclusions

In this paper, the concept of installation of a slowdown warning system in automobiles is discussed. While driving on a highway, if a car in a platoon decelerates abruptly, then such a system provides advance information, near simultaneously, to all the drivers behind that car. This advance information gives these drivers more time to react in preparation of the impending slow-down. This additional available time can alleviate collisions. Furthermore, it is shown that even if only a fraction of the cars in a platoon are equipped with such a system, it is still possible to alleviate crashes even in the unequipped cars. This is because each equipped car, with its increase in headway, acts as an attenuator that arrests the amplification of the velocity perturbation of the lead car as it propagates through the line of cars; and therefore, even with a few equipped cars, it is possible to keep the level of amplification below the threshold that leads to car pile-ups. Equipping cars with a slowdown warning system can also reduce the intensity of the shock wave that otherwise occurs when all the cars are unequipped.

B. Future Works

Future work will involve using more sophisticated PDE models to evaluate the macroscopic effects of the slowdown warning system in an environment with partial equipage. It is planned to use more accurate second order models that will enable the desired velocity of each driver to be a function of not just the density at his location, but also the density gradient at his location. Additionally, it is planned to use the momentum equation also to take into account the finite reaction time it takes a driver to adjust his current velocity to the value of his desired velocity.

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