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Output Regulation of Linear Systems with Input Constraints

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Abstract— **This paper studies the output regulation problem of general linear systems with input saturation. The asymptotically regulatable region which is the set of all initial states of the plant and the exosystem is given. A feedback controller based on a stabilizing law is given such that for given initail pair in the regulatable region, the controller ensures exponential output regulation.**

I. Introduction

In this paper we consider the regulation problem of linear system subject to actuator saturation. Such a system has a control input and an exogenous input which is generated by an autonomous exosystem. Solving this problem involves constructing an appropriate state feedback controller that renders the closed loop system internally stable while the tracking error tends to zero as time tends to in£nity. The classical output regulation problem has been well studied by Davison, Francis, Wonham [7], [8], [9]. More recently, there has been a great deal of renewed interest in the study of this problem subject to input saturation, which is motivated from practical considerations of the inherent constraints on actuators..

It is well known that output regulation inherently requires internal stabilization and in general, linear feedback control laws can not be used for the purpose of global asymptotic stabilization of linear systems subject to input saturation [2], [3]. A particular nonlinear feedback law using multiple saturation functions for the global asymptotic stabilization of such systems was studied in [4], [5] and the research on regulation problem was achieved in [6].

However, it should be pointed that most of the studies are subjected to the semi-stable systems, i.e. the eigenvalues of the system are in the closed unit circle. For arbitrary linear saturated system with unstable modes, it's very dif£cult to deal with, since the null-controllable region is not the whole state space. It was shown that nonlinear controllers could be designed to make the closed loop system globally stable [5]. Recently, it was shown in [1] that a straightforward design procedure and an easily implementable controller can achieve exponential convergence.

In[10], unstable linear system output regulation with actuator saturation was studied. Based on a local solution and the initial conditions which are a little neighborhood of origin, a nonlinear controller attempts to enlarge the set of initial conditions.. A recent work[11] studies the problem of general linear system. The asymptotically regulatable region was studied. However, it was de£ned on a transformed state of the plant and exosystem and hence did not give the whole regulatable region if the system does not have unique equilibrium point. A feedback law was constructed assuming that a stabilizing control law has been designed. On a sequence of subsets in the state space of plant and exosystem, a state feedback law was de£ned on each of these sets and the control was executed by choosing one from this sequence to ensure the transformed state converge to the origin and achieve regulation.

The objective of this paper is to study the output regulation problem of general linear systems with input saturation and characterize the asymptotically regulatable region which is the set of all initial conditions of the plant and the exosystem. We then design a simple feedback controller based on the stabilizing law in [1] such that for given initial pair in the regulatable region, the controller ensures exponential output regulation.

II. Problem statement and preliminaries

Consider the system

$$\begin{aligned} x(k+1) &= Ax(k)+Bu(k)+Pw(k) \qquad (1)\\ w(k+1) &= Sw(k) \\ e(k) &= Cx(k)+Qw(k) \end{aligned}$$

where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $P \in \mathbb{R}^{n\times s}$, $S \in \mathbb{R}^{s\times s}$, $C \in \mathbb{R}^{p\times n}$, $Q \in \mathbb{R}^{p\times s}$. The plant with state $x \in \mathbb{R}^n$, input $u(k) \in \mathbb{R}^m$ and $|u(k)|_\infty \le 1$, subject to the effect of an exogenous disturbance represented by $Pw(k)$, where $w \in \mathbb{R}^s$ is the state of an exosystem. $e(k)$ represents the error between the output $Cx(k)$ and reference signal $-Qw(k)$.

The problem of constrained output regulation to be addressed in this paper is the following:

(I) Characterize of the regulatable region : Due to the constrained input, it's well known that the initial states of the plant and the exosystem can not be in the whole space, we will characterize the set of all initial states (x_0, w_0) on which the problem of constrained output regulation is solvable.

(II) Design of constrained state feedback controller : If possible, £nd a state feedback law $u = f(x,w)$, with $|f(x,w)|_\infty \le 1$ and $f(0,0) = 0$,such that the following conditions hold:

(1). The equilibrium point $x = 0$ of the system $x(k+1) = Ax(k) + Bf(x,0)$ is asymptotically stable.

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(2). For any initial pair (x_0, w_0) in the regulatable region, the closed-loop system satisfies $\lim_{k\to\infty} e(k) = 0$.

To begin with, The following assumptions are made:

A1. The pair (A, B) is stabilizable.

A2. S has all its eigenvalues on the unit circle and S is diagonalizable.

A3. There exist matrices Π and Γ that solve the linear matrix equations

$$\begin{aligned} \Pi S &= A\Pi + B\Gamma + P \\ 0 &= C\Pi + Q \end{aligned} \quad (2)$$

Notes that the stabilizability of (A, B) is necessary for stabilization of the plant. Due to bounded controls, the exosystem must be bounded and S can not have unstable modes, the stable modes can be ignored. So, we assume S has all its eigenvalues on the unit circle, but this is not enough, since for any non-diagonalizable form

$$J = \begin{bmatrix} \lambda & 1 \\ & \lambda \end{bmatrix}, \text{ with } |\lambda| = 1,$$

we have $\lim_{k\to\infty} |J^k| = \infty$, which is unbounded.

The third assumption, due to Francis [8], is the necessary and sufficient conditions for the existence of solutions to the unconstrained output regulation problem via state feedback control.

III. The Regulatable Region

Consider system (1), a control signal u is said to be admissible if $|u(k)|_\infty \le 1$.

Definition 3.1: A pair $(x_0, w_0) \in \mathbb{R}^n \times \mathbb{R}^s$ is said to be regulatable if there exists an admissible control so that the time response of (1) satisfies $e(K) = 0$ for some $K > 0$. The set of all regulatable pair (x_0, w_0) is called the regulatable region and is denoted by $\mathcal{R}_g$.

To begin with, we observe that equation (2) is an "equilibrium point" description or zero error state equation. At the equilibrium point $x = \Pi w$ with $u = \Gamma w$, the error $e = 0$. Due to the restriction that $|u(k)|_\infty \le 1$, it is clear that $e(k)$ goes to zero asymptotically at this equilibrium point only if

$$\sup_{k\ge 0} \left|\Gamma S^k w_0\right|_\infty < 1.$$

So, it's necessary to restrict the exosystem initial conditions corresponding to this equilibrium point in the following compact set

$$\mathcal{W}_\Gamma = \left\{w_0 \in \mathbb{R}^s : \left|\Gamma S^k w_0\right|_\infty \le \rho, \forall k \ge 0\right\},$$

for some $\rho \in (0, 1)$.

Also note that in general case $(m > p)$, equation (2) has infinite solutions, so there are infinite equilibrium points at which the regulation can be achieved. For each of these equilibrium point, the corresponding initial exosystem state sets are different. So, the overall regulatable initial exosystem state set should be the union of all these sets.

$$\mathcal{W}_0 = \bigcup_\Gamma \mathcal{W}_\Gamma.$$

Now, we will study the initial conditions of system state x_0 and describe the regulatable region $\mathcal{R}_g$ in terms of $\mathcal{W}_0$ and the null controllable region $\mathcal{C}(A, B)$ of the system

$$x(k+1) = Ax(k) + Bu(k)\ ,\ |u(k)|_\infty \le 1.$$

Definition 3.2: A state $x_0 \in \mathbb{R}^n$ is said to be null controllable if there exists an admissible control so that the time response satisfies $x(K) = 0$ for some $K > 0$. The set of all null controllable states x_0 is called the null controllable region and is denoted by $\mathcal{C}(A, B)$.

By carrying out a similarity transformation if necessary, we may assume that A and B are of the form

$$\begin{aligned} A &= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \in \mathbb{R}^{(n_1+n_2)\times(n_1+n_2)} \\ B &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \in \mathbb{R}^{(n_1+n_2)\times m} \end{aligned} \quad (3)$$

with A_1 having all eigenvalues outside the unit circle, and A_2 having all eigenvalues on or inside the unit circle. It is well known that the null-controllable region is $\mathcal{C}(A, B) = \mathcal{C}(A_1, B_1) \oplus \mathbb{R}^{n_2}$.

Corresponding to (3), we let

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, Q = \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix}.$$

Theorem 1. *Let $V_0 \in \mathbb{R}^{n_1\times s}$ be the unique solution to the linear matrix equation*

$$V_0 S - A_1 V_0 = P_1,$$

then the regulatable region $\mathcal{R}_g$ is given by

$$\begin{aligned} \mathcal{R}_g = \{&(x_{10}, x_{20}, w_0) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathcal{W}_0 : \\ &x_{10} - V_0 w_0 \in \mathcal{C}(A_1, B_1)\}. \end{aligned}$$

Proof. Since A_1 and S have no common eigenvalues, the uniqueness of V is obvious.

We first show that

$$\begin{aligned} \mathcal{R}_g \subseteq \{&(x_{10}, x_{20}, w_0) \in \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \mathcal{W}_0 : \\ &x_{10} - V_0 w_0 \in \mathcal{C}(A_1, B_1)\} \\ := {}& \mathcal{R}. \end{aligned}$$

Given initial pair $(x_0, w_0) \in \mathcal{R}_g$, assume there exists an admissible control $u(k)$ with $|u(k)|_\infty \le 1$ such that $\lim_{k\to\infty} e(k) = 0$. we have

$$\begin{aligned} x(k) &= A^k x_0 + \sum_{l=0}^{k-1} A^{k-l-1}[Bu(l) + Pw(l)] \\ &= A^k x_0 + \sum_{l=0}^{k-1} A^{k-l-1} Bu(l) + \sum_{l=0}^{k-1} A^{k-l-1} PS^l w_0. \end{aligned}$$

So, we get

$$e(k) = Cx(k) - QS^k w_0$$

or

$$\begin{bmatrix} e_1(k) \\ e_2(k) \end{bmatrix} = \begin{bmatrix} Cx_1(k) - Q_1S^kw_0 \\ Cx_2(k) - Q_2S^kw_0 \end{bmatrix}.$$

Since Q_1S^k is bounded for all k and $A_1^k \to \infty$ as $k \to \infty$. so $\lim_{k\to\infty} e(k) = 0$ only if

$$x_{10} + \sum_{l=0}^{\infty} A_1^{-l-1}P_1S^lw_0 + \sum_{l=0}^{\infty} A_1^{-l-1}B_1u(l) = 0.$$

De£ne $V_0 = -\sum_{l=0}^{\infty} A_1^{-l-1}P_1S^l$, it's easy to check that V_0 satis£es $V_0S - A_1V_0 = P_1$, and we get $x_{10} - V_0w_0 \in C(A_1, B_1)$. This proved that $\mathcal{R}_g \subseteq \mathcal{R}$.

If for any initial pair $(x_0, w_0) \in \mathcal{R}$ we can £nd an admissible control $u(k)$ with $|u(k)|_\infty \leq 1$ such that $\lim_{k\to\infty} e(k) = 0$ then we will end the prove. This will be done in next section. ■

Remark 3.1: Note the difference between theorem 1 and the theorem in [11], since Γ may not be unique, the set of initial state x_0 corresponding to a given w_0 is also different, the former set is big then the later in [11].

IV. State Feedback Controller Design

In this section, we will construct a feedback controller that solves constrained output regulation problem . We will consider two cases of the given system.

Case I. Assume there exists matrix V which satis£es $VS - AV = P$

We will rearrange A and B into

$$\begin{aligned} A &= \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \in R^{(n_1'+n_2')\times(n_1'+n_2')} \quad (4) \\ B &= \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} \in R^{(n_1'+n_2')\times m} \end{aligned}$$

with A_1 having all eigenvalues on or outside the unit circle, and A_2 having all eigenvalues inside the unit circle. Notice the difference in partition (4) and partition (3). Corresponding to (4), we let

$$x = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \Pi = \begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix}, P = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}, V = \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

and the above equation becomes

$$\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} S - \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}.$$

Lemma 1 [1] Let $\lambda \in (0, 1)$, for every initial condition $\tilde{x}_0 \in \mathcal{C}_\lambda \triangleq \mathcal{C}(\lambda^{-1}A_1, \lambda^{-1}B_1)$, there exists a state feedback law $u(k) = h[x(k)]$ such that the solution of $x(k+1) = A_1x(k) + B_1u(k)$ satis£es $x(k) \in \lambda^k\rho_{\mathcal{C}_\lambda}(x_0)\bar{\mathcal{C}}_\lambda$, and the control signal satis£es $|u(k)|_\infty \leq \lambda^k\rho_{\mathcal{C}_\lambda}(x_0) \leq \lambda^k$.

The construction of this state feedback controller can be found in [1]. We will construct a control law which is based on this feedback controller to achieve regulation.

Theorem 2 *Assume there exists matrix V which satis£es $VS - AV = P$, for every initial pair $(x_0, w_0) \in \left\{(x_{10}, x_{20}, w_0) \in \mathbb{R}^{n_1'} \times \mathbb{R}^{n_2'} \times \mathcal{W}_\Gamma : x_{10} - V_1w_0 \in \mathcal{C}_\lambda\right\}$, under the following controller*

$$\begin{aligned} u(k) &= h[x(k) - \lambda^kV_1w(k) - (1-\lambda^k)\Pi_1w(k)] \\ &\quad + (1-\lambda^k)\Gamma w(k) \end{aligned}$$

the closed-loop system satis£es $\lim_{k\to\infty} e(k) = 0$.

Proof. Corresponding to (4), we can divide system (1) into two subsystems

$$\begin{aligned} x_1(k+1) &= A_1x(k) + B_1u(k) + P_1w(k) \quad (5) \\ x_2(k+1) &= A_2x(k) + B_2u(k) + P_2w(k) \quad (6) \\ w(k+1) &= Sw(k) \\ e(k) &= Cx(k) + Qw(k) \end{aligned}$$

De£ne

$$\tilde{x}(k) = x(k) + \lambda^kVw(k) + (1-\lambda^k)\Pi w(k)$$

or

$$\tilde{x}_i(k) = x_i(k) - \lambda^kV_iw(k) - (1-\lambda^k)\Pi_iw(k),$$

for $i = 1, 2$. Then we can get

$$\begin{aligned} e(k) &= C[\tilde{x}(k) + \lambda^kVw(k) + (1-\lambda^k)\Pi w(k)] \\ &\quad + Qw(k) \\ &= C\tilde{x}(k) + \lambda^kC[Vw(k) - \Pi w(k)]. \end{aligned}$$

Since $VS - AV = P$, we have $AV - VS + \Pi S - A\Pi = B\Gamma$ or

$$\begin{aligned} A_1V_1 - V_1S + \Pi_1S - A_1\Pi_1 &= B_1\Gamma \\ A_2V_2 - V_2S + \Pi_2S - A_2\Pi_2 &= B_2\Gamma \end{aligned}$$

so, for $i = 1, 2$, we have

$$\tilde{x}_i(k+1) = A_i\tilde{x}(k) - (1-\lambda^k)B_i\Gamma w(k) + B_iu(k).$$

We now construct a controller which is based on the controller in Lemma 1.

$$u(k) = h[\tilde{x}_1(k)] + (1-\lambda^k)\Gamma w(k).$$

Apply the above controller to these two subsystems, we get

$$\begin{aligned} \tilde{x}_1(k+1) &= A_1\tilde{x}_1(k) + B_1h[\tilde{x}_1(k)] \\ \tilde{x}_2(k+1) &= A_2\tilde{x}_2(k) + B_2h[\tilde{x}_1(k)] \end{aligned}$$

Notes that $\tilde{x}_{10} = x_{10} - V_1w_0 \in \mathcal{C}_\lambda$, by Lemma 1 , we get

$$\begin{aligned} \lim_{k\to\infty} \tilde{x}_1(k) &= 0 \\ |h[\tilde{x}_1(k)]|_\infty &\leq \lambda^k \end{aligned}$$

Since A_2 is stable and $|h[\tilde{x}_1(k)]|_\infty \leq \lambda^k$, it follows that $\tilde{x}_2(k)$ also converges to the origin. It is easy to see that

$$\begin{aligned} |u(k)|_\infty &= \left|h[\tilde{x}_1(k)] + (1-\lambda^k)\Gamma w(k)\right|_\infty \leq 1 \\ \lim_{k\to\infty} e(k) &= \lim_{k\to\infty} \left[C\tilde{x}(k) + \lambda^kC[Vw(k) - \Pi w(k)]\right] \\ &= \lim_{k\to\infty} \lambda^kC(V-\Pi)S^kw_0 \\ &= 0 \end{aligned}$$

I.e. the closed-loop system satis£es $\lim_{k\to\infty} e(k) = 0$. ∎

Case II. Assume there doesn't exist any matrix V which satis£es $VS - AV = P$

In this case, we use partition (3) and

$$\begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} S = \begin{bmatrix} A_1 & 0 \\ 0 & A_2 \end{bmatrix}\begin{bmatrix} \Pi_1 \\ \Pi_2 \end{bmatrix} + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix}\Gamma + \begin{bmatrix} P_1 \\ P_2 \end{bmatrix}.$$

Let V_0 be the unique solution of $V_0 S - A_1 V_0 = P_1$.

By lemma 1 , there exits a control $u = \delta\ h(v/\delta)$ such that the origin of

$$v(k+1) = A_1 v(k) + \delta\ B_1\ h(v/\delta)$$

has a domain of attraction $\delta\mathcal{C}(A_1, B_1)$. According to [12], there exists a control $u = \delta\ \text{sat}(f(v))$ such that the origin of

$$v(k+1) = Av(k) + \delta\ B\ \text{sat}(f(v(k)))$$

has a domain of attraction $S_\delta = \delta\mathcal{C}(A_1, B_1) \times R^{n_2}$.

for any initial pair $(x_0, w_0) \in \left\{(x_{10}, x_{20}, w_0) \in \mathbb{R}^{n_1'} \times \mathbb{R}^{n_2'} \times \mathcal{W}_\Gamma : x_{10} - V_1 w_0 \in \mathcal{C}_\lambda\right\}$, de£ne a new state $z = x - \Pi w$, or $z_1 = x_1 - \Pi_1 w$, $z_2 = x_2 - \Pi_2 w$.

Let $\tilde{x}_1(k) = x_1(k) - \lambda^k V_0 w(k) - (1-\lambda^k)\Pi_1 w(k)$, note that $\tilde{x}_1(k) = z_1(k) - \lambda^k(V_0 - \Pi_1)w(k)$ and

$$\lim_{k\to\infty} \tilde{x}_1(k) = z_1(k).$$

By theorem 2, under the controller

$$u(k) = h[x(k) - \lambda^k V_0 w(k) - (1-\lambda^k)\Pi_1 w(k)] + (1-\lambda^k)\Gamma w(k)$$

we can get

$$\lim_{k\to\infty} \tilde{x}_1(k) = 0.$$

So there is a £nite K such that $z(K) \in S_\delta$. Finally, let the overall controller to be

$$u(k) = h[x(k) - \lambda^k V_0 w(k) - (1-\lambda^k)\Pi_1 w(k)] + (1-\lambda^k)\Gamma w(k),$$

if $x(k) - \Pi w(k) \notin S_\delta$;

$$u(k) = \Gamma w(k) + \delta\ \text{sat}\{f[x(k) - \Pi w(k)]\}, x(k) - \Pi w(k) \in S_\delta.$$

then under this control,

$$\lim_{k\to\infty} [x(k) - \Pi w(k)] = 0$$

and

$$\lim_{k\to\infty} e(k) = 0.$$

Remark 4.1: Given any $(x_0, w_0) \in \mathcal{R}_g$, there always exist Γ and λ such that $(x_0, w_0) \in \left\{(x_{10}, x_{20}, w_0) \in \mathbb{R}^{n_1'} \times \mathbb{R}^{n_2'} \times \mathcal{W}_\Gamma : x_{10} - V_1 w_0 \in \mathcal{C}_\lambda\right\}$. So, we can always £nd an admissible control $u(k)$ with $|u(k)|_\infty \le 1$ such that $\lim_{k\to\infty} e(k) = 0$. This end the prove of Theorem 1.

V. Examples

In this section, we will give two examples using the controller developed above.

Example 1. A second-order antistable system is described by

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1.4 & 0 \\ 0.2 & 1.2 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u(k) \\ &\quad + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} w(k) \\ w(k+1) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w(k) \\ e(k) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(k) - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w(k). \end{aligned}$$

with $x_0 = [\ -1 \quad -1\ ]'$ and $w_0 = [\ 2 \quad 2\ ]'$.It's easy to see that equation (2) has unique solution

$$\Pi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \Gamma = S - A - P = \begin{bmatrix} -0.5 & 0 \\ -0.2 & -0.3 \end{bmatrix},$$

and the unique solution to $VS - AV = P$ is

$$V = \begin{bmatrix} -0.25 & 0 \\ 0.25 & -0.5 \end{bmatrix}.$$

We apply the following controller to the system

$$\begin{aligned} u(k) &= h[x(k) - 0.99^k V w(k) - (1-0.99^k)\Pi w(k)] \\ &\quad + (1-0.99^k)\Gamma w(k), \end{aligned}$$

the closed loop response and control action are plotted in Fig.1.

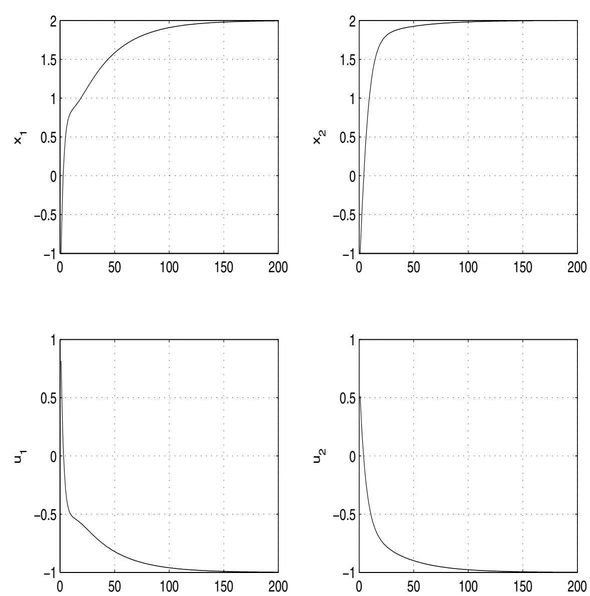

Fig. 1. Response of system in example 1

Example 2. A second-order antistable system is described by

$$\begin{aligned} x(k+1) &= \begin{bmatrix} 1.4 & 0 \\ 0.2 & 1.2 \end{bmatrix} x(k) + \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} u(k) \\ &\quad + \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix} w(k) \\ w(k+1) &= \begin{bmatrix} 0 & 1 \\ -1 & 1.99 \end{bmatrix} w(k) \\ e(k) &= \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} x(k) - \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} w(k). \end{aligned}$$

with $x_0 = [\ 1 \quad -1\ ]'$ and $w_0 = [\ 0.108 \quad 0.32\ ]'$.It's easy to see that equation (2) has unique solution

$$\Pi = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \Gamma = S - A - P = \begin{bmatrix} -1.5 & 1 \\ -1.2 & -0.89 \end{bmatrix},$$

and the unique solution to $VS - AV = P$ is

$$V = \begin{bmatrix} 0.3391 & -0.5747 \\ 0.7429 & -0.9593 \end{bmatrix}.$$

We apply the following controller to the system

$$\begin{aligned} u(k) &= h[x(k) - 0.99^k V w(k) - (1 - 0.99^k)\Pi w(k)] \\ &\quad + (1 - 0.99^k)\Gamma w(k), \end{aligned}$$

the closed loop response and control action are plotted in Fig.2.

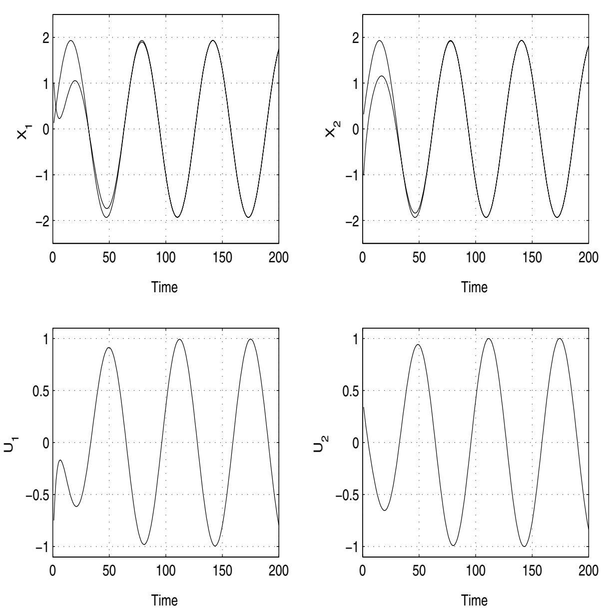

Fig. 2. Response of system in example 2

VI. Conclusions

In this paper we studied the regulation problem of linear system subject to actuator saturation. The asymptotically regulatable region which is the set of all initial conditions of the plant and the exosystem is given. A simple feedback controller based on a stabilizing law is also given. For every arbitrary large compact subset of the regulatable region, the controller ensures exponential output regulation.

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