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A Polynomial Adaptive Controller for Discretely Parameterized Systems

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***Abstract*— This paper deals with a class of uncertain systems, in which the unknown parameters belong to a discrete set and occur nonlinearly. Such a system represents a nonlinearly parameterized system and, therefore, application of the traditional linear-in-parameters adaptive controllers is not feasible. In this paper, a new Polynomial Adaptive Controller (PAC) is proposed and its performance for control of systems with unknown discrete parameters that occur nonlinearly is shown.**

I. Introduction

Adaptive control of linearly parameterized (LP) systems has been well investigated for many years [1]. Extensions of adaptive estimation and control to nonlinearly parameterized systems have been carried out in [2]–[10]. In these papers, often there are limitations and restrictions due to the inadequacy of the gradient algorithm and its variant, which is a min-max algorithm, for general nonlinearly parameterized (NLP) systems. The min-max algorithm in [5], [7], for instance, is either applicable only for concave/convex parameterizations or for special classes of general NLP systems. In both cases, a nonlinear optimization problem needs to be solved at each step, and can introduce a significant computational burden. The objective in this paper is to develop an alternative approach to adaptive control of general NLP systems that is as general in scope as that of LP systems as in [1]. Our focus, in particular, is on a class of NLP systems of the form

$$\dot{y}(t) = f(y(t), \theta) + h(u(t)), \quad y(0) = y_0, \quad t \geq 0\,, \tag{1}$$

where $u \in \mathbb{R}$ is the control signal to be determined, $y \in \mathbb{R}$ is the system state available as a measurement, h is an invertible function, f is a known map $f : (\mathbb{R}, \Theta) \rightarrow \mathbb{R}$, while θ represents an unknown constant parameter that can take values from a known discrete set $\Theta = \{\theta_1, \cdots, \theta_N\}$ of size N.

We note that the system in (1) captures the special class of nonlinear systems in which the uncertainties in θ can be associated with N unknown switching modes. That is,

$$f(y, \theta_i) \quad \triangleq \quad f_i(y) \qquad \text{for } i = 1, \ldots, N \tag{2}$$

where $f_i : \mathbb{R} \rightarrow \mathbb{R}$, is a known map, and corresponds to different operational modes of the system in (1). It is assumed that the system can operate in only in one of the N modes at any time instant, with the actual operating mode unknown. We note that these different modes in (2) can be associated with different operating points of the system dynamics, with the changes between modes occuring due to changes in the environment, or due to component failures. In such cases, a control signal that is appropriate for one mode may destabilize the system in another mode, thereby warranting the need for an adaptive solution. It is the objective of this paper to develop an adaptive controller for systems of the form of (1), (2). The controller will be shown to lead to stability by treating the unknown working mode as an unknown parameter θ belonging to a discrete set Θ.

In [11], [12] and the references therein, multiple model adaptive controllers are proposed which also consider systems with multiple modes as discussed in this paper. The multiple model adaptive controller proceeds by online identification of the model and switches to the "most suitable" controller among all the *a priori* defined ones. In [11], the multiple models are related via linear relationships. In [12], it is assumed that the multiple models are LTI systems, and excitation of reference signal is introduced to ensure identification of the true operational mode for switching to the right controller. In this paper, a Lyapunov-based Polynomial Adaptive Controller (PAC) is proposed which deals unknown operational modes. The PAC removes the requirement of the LTI models needed in the multiple model approach and considers general nonlinear dynamical systems. Furthermore, the unknown parameter can occur nonlinearly, which is a significant improvement over the current linear-in-parameter adaptive controllers. The PAC can deal with a class of nonlinearly parameterized systems as the Model Reference Adaptive Controller (MRAC) for linearly parameterized systems. Similar to MRAC, PAC ensures asymptotic convergence of the output error to zero independent of the parameter convergence, and therefore no persistent excitation of reference signals is required.

In [13], a new Polynomial Adaptive Estimator (PAE)

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is proposed that uses the technique of auxiliary estimates to form a polynomial approximation instead of the linear approximation used in the Linear Adaptive Estimator. Hence, the PAE extends the adaptive estimator to systems with unknown parameters occurring in a higher order polynomial as opposed to a linear function, which can be seen as a first order polynomial. The Polynomial Adaptive Controller (PAC) proposed in this paper is based on the technique of polynomial adaptive estimator. It serves as a general tool for adaptive control of discretely NLP systems. The candidate Lyapunov function is not a quadratic function, but a higher order polynomial function. This leads to a new adaptive law that ensures approximation of polynomial nonlinearities in the system as opposed to conventionally adopted adaptation for unknown constant parameters (the latter can be viewed as first order polynomial). This provides extra freedom to the adaptive controller and enables an extension to NLP systems. In particular it solves the problem of systems with unknown discrete parameters occurring nonlinearly, which is the topic of this paper.

The paper is organized as follows. Section II gives the problem formulation of a nonlinearly discretely parameterized system. For simplicity, a scalar system is considered. The PAC for this scalar discretely parameterized system is proposed in Section III. Properties, lemmas and convergence results of PAC are proposed in Section IV. In section V, simulation results are presented, and Section VI concludes the paper.

II. Problem Formulation

In this section, we will introduce a standard formulation of a class of discretely parameterized systems:

$$\begin{aligned} \dot{y}(t) &= f(y(t),\theta) + h(u(t)), \quad y(0) = y_0, \quad t \geq 0\,, \\ \theta &\in \Theta = \{\theta_1, \cdots, \theta_N\}, \\ \theta_i &= \frac{(i-1)\Theta_{\max}}{N-1}, \end{aligned} \tag{3}$$

where $u \in \mathbb{R}$ is the control signal to be determined, $y \in \mathbb{R}$ is the system state available as a measurement, h is an invertible function, Θ is a known discrete set of size N, f is a known map $f : \mathbb{R} \times \Theta \to \mathbb{R}$, θ represents an unknown constant parameter that can take values from Θ, and $\Theta_{\max}$ is an arbitrary positive constant. We note without loss of generality that we can replace Θ with another set whose elements are not necessarily uniformly distributed. The goal is to design a controller to ensure that $y(t)$ tracks a given bounded command signal $r(t)$.

The system in (3) represents a fairly general class of discretely parameterized systems including f defined as in (2), with θ_i belonging to a discrete set Θ defined as

$$\Theta = \{1, .., N\}.$$

In contrast to the problem formulation in (3), an LP control structure will require that there exist constants $c_i, i = 1, .., N$ such that

$$c_1 f_1(y(t)) = c_2 f_2(y(t)) = ... = c_N f_N(y(t)), \quad \forall t \geq 0 \tag{4}$$

and hence f_i must be a function of the same structure for all i. However, the adaptive controller proposed in this paper can deal with a vector of unknown parameters θ that occurs nonlinearly, and hence remove the constraint in (4).

III. Polynomial Adaptive Controller

In this section, we will develop a Polynomial Adaptive Controller (PAC) for the system in (3). We define

$$k \quad = \quad 1/\Theta_{\max}, \tag{5}$$

where

$$g_i(x) = \begin{cases} x^i, & \text{if } i \text{ is odd;} \\ x^{i-1} + kx^i, & \text{if } i \text{ is even.} \end{cases} \tag{6}$$

Introduce the following companion model:

$$\begin{aligned} \dot{\hat{y}}(t) &= -\alpha\tilde{y}(t) + \phi_0(y(t),\hat{\theta}(t)) + h(u(t)), \\ \tilde{y}(t) &= \hat{y}(t) - y(t), \end{aligned} \tag{7}$$

where α is an arbitrary positive constant, $\hat{\theta}(t) \triangleq [\hat{\theta}_1(t), .., \hat{\theta}_i(t), .., \hat{\theta}_N(t)]^\top$ are adaptive parameters, and $\phi_0(y(t),\hat{\theta}(t))$ is the first element of the following vector:

$$\phi \triangleq [\phi_0, .., \phi_i, .., \phi_{N-1}]^\top = A_\phi^{-1}(\hat{\theta}(t))c_\phi(y(t)). \tag{8}$$

Here $A_\phi(\hat{\theta}(t))$ is a $N \times N$ matrix with its ith row jth column element a_{ij} as:

$$a_{ij} = \begin{cases} 1, & j = 1, 1 \leq i \leq N \\ g_{j-1}(\widehat{\theta}_{j-1}(t) - \theta_i), & 2 \leq j \leq N, 1 \leq i \leq N, \end{cases}$$

where g_j is defined as in (6), while the i^{th} element of the vector $c_\phi(y(t))$ is

$$c_{\phi_i}(y(t)) = f(y(t),\theta_i), \tag{9}$$

and the adaptive parameters $\hat{\theta}_i(t)$ are governed by the following laws:

$$\begin{aligned} \dot{\hat{\theta}}_i(t) = &\begin{cases} 0, \text{ if } \tilde{y}\phi_i(y,\hat{\theta}) > 0 \text{ and } \widehat{\theta}_i(t) \geq \Theta_{\max}; \\ 0, \text{ if } \tilde{y}\phi_i(y,\hat{\theta}) < 0 \text{ and } \widehat{\theta}_i(t) \leq 0; \\ \tilde{y}(t)\phi_i(y(t),\hat{\theta}(t)), \qquad \text{otherwise,} \end{cases} \\ &\forall i = 1, .., N-1. \end{aligned} \tag{10}$$

The control law for the system in (3) is defined as:

$$u = h^{-1}(\alpha\tilde{y}(t) - \phi_0(y(t),\hat{\theta}(t)) - \beta(\hat{y}(t) - (t))), \tag{11}$$

where $r(t)$ is the bounded reference signal and $\beta > 0$ is a positive design gain.

The complete PAC consists of the companion model in (7), the adaptive law in (10), the control law in (11) and the related algebraic functions in (8).

IV. Stability Proof

To avoid the singularity in the PAC, we have to show that the matrix A_ϕ in (9) is non-singular.

Lemma 1: Matrix $A_\phi(\hat{\theta}(t)),\ t \geq 0$, defined in (9), is full rank.

Proof of Lemma 1: In what follows, we show that by a series of column scaling and add/subtract operations, matrix A_ϕ can be transformed into a Vendemonde' matrix which is known to be full rank.

We denote the i^{th} column of A_ϕ as A_i. First, let us consider A_2 which is $[(\widehat{\theta}_1 - \theta_1)\ ..\ (\widehat{\theta}_1 - \theta_i)\ ..\ (\widehat{\theta}_1 - \theta_N)]^\top$. Subtracting $\widehat{\theta}_1 A_1$ from A_2 and scaling column 2 by -1, we get: $\bar{A}_2 = [\theta_1\ \ ..\ \ \theta_i\ \ ..\ \ \theta_N]^\top$. We can continue this process through columns 3 to N. For $(j+1)^{th}$ column, we assume that the columns 1 to j have already been transformed into

$$[\bar{A}_1\ ..\ \bar{A}_j] \;=\; \begin{bmatrix} 1 & \theta_1 & .. & \theta_1^{j-1} \\ : & : & :: & : \\ 1 & \theta_i & .. & \theta_i^{j-1} \\ : & : & :: & : \\ 1 & \theta_N & .. & \theta_N^{j-1}, \end{bmatrix}.$$

Subtracting weighted $\bar{A}_1$ through $\bar{A}_i$ from A_{i+1}, the new $(i+1)^{th}$ column can be scaled to be $\bar{A}_{i+1} = [\theta_1^j\ \ ..\ \ \theta_i^j\ \ ..\ \ \theta_N^j]^\top$. Repeating this process until the last column, the new matrix takes the form:

$$[\bar{A}_1\ ..\ \bar{A}_N] \;=\; \begin{bmatrix} 1 & \theta_1 & .. & \theta_1^{N-1} \\ : & : & :: & : \\ 1 & \theta_i & .. & \theta_i^{N-1} \\ : & : & :: & : \\ 1 & \theta_N & .. & \theta_N^{N-1} \end{bmatrix}.$$

This matrix is Vendemonde' matrix and is therefore full rank. Since these linear transformations do not affect the rank of the matrix, A_ϕ must be full rank too.

Remark 1: Though A_ϕ is a matrix dependent upon k, Θ and $\widehat{\theta}_i(t), i = 1,..,N$, we can check easily that $\det(A_\phi)$ just depends on k and Θ and not on the time-varying auxiliary estimates $\widehat{\theta}_i(t), i = 1,..,N$. Since k and Θ are pre-defined PAC parameters, $\det(A_\phi(t))$ is constant during the operation of the PAC.

The following Lemma (2) shows that the auxiliary estimates $\widehat{\theta}_i(t)$ are bounded.

Lemma 2: For the system in (3) and the PAC in (7), (10), (11) the following upper bounds are true:

$$0 \leq |\hat{\theta}_i(t)| \leq \Theta_{\max}, \qquad i = 1, \cdots, N-1,\ \ t \geq 0.$$

Proof of Lemma 2: For any $i = 1,..,N-1$, it follows from (10) that

$$\begin{aligned} \dot{\widehat{\theta}}_i(t) &\leq 0, \text{ if } \widehat{\theta}_i(t) \geq \Theta_{\max}; \\ \dot{\widehat{\theta}}_i(t) &\geq 0, \text{ if } \widehat{\theta}_i(t) \leq 0, \end{aligned}$$

and this concludes the proof.

From (3) and (7), the error model between the system and the companion model can be derived:

$$\dot{\tilde{y}}(t) = -\alpha\tilde{y}(t) + \phi_0(y(t), \hat{\theta}(t)) - f(y(t), \theta). \quad (12)$$

The closed-loop system of the companion model (7) with the control law (11) is

$$\dot{\hat{y}}(t) = -\beta(\hat{y}(t) - r(t)). \quad (13)$$

It follows from (10) and (12) that the error dynamics of the entire system can be rewritten as

$$\begin{aligned} \dot{\tilde{y}}(t) &= -\alpha\tilde{y}(t) + \phi_0(y(t), \hat{\theta}(t)) - f(y(t), \theta), \\ \dot{\widehat{\theta}}_i(t) &= \tilde{y}(t)\phi_i(y(t), \hat{\theta}(t)) + v_i(t), \\ \tilde{\theta}_i(t) &= \widehat{\theta}_i(t) - \theta, \qquad i = 1, \cdots, N-1, \end{aligned} \quad (14)$$

where

$$\begin{cases} v_i(t) = 0 & \text{if} \quad \widehat{\theta}_i(t) \in (0, \Theta_{\max}); \\ v_i(t) \leq 0 & \text{if} \quad \widehat{\theta}_i(t) \geq \Theta_{\max}; \\ v_i(t) \geq 0 & \text{if} \quad \widehat{\theta}_i(t) \leq 0 \\ & i = 1,..,N-1, \end{cases} \quad (15)$$

and the remaining algebraic relationships are the same as in (8). We define

$$p_i(x) = \begin{cases} x^{i+1}/(i+1) \text{ if } i \text{ is odd;} \\ kx^{i+1}/(i+1) + x^i/i, \text{ if } i \text{ is even,} \end{cases} \quad (16)$$

and consider a Lyapunov function candidate as

$$V(t) = \frac{\tilde{y}(t)^2}{2} + \sum_{i=1}^{N-1} p_i(\tilde{\theta}_i(t)), \quad (17)$$

where $\tilde{\theta}_i$ is defined as in (14).

Remark 2: Notice that the Lyapunov function candidate (17) is not globally positive definite but only locally positive definite in a set defined as:

$$|\tilde{\theta}_i| \leq \Omega_{\max}, \qquad \forall i = 1,..,N. \quad (18)$$

Lemma 2 ensures that $|\tilde{\theta}_i(t)| \leq \Omega_{\max},\ \ \forall t > 0$, hence such a choice of a Lyapunov function candidate in (17) is reasonable.

The following Theorem 1 states that V is a Lyapunov function.

Theorem 1: For the system in (3) and the PAC in (7), (10) and (11), we have

$$\dot{V}(t) \leq -\alpha\tilde{y}(t)^2,$$

where V is defined in (17).

Proof of Theorem 1: Using (6) and (16), it can be checked easily that for any positive i,

$$\frac{dp_i(x)}{dx} = g_i(x). \tag{19}$$

It follows from (14), (17) and (19) that

$$\dot{V} = -\alpha\tilde{y}^2 + \sum_{i=1}^{N-1} g_i(\tilde{\theta}_i)(\tilde{y}\phi_i + v_i) + \tilde{y}\left(\phi_0 - f(y,\theta)\right). \tag{20}$$

Lemma 2 implies that

$$|\tilde{\theta}_i(t)| \leq \Theta_{\max}. \tag{21}$$

Using(5) and (21), it follows from (6) that

$$g_i(\tilde{\theta}_i(t)) \geq 0 \tag{22}$$

when $\widehat{\theta}_i(t) = \Theta_{\max}$. It follows from (15) and (22) that

$$g_i(\tilde{\theta}_i(t))v_i(t) \leq 0, \qquad \text{when } \widehat{\theta}_i(t) = \Theta_{\max}. \tag{23}$$

Using the same methodology, it can be shown that

$$g_i(\tilde{\theta}_i(t))v_i(t) \leq 0, \qquad \text{when } \widehat{\theta}_i(t) = 0. \tag{24}$$

Since $v_i(t) = 0$ when $0 < \widehat{\theta}_i(t) < \Theta_{\max}$, it can be verified easily that

$$g_i(\tilde{\theta}_i(t))v_i(t) = 0, \qquad \text{when } \widehat{\theta}_i(t) \in (0,\ \Theta_{\max}). \tag{25}$$

It follows from Lemma 2 that

$$\widehat{\theta}_i(t) \in [0,\ \Theta_{\max}],\ \forall i = 1,..,N-1, \text{ and } t \geq 0. \tag{26}$$

Combining (23), (24), (25) and (26), one obtains:

$$g_i(\tilde{\theta}_i(t))v_i(t) \leq 0, \qquad \forall i = 1,..,N-1,$$

and hence

$$\sum_{i=1}^{N-1} g_i(\tilde{\theta}_i(t))v_i(t) \leq 0. \tag{27}$$

From (20) and (27), it follows that

$$\dot{V} \leq -\alpha\tilde{y}^2 + \tilde{y}\left(\phi_0 + \sum_{i=1}^{N-1} g_i(\tilde{\theta}_i)\phi_i - f(y,\theta)\right). \tag{28}$$

The equation $A_\phi\phi = c_\phi$ in (8) implies that

$$\phi_0 + \sum_{i=1}^{N-1} g_i(\widehat{\theta}_i - \theta_j)\phi_i - f(y,\theta_j) = 0, \quad \forall \theta_j \in \Theta. \tag{29}$$

Since $\theta \in \Theta$, equation (29) implies that for any $t \geq 0$,

$$\phi_0(t) + \sum_{i=1}^{N-1} g_i(\tilde{\theta}_i(t))\phi_i(t) - f(y(t),\theta) = 0. \tag{30}$$

It follows from (28) and (30) that $\dot{V}(t) \leq -\alpha\tilde{y}(t)^2$, which proves Theorem 1. •

Next we show that the tracking error goes to zero asymptotically.

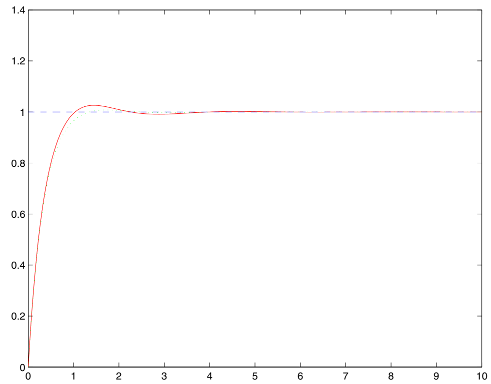

Fig. 1. Trajectories of $y(t)$ (solid), $\hat{y}(t)$ (dotted), and $r(t)$ (dashed)

Lemma 3: For the system in (3) and the PAC in (7), (10) and (11), if the reference input $r(t)$ is bounded, then $\lim_{t\to\infty} \tilde{y}(t) = 0$.

Proof of Lemma 3: If $r(t)$ is bounded, it follows from (13) that $\hat{y}(t)$ is bounded. Further, it follows from Lemma 1 that $V(t)$ is bounded and therefore $\tilde{y}(t)$ is bounded. Hence, $y(t)$ is bounded. Lemma 2 implies that all the auxiliary estimates are bounded. Since A_ϕ is non-singular with constant determinant and $\phi(t)$ is obtained through (8) using bounded signals, then $\phi(t)$ is bounded, as well as $\phi_0(t)$. Therefore, $\dot{\tilde{y}}(t)$ is bounded and it follows from Barbalat's lemma that $\lim_{t\to\infty} \tilde{y}(t) = 0$ The proof is complete. •

In the following Lemma 4, we will show that if $r(t) = r = const$, then $y(t)$ converges to r asymptotically.

Lemma 4: For the system in (3) and the PAC in (7), (10), and (11), if the reference signal r is a constant, then $\lim_{t\to\infty} y(t) = r$.

Proof of Lemma 4: Since r is bounded, it follows from Lemma 3 that

$$\lim_{t\to\infty} \tilde{y}(t) = 0. \tag{31}$$

For a constant r, it follows from (13) that $\lim_{t\to\infty} \hat{y}(t) = r$, which along with (31) proves Lemma 4.

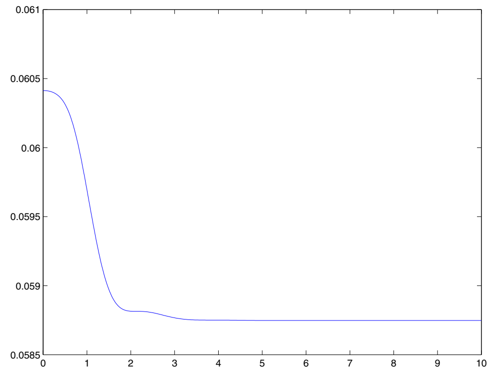

Fig. 2. Case 1: Trajectory of Lyapunov function $V(t)$

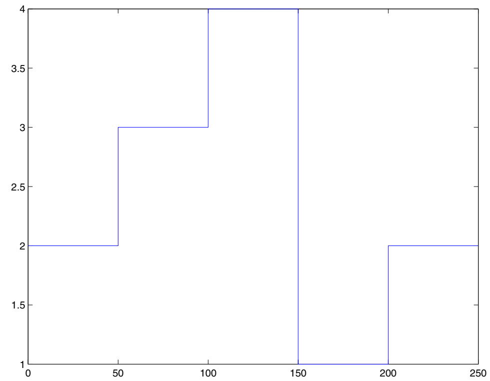

Fig. 4. Trajectory of unknown switching operational mode $q(t)$.

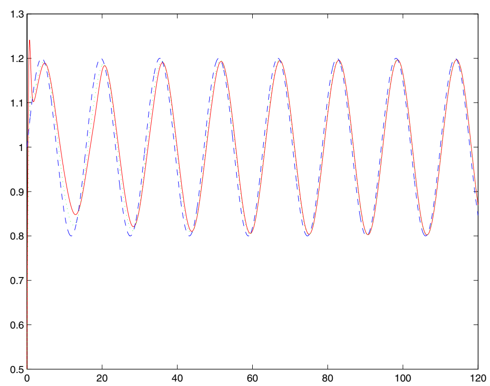

Fig. 3. Trajectories of $y(t)$ (solid), $\hat{y}(t)$ (dotted), and $r(t)$ (dashed)

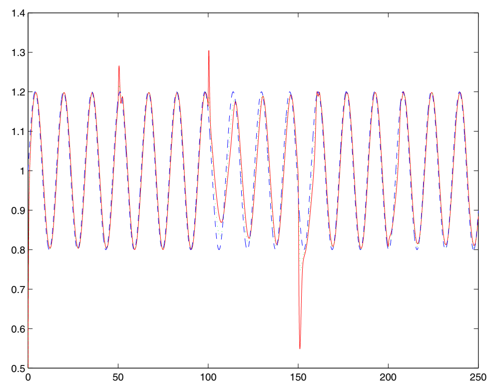

Fig. 5. Trajectories of $y(t)$ (solid), $\hat{y}(t)$ (dotted), and $r(t)$ (dashed)

V. Simulation Results

In this section, we consider a simulation example as:

$$\dot{y} \quad = \quad \bar{f}(y,\theta)+u \quad y(0)=y_0, \quad t\geq 0\,, \qquad (32)$$

where

$$\bar{f}(y,q)=\begin{cases} y^2, & q=1 \\ \sin(y), & q=2 \\ |y|, & q=3 \\ \exp(y/2), & q=4, \end{cases} \qquad (33)$$

where $u\in\mathbb{R}$ is the control signal to be determined, $y\in\mathbb{R}$ is the state measurement, $\bar{f}$ is a known map and q is the unknown operating mode. The goal is to design a controller to ensure that $y(t)$ tracks some bounded reference signal $r(t)$. Construct a set Θ:

$$\Theta=[\theta_1,\theta_2,\theta_3,\theta_4]=\left\{0,\frac{\Theta_{\max}}{3},\frac{2\Theta_{\max}}{3},\Theta_{\max}\right\}, \qquad (34)$$

where $\Theta_{\max}=1.5$. It is straightforward to consider the following one-to-one mapping $H:\Theta\to\{1,2,3,4\}$: $H(\theta_i)=i, \quad i=1,2,3,4$. Hence, the system in (32) can be rewritten as

$$\begin{aligned} \dot{y} &= f(y,\theta)+u, \quad \theta\in\Theta, \\ f(y,\theta_i) &= \bar{f}(y,H(\theta_i))=\bar{f}(y,i), \end{aligned}$$

where $y\in\mathbb{R}$, $u\in\mathbb{R}$ are the same as in (32), Θ is a known discrete set as in (34), θ is unknown and f is a known map $f:\mathbb{R}\times\Theta\to\mathbb{R}$. We note that the system in (32) cannot be formulated as a linearly parameterized system, and the control objective cannot be addressed using the current available linear-in-parameter adaptive controller.

For the application of PAC, the companion model is selected as

$$\begin{aligned} \dot{\hat{y}}(t) &= -2\tilde{y}(t)+\phi_0(y(t),\hat{\theta}(t))+u(t) \\ \tilde{y}(t) &= \hat{y}(t)-y(t) \end{aligned}$$

The adaptive law is the same as (10) and the control law is:

$$u(t)=-\beta(\hat{y}(t)-r(t))-\phi_0(y(t),\hat{\theta}(t))$$

where $\beta=3$.

Three different simulation scenarios have been considered. At first, we consider the system in (33) with constant reference signal $r(t) = 1$ and constant unknown parameter $\theta = \theta_4$, which implies that the unknown operation mode is $q = 4$. The simulation results are illustrated in Figs 1, and 2. Tracking performance of r by $\hat{y}(t)$ and $y(t)$ is shown in Fig. 1. Fig. 2 plots the trajectory of the non-increasing Lyapunov function $V(t)$. Next, we consider time-varying reference signal

$$r(t) = 1 + 0.2\sin(0.4t) \tag{35}$$

for unknown operation mode $q = 3$. The simulation results are illustrated in Fig. 3. Last, we consider tracking of the time-varying signal in (35) with a time-varying unknown parameter $\theta(t)$, i.e. when the unknown operational mode q switches value as time advances. Fig. 4 plots the switching unknown operational mode q as a function of time t. The trajectories of $r(t)$, $\hat{y}(t)$ and $y(t)$ are illustrated in Fig. 5. PAC adjusts adaptively to changes in the operational mode and drives the output signal back into the desired position upon some transient error.

VI. Conclusion

In this paper, a Polynomial Adaptive Controller (PAC) is presented, which is a general tool for controlling nonlinearly discretely parameterized systems. Extension to general higher-dimensional nonlinear systems will be reported shortly.

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