

Adaptive Dynamic Inversion Control of a Linear Scalar Plant with Constrained Control Inputs

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Abstract— This paper develops a methodology for stable adaptation in the presence of control position limits, for a class of linear time invariant systems with uncertain parameters. The central idea is to modify the desired reference by the best estimated deficiency in the state derivative, because of the saturation. The modified reference is used in determining the modified tracking error which is further used in the parameter update laws. A notion of ‘points of no return’ is presented for unstable systems with bounded control, and a switching control strategy is devised to address this problem of containing the state within the points of no return. The paper presents stability proofs for the control scheme and numerical simulations to demonstrate the performance of the controller.

I. INTRODUCTION

Traditionally, adaptive control assumes full authority control and lacks an adequate theoretical treatment for control in the presence of actuator saturation limits. Saturation becomes more critical for adaptive systems than non-adaptive systems, because continued adaptation in the presence of saturation may lead to instability. In recent years, there has been a lot of research effort for adaptive control design in the presence of input saturation constraints [1]. A modification to the adaptive control structure to counteract the adverse effects of control saturation was suggested by Monopoli [2], but no formal proof of stability was provided. Karason and Annaswamy presented the concept of modifying the error, proportional to the control deficiency [3]. They laid out a rigorous mathematical proof of asymptotic stability for a model reference framework and identified the largest set of initial conditions of the plant and the controller for which a stable controller could be realized. Johnson and Calise introduced the concept of ‘pseudo-control hedging’ which is a fixed gain adjustment to the reference model, proportional to the control deficiency [4]. Recently Lavretsky and Hovakimyan proposed a new design approach called ‘positive μ -modification’ which guarantees that the control never incurs saturation [5].

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In this paper, we extend the ideas in [3] and [4] for the adaptive dynamic inversion framework. In some of our earlier papers, we have also applied this methodology for control of nonlinear systems affine in control, with the unknown parameters appearing linearly. This method has been successfully applied for trajectory tracking of aggressive maneuvers for aircraft [6] and spacecraft [7], [8]. However stability is not rigorously proved for these cases.

The paper is organized as follows. Section II defines the control problem to be solved. Section III characterizes the trackable regions in the state space with respect to the system parameters and the control constraints. Section IV analyzes the case when the control becomes unsaturated and stays unsaturated for infinite time. Section V analyzes stability for the case when the control shifts from being unsaturated to saturated. The proofs in Sections IV and V ensure stability in the presence of successive transitions of the control from the saturated to the unsaturated regimes and vice versa. Numerical simulations are presented in Section VI, which validate the control methodology. Finally conclusions and future work are presented in Section VII.

II. PROBLEM DEFINITION

Consider a linear scalar plant

$$\dot{x} = -a^*x + b^*u_a \quad (1)$$

where $x \in \mathbb{R}^1$ and a^* , b^* are unknown scalars with $b^* \neq 0$. The nominal value for a^* and b^* is known, namely a_n , b_n with some uncertainty which defines the bounds $[a_{max}, a_{min}]$ and $[b_{max}, b_{min}]$ respectively. The applied control u_a is bounded symmetrically as, $u_a \in [-u_m, +u_m]$, where $u_m > 0$ is known.

The control objective is to track any feasible reference trajectory that can be tracked within control limits. For trajectories which are not feasible with respect to the control limits, the objective is to track as close as possible, maintain stability and ensure that all signals remain bounded.

III. CHARACTERIZING THE FEASIBILITY OF TRACKING A REFERENCE TRAJECTORY IN THE PRESENCE OF CONTROL LIMITS

A. Case 1: Stable Plant ($a^* > 0$)

If the actual plant trajectory has to exactly follow the reference trajectory we must have

$$\dot{x}_r = -a^*x + b^*u_a \quad (2)$$

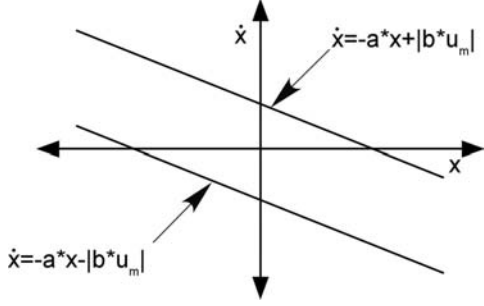


Fig. 1. Phase Plot Showing Domain for a Trackable Reference Trajectory for a Stable Plant

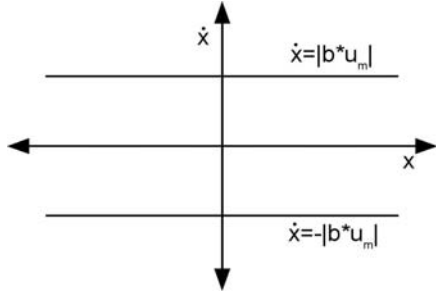


Fig. 2. Phase Plot Showing Domain for a Trackable Reference Trajectory for a Neutrally Stable Plant

Enforcing the control limits, $|u_a(t)| \leq u_m, \forall t \in \mathbb{R}^+$

$$\left| \frac{\dot{x}_r + a^* x_r}{b} \right| \leq u_m \quad (3)$$

Hence, any trajectory lying within the lines shown in Fig.1 can be tracked with an admissible control. Also note that a stable plant has a restoring tendency to return to the origin in the absence of any control.

B. Case 2: Neutrally Stable Plant ($a^* = 0$)

A neutrally stable plant has neither a restoring nor a destabilizing tendency due to the value of the current state. The derivative of the state is only affected by the control. The plant model is

$$\dot{x} = b^* u \quad (4)$$

and the limits for a trackable reference are

$$|\dot{x}_r| \leq b^* u_m \quad (5)$$

The domain for a trackable reference trajectory for such a plant is shown in Fig.2.

C. Case 3: Unstable Plant ($a^* < 0$)

For an unstable plant, if the current state is (x), the unforced response ($-a^*x$) tries to drive the plant away from the state $x = 0$. If the plant reaches a state where the destabilizing tendency becomes greater than the maximum restoring contribution the control can provide, then the state continues to diverge. So the plant state $x \rightarrow \infty$ if $|x| > \left| \frac{b^* u_m}{a^*} \right|$.

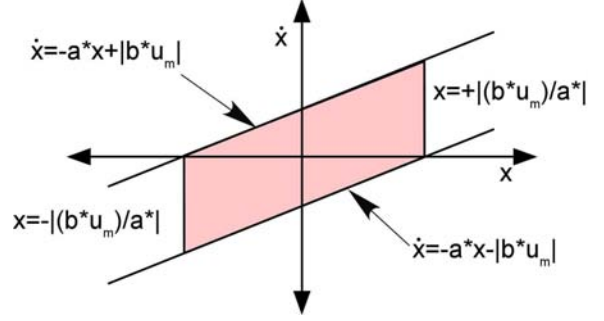


Fig. 3. Phase Plot Showing Domain for a Trackable Reference Trajectory for an Unstable Plant

These are the points of no return. If the value of the state crosses these points, the stability of the system cannot be recovered. So for an unstable plant, in addition to the constraint given by (3) we have

$$|x_r| < \left| \frac{b^* u_m}{a^*} \right| \quad (6)$$

IV. FROM SATURATED TO UNSATURATED

This section analyzes the stability of the adaptive control scheme as the control shifts from being saturated to unsaturated. The time origin is placed at the instant the control becomes unsaturated. We assume that all of the signals have finite values at $t = 0$ and the the control remains unsaturated as $t \rightarrow \infty$.

For the plant given in (1), the control objective is to track a reference trajectory specified in terms of continuous functions x_r and \dot{x}_r such that $x_r, \dot{x}_r \in \mathcal{L}_\infty$. The tracking error is defined as

$$e_1 \triangleq x - x_r \quad (7)$$

Differentiating (7) with respect to time and using (1)

$$\dot{e}_1 = -a^* x + b^* u_a - \dot{x}_r \quad (8)$$

We want to prescribe the dynamics $\dot{e}_1 = -\lambda e_1$ to the tracking error, where $\lambda > 0$. Adding and subtracting λe_1 ,

$$\dot{e}_1 = -\lambda e_1 + (\lambda e_1 - a^* x + b^* u_a - \dot{x}_r) \quad (9)$$

The value of control which ensures that the error e_1 follows the prescribed dynamics is

$$u_c = \frac{1}{b^*} (a^* x + \dot{x}_r - \lambda e_1) \quad (10)$$

Note that $b^* \neq 0$. Also, this control law requires accurate knowledge of the system parameters a^* and b^* , which are unknown. Hence adaptive parameters \hat{a} and \hat{b} are used which are updated in real-time by the adaptive law. The calculated control is

$$u_c = \frac{1}{\hat{b}} (\hat{a} x + \dot{x}_r - \lambda e_1) \quad (11)$$

To get finite values for u_c , we have to ensure that the $\hat{b} \neq 0$. Substituting (11) in (9)

$$\dot{e}_1 = -\lambda e_1 + (\hat{a} - a^*)x - \hat{b}u_c + b^*u_a \quad (12)$$

Let $\delta \triangleq u_c - u_a$ be the difference between the calculated and the applied control. Adding and subtracting $\hat{b}u_a$ from the right hand side and using the definition for δ

$$\dot{e}_1 = -\lambda e_1 + (\hat{a} - a^*)x - (\hat{b} - b^*)u_a - \hat{b}\delta \quad (13)$$

Let $\tilde{a} \triangleq \hat{a} - a^*$ and $\tilde{b} \triangleq \hat{b} - b^*$. Equation(13) becomes

$$\dot{e}_1 = -\lambda e_1 + \tilde{a}x - \tilde{b}u_a - \hat{b}\delta \quad (14)$$

Let $e_2 \triangleq x - x_m$ and $e_3 \triangleq x_m - x_r$. Therefore $e_1 = e_2 + e_3$. Defining the modified reference x_m as

$$\dot{x}_m = x_r - \hat{b}\delta - \lambda(x_m - x_r) \quad (15)$$

Writing the left hand side of (14) as $\dot{e}_2 + \dot{e}_3$ and using (15), (14) becomes

$$\dot{e}_2 = -\lambda e_2 + \tilde{a}x - \tilde{b}u_a \quad (16)$$

Now consider the candidate Lyapunov function

$$V = \frac{e_2^2}{2} + \frac{\tilde{a}^2}{2\gamma_1} + \frac{\tilde{b}^2}{2\gamma_2} \quad (17)$$

where $\gamma_1, \gamma_2 > 0$ Taking the derivative of the Lyapunov function along the system trajectories results in

$$\dot{V} = e_2\dot{e}_2 + \frac{\tilde{a}\dot{\tilde{a}}}{\gamma_1} + \frac{\tilde{b}\dot{\tilde{b}}}{\gamma_2} \quad (18)$$

Substituting \dot{e}_2 from (16) produces

$$\dot{V} = -\lambda e_2^2 + \tilde{a}(e_2x + \frac{\dot{\tilde{a}}}{\gamma_1}) + \tilde{b}(-e_2u_a + \frac{\dot{\tilde{b}}}{\gamma_2}) \quad (19)$$

Setting the sign indefinite terms to zero, and noting that the true system parameters a^* and b^* are constant, the update laws for the adaptive parameters are

$$\dot{\tilde{a}} = \dot{\hat{a}} = -\gamma_1 e_2 x, \quad \dot{\tilde{b}} = \dot{\hat{b}} = \gamma_2 e_2 u_a \quad (20)$$

Since the bounds on the parameters are known, the adaptive parameter can be restricted within these bounds by using parameter projection [9]. The bounds should be applied such that \hat{b} does not cross zero. If $\hat{b} = 0$ the control u_c in (11) is not defined. Parameter projection retains the same stability properties established in the absence of projection, if the true parameter lies within the specified bounds.

Note that the control is calculated using the error e_1 between the true trajectory and the desired reference trajectory as shown in (11). The parameter updates are calculated by using the error e_2 between the plant trajectory and the modified reference as shown by (20).

If the control is unsaturated, $\delta = 0$ and using the definition of e_3 , (15) becomes

$$\dot{e}_3 = -\lambda e_3 \quad (21)$$

Hence we conclude that $e_3, x_m \in \mathcal{L}_\infty$ and $x_m \rightarrow x_r$ as $t \rightarrow \infty$. The adaptive law causes \dot{V} to be negative semi-definite. Thus $e_2, x, \tilde{a}, \tilde{b}, \hat{a}, \hat{b} \in \mathcal{L}_\infty$ and $e_2 \in \mathcal{L}_2$. From (16) we conclude that $\dot{e}_2 \in \mathcal{L}_\infty$. Thus, from Barbalat's lemma [10] we conclude that $e_2 \rightarrow 0$ as $t \rightarrow \infty$. Thus $x \rightarrow x_m$ as $t \rightarrow \infty$.

We show that the modified reference converges to the original desired reference asymptotically, when the control is unsaturated. Also, the plant trajectory converges asymptotically to the modified reference and subsequently to the original desired reference. All signals in the closed-loop are bounded and the control objective is met.

V. FROM UNSATURATED TO SATURATED

This section analyzes the stability of the adaptive control scheme as the control shifts from being unsaturated to saturated. The time origin is placed at the instant the control becomes saturated. We assume that all of the signals have finite values at $t = 0$ and the control remains saturated as $t \rightarrow \infty$.

The stability properties of the adaptive scheme when the control is saturated depend heavily on the open loop stability characteristics of the plant. Let us consider the various possibilities.

A. Sub-Case 1: Stable Plant : $a^* > 0$

When the control is saturated, $|u_c| > u_m \Rightarrow u_a = u_m \text{sign}(u_c)$. Substituting in (1)

$$\dot{x} = -a^*x + b^*(\pm u_m) \quad (22)$$

Calculating the new equilibrium point

$$x_e = \frac{\pm b^* u_m}{a^*} \quad (23)$$

Let $\varepsilon \triangleq x - x_e$ be the error between the plant state and the equilibrium point. Differentiating with respect to time

$$\dot{\varepsilon} = -a^*(x - \frac{\pm b^* u_m}{a^*}) = -a^*\varepsilon \quad (24)$$

Thus we conclude that $\varepsilon, x \in \mathcal{L}_\infty$ and $\varepsilon \rightarrow 0$ as $t \rightarrow \infty$. From the derivative of the Lyapunov function in (19) and the parameter update law (20) we have, $e_2, \tilde{a}, \tilde{b}, \hat{a}, \hat{b} \in \mathcal{L}_\infty$ and $e_2 \in \mathcal{L}_2$. Since $x, x_r, e_2 \in \mathcal{L}_\infty$, we have $x_m, e_1, e_3 \in \mathcal{L}_\infty$. From (16) we conclude that $\dot{e}_2 \in \mathcal{L}_\infty$. Thus, from Barbalat's lemma we conclude that $e_2 \rightarrow 0$ as $t \rightarrow \infty$. Thus $x \rightarrow x_m$ as $t \rightarrow \infty$. From (11), we have $u_c, \delta \in \mathcal{L}_\infty$.

Consider the update law for the modified reference given in (15). The hedging signal $\hat{b}\delta$ acts as a bounded disturbance, causing the modified reference to diverge from the original desired reference. Since the hedging signal $\hat{b}\delta \neq 0$ as long as the control remains saturated, we cannot ensure that $x_m \rightarrow x_r$. We can only prove that $x_m \in \mathcal{L}_\infty$.

When the control is saturated, the plant trajectory approaches $\pm b^* u_m / a^*$ asymptotically for a stable plant. Thus, we have bounded response on saturation. All signals in the closed-loop are bounded and the control objective is met.

B. Sub-Case 2: Neutrally Stable Plant : $a^* = 0$

The plant model for a neutrally stable plant is

$$\dot{x} = b^* u_a \quad (25)$$

The expression for the calculated control given by (11) becomes

$$u_c = \frac{1}{\hat{b}} (\{\dot{x}_r + \lambda x_r\} - \lambda x) \quad (26)$$

Since we are analyzing the case when the control is saturated, two conditions can be possible: $u_c > u_m$ or $u_c < -u_m$. Let us assume $\hat{b} > 0$. Similar results can be proved when $\hat{b} < 0$. Let us first consider the condition when $u_c > u_m$.

$$u_c = \frac{1}{\hat{b}} (\{\dot{x}_r + \lambda x_r\} - \lambda x) \geq u_m \quad (27)$$

The adaptive parameter \hat{b} is bounded by b_{min} and b_{max} . Since $\dot{x}_r, x_r \in \mathcal{L}_\infty$ there exists a maximum value that $\dot{x}_r + \lambda x_r$ can take. We conclude that $\exists X_{max}$ such that, if $x > X_{max}$ then $u_c \not\geq u_m$. As soon as $x > X_{max}$, the control becomes unsaturated. Thus, as long as the control is saturated, $x < X_{max}$. We have already shown that $x \in \mathcal{L}_\infty$ when the control is unsaturated. By considering the second condition $u_c < -u_m$ we can prove that $\exists X_{min}$ such that, as long as the control is saturated, $x > X_{min}$. Thus we have proved that the state x is bounded. Having shown this, we can continue the proof as in Section V-A and establish similar properties.

C. Sub-Case 3: Unstable Plant : $a^* < 0$

In the case of control of an unstable plant the points of no return play a crucial role. The central idea here is to prevent the state from crossing the points of no return by applying a control such that the derivative of the state goes to zero. To be more precise, the derivative should not be exactly equal to zero. It should have a small value so that the state has a restoring tendency towards $x = 0$. This ensures that the state does not stagnate at the point of no return.

Another important issue here is to show that if $|x| < |b^* u_m / a^*|$ there exists an admissible control which can prevent the state from crossing the points of no return. Consider the plant given in (1) with $a < 0$. The point of no return is given by

$$p_{nr} = \frac{\pm b^* u_m}{a^*} \quad (28)$$

Suppose that the state x is approaching the point of no return. The control required to restrict the state at the point of no return is

$$u_s = \frac{a^* x}{b^*} \quad (29)$$

Substituting (28) in (29), we get that if $|x| < |p_{nr}|$, then $|u_s| < |u_{max}|$. Thus the control u_s is always admissible. To keep a margin of safety and to ensure a small restoring tendency towards $x = 0$, we will try to restrict the state to $0.98(b^* u_m / a^*)$.

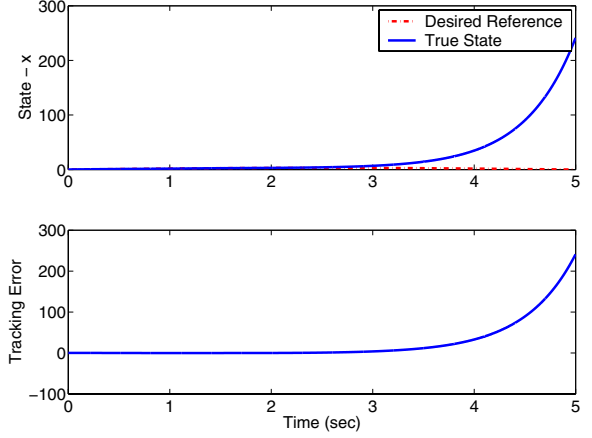


Fig. 4. Time Histories of State and Tracking Error when Instability Protection is Switched Off

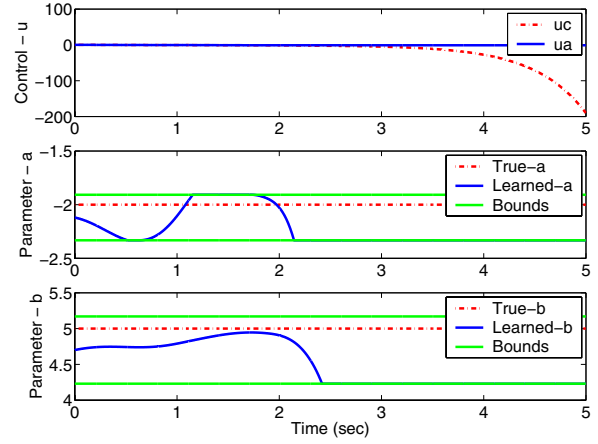


Fig. 5. Time Histories of Control and Adaptive Parameters when Instability Protection is Switched Off

To calculate p_{nr} and u_s the knowledge of system parameters a^* and b^* is required. When the true system parameters are unknown, the conservative estimates for p_{nr} and u_s are

$$p_{nr} = \frac{\pm \bar{b} u_m}{\bar{a}}, \quad u_s = \frac{\bar{a} x}{\bar{b}} \quad (30)$$

where $\bar{a} = a_{min}$, $\bar{b} = b_{max}$ if $b^* < 0$ and $\bar{b} = b_{min}$ if $b^* > 0$. This enforces stricter bounds on the value of x . The state is restricted much closer than the points of no return.

Normally, to satisfy the tracking objective the control would be calculated from (11). Let this control be denoted by u_t . As the state approaches the point of no return we transition from u_t to u_s . To avoid a sudden jump from u_t to u_s and thereby avoid excessive control rate, the transition is as follows

$$u_c = \begin{cases} u_t & \text{if } |x| < 0.88 p_{nr} \\ u_s & \text{if } |x| > 0.98 p_{nr} \end{cases} \quad (31)$$

If $0.88|p_{nr}| < |x| < 0.98|p_{nr}|$, then the control is linearly interpolated between u_t and u_s as

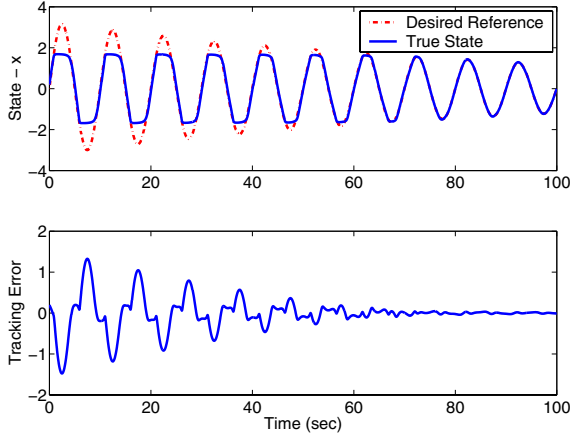


Fig. 6. Time Histories of State and Tracking Error when Reference Modification is Switched Off

$$u_c = u_t + 10\{|x| - 0.88|p_{nr}|\}(u_s - u_t) \quad (32)$$

This idea of switching the control from the tracking objective to the stability objective and restricting the state within the points of no return is termed as ‘Instability Protection’. The above control strategy restricts the control to lie between the points of no return $|x| < |\bar{b}u_m/\bar{a}|$. Thus $x \in \mathcal{L}_\infty$. Following similar arguments as presented in Section V-A, we conclude stability of the adaptive control scheme.

VI. NUMERICAL EXAMPLE

The numerical simulation simulates the response of an unstable plant when the reference trajectory demanded is not only untrackable for some duration, but also crosses the points of no return. Three different test cases are presented.

A. Case 1: Instability Protection Switched Off

When the instability protection is switched off and the reference trajectory crosses the points of no return, the controller performs the tracking objective and the state crosses the point of no return too. Now, the diverging tendency due to the state dominates the restoring tendency that the control can provide, and the state diverges to infinity as seen in Fig.4. From Fig.5 we see that the control also diverges to infinity and the adaptive parameters settle down to one of the parameter bounds.

B. Case 2: Reference Modification Switched Off

In this simulation the instability protection is on, but the reference modification is turned off. From Fig.6 we see that the tracking error is bounded and asymptotically approaches zero as the reference trajectory lies in the trackable region and within the estimated points of no return. Fig.7 shows that the state is restricted well within the points of no return, because the points of no return p_{nr} and the stabilizing control u_s are conservatively estimated from \bar{a} and \bar{b} . This is the best that can be done with the imprecise knowledge of the system parameters a^* and b^* . The adaptive parameters

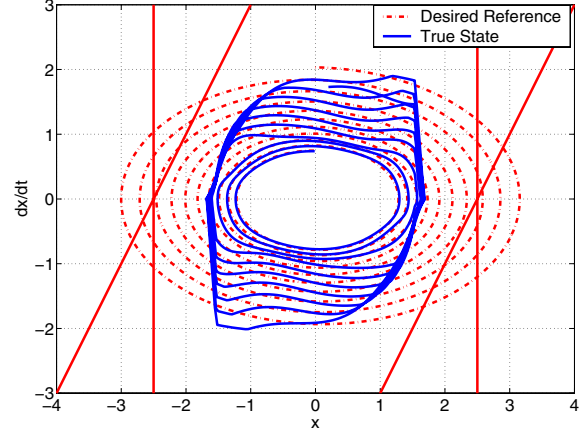


Fig. 7. Phase Portrait when Reference Modification is Switched Off

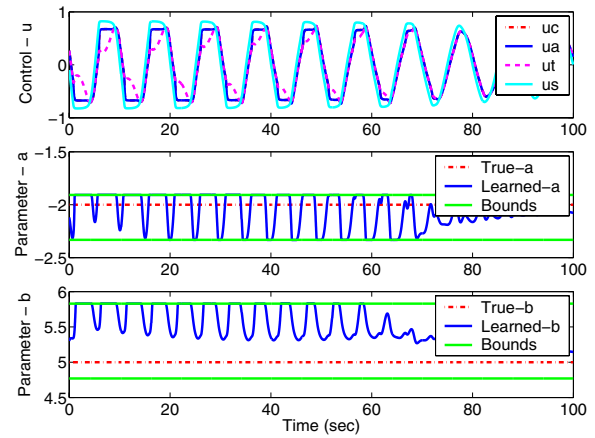


Fig. 8. Time Histories of Control and Adaptive Parameters when Reference Modification is Switched Off

oscillate haphazardly between the bounds as seen from Fig.8. However, the transition of the control u_c between u_t and u_s is smooth and no control chattering is seen.

C. Case 3: Instability Protection & Reference Modification Switched On

In this simulation both the instability protection and the reference modification are turned on. The tracking error is bounded and asymptotically approaches zero as the original desired reference lies in the trackable region and within the points of no return, as shown in Fig.9. From Fig.10 we see that the controller does a better job of tracking than in Fig.7. Fig.11 shows that the estimated parameters converge to the true parameters. This is because the reference is persistently exciting and may not be true otherwise.

VII. CONCLUSIONS AND FUTURE RESEARCH

This paper presented a methodology for stable adaptation in the presence of control position limits for scalar linear time invariant systems with uncertain parameters. For unstable systems, the paper identifies the points of no return and proposes a switching control strategy to restrict the state

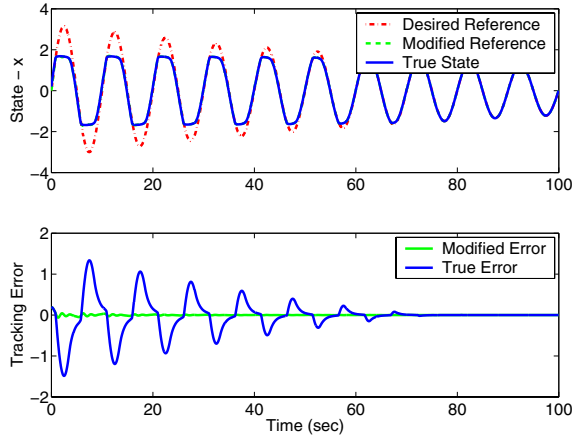


Fig. 9. Time Histories of State and Tracking Error when Reference Modification and Instability Protection are Switched On

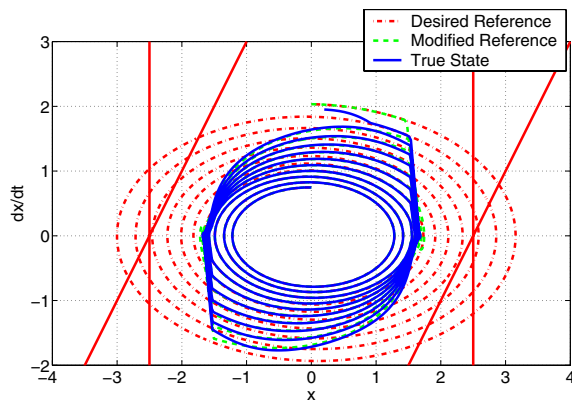


Fig. 10. Phase Portrait when Reference Modification and Instability Protection are Switched On

within the points of no return. Formal proofs for asymptotic tracking when the control is unsaturated and for bounded tracking errors when the control is saturated, are provided. Based on the stability analysis and the numerical simulations presented in the paper, the following conclusions are made:

1. If the control is unsaturated, the tracking error asymptotically goes to zero and all signals in the closed-loop are bounded. If the control is saturated, all signals are bounded and the state asymptotically approaches the modified reference.
2. The switching control strategy successfully restricts the state within the points of no return and the control does not show any chattering.
3. If the reference trajectory is persistently exciting, the adaptive parameters converge to the true parameters, even in the presence of control saturation.

Current efforts are directed toward extending the results to multi-dimensional SISO and MIMO, linear and affine in control nonlinear systems. It should be noted that this extension is not straightforward and leads to considerable

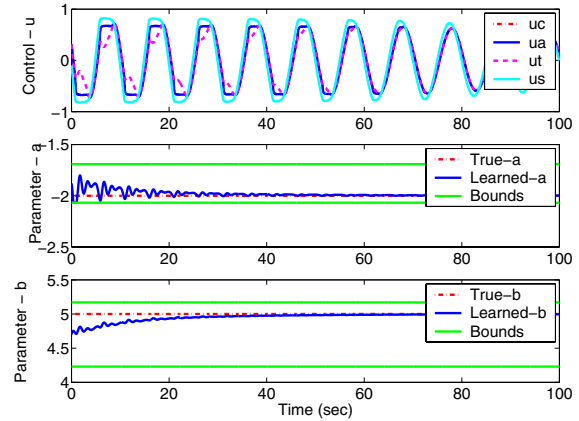


Fig. 11. Time Histories of Control and Adaptive Parameters when Reference Modification and Instability Protection are Switched On

increase in complexity. Further extension will address rate saturation as well as position saturation.

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REFERENCES

- [1] D. Bernstein and A. A. Michel, "Chronological bibliography on saturating actuators," *International Journal of robust and nonlinear control*, vol. 5, pp. 375–380, 1995.
- [2] R. V. Monopoli, "Adaptive control for systems with hard saturation," in *Proceedings of the IEEE Conference on Decision and Control*, Houston, Tx, 1975, pp. 841–843.
- [3] S. P. Karason and A. M. Annaswamy, "Adaptive control in the presence of input constraints," *IEEE Transactions on Automatic Control*, vol. 39, no. 11, pp. 2325–2330, November 1994.
- [4] E. N. Johnson and A. J. Calise, "Limited authority adaptive flight control for reusable launch vehicles," *Journal of Guidance Control and Dynamics*, vol. 26, pp. 906–913, 2003.
- [5] E. Lavretsky and N. Hovakimyan, "Positive μ -modification for stable adaptation in the presence of input constraints," in *Proceedings of the 2004 American Control Conference*, Boston, MA, June 30 - July 2 2004, pp. 2545–2550.
- [6] M. D. Tandale and J. Valasek, "Structured adaptive model inversion control to simultaneously handle actuator failure and actuator saturation," *Journal of Guidance Control and Dynamics*, vol. (accepted; to appear), 2005.
- [7] M. D. Tandale, K. Subbarao, J. Valasek, and M. R. Akella, "Structured adaptive model inversion control with actuator saturation constraints applied to tracking spacecraft maneuvers," in *Proceedings of the American Control Conference*, Boston, MA, 2 July 2004.
- [8] M. D. Tandale and J. Valasek, "Adaptive dynamic inversion control with actuator saturation constraints applied to tracking spacecraft maneuvers," *Journal of Astronautical Sciences*, vol. (accepted; to appear), 2005.
- [9] P. A. Ioannou and J. Sun, *Robust Adaptive Control*. Upper Saddle River, NJ: Prentice Hall, 1995, pp. 203–208.
- [10] H. K. Khalil, *Nonlinear Systems*, 2nd ed. Upper Saddle River, NJ: Prentice Hall, 1996.