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# Pad Conditioning Density Distribution in CMP Process with Diamond Dresser

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***Abstract*— In the chemical mechanical polishing (CMP) process, the pad conditioning density distribution plays a determinant role in the pad wear. A precise model and detail analysis of conditioning density function is required. To this end, at first we construct the mathematic model of polishing trajectories on the pad, then under the assumptions that the diamond grains are uniformly distributed, and slowly sweeping motion during the dressing, the conditioning density distribution for pad in CMP process is determined. This conditioning density function is verified through numerous numerical examples. In the mean time, it was also observed that to have flat distribution of pad wear rate we have to make the ratio of disk-radius-to-pad-radius small, and the effect of the pattern of grain distribution on conditioning density function is insignificant, which also agree with known results from literatures.**

## I. Introduction

In the manufacture of semiconductor featuring enhanced functions, an extremely important requisite is the flatness of wafers. In this regard, the chemical mechanical polishing (CMP) process is increasingly being adopted as the technology to provide flatness. It is known that in CMP process, the pad deformation may occur during wafer polishing [1]. As a result the wafer removal rate will be degraded. Dressing of the pad by a conditioner so as to regenerate a new pad surface and recover its role in the polishing process is thus necessary.

Another main reason for pad conditioning is due to the "glazing" phenomenon. The polishing pad in CMP is composed of multi-holes blown polyurethane and lubricated by slurry. The polishing grains in the slurry and by-productions of polishing will be buried in and block the holes and grooves of polishing pad. Thus the slurry can no longer distribute uniformly on the pad surface. To overcome the glazing problem and extend the lifetime and condition of polishing pad, the polishing pad has to be pruned periodically to remove the debris and by-products in the pad. The present pruning technique includes liquid rinsing, gas blowing and diamond abrasive wheel polishing. The effect of diamond abrasive wheel polishing is so far the best [2]. The CMP pad conditioner renews the pad surface by grinding off the loading on the pad surface by means of diamond grit bonded on stainless steel substrate, but in doing so induces pad wear [3].

Generally, the conditioning disc is mounted on a powered rotating chuck that can be lowered onto the pad surface. The conditioning disc is either swept across the pad surface in a controlled manner, or is of sufficient dimension to condition the entire surface of the pad where it is in contact with a wafer (see Fig. 1).

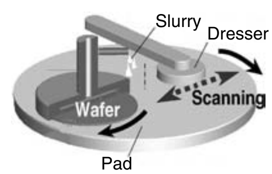


Fig. 1. CMP Schematic

In words, merits of pad conditioner are: (1) lessen pad damages and realized long duration life, (2) stabilize wafer grinding rates. To achieve the goals, in practical application, lots of parameters need to be considered, including the dimension of the dresser base, the size and property of diamond grain, for example, the crystallization, the density of the diamond grain distribution on the dresser base surface and the protrusion height of the diamond grains [4]. Although the key factor in controlling the polishing pad removal rate is the diamond type (nominal abrasive, less aggressive abrasive or more aggressive abrasive), not the abrasive configuration (random or structured, see Fig. 2) [5]. It was also observed that structured configuration provides stable removal rate of dressing [6]. Hence, in this paper we adopt uniform distribution for the diamond grains.

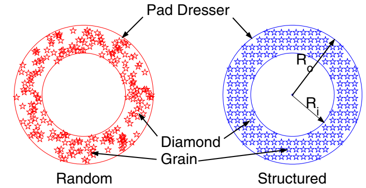


Fig. 2. Diamond Dresser Grain Configuration

Although in the last decade, the modelling of CMP has been progressed to a certain extent. Emphases were placed on the physical interactions among the wafer, slurry, and pad. The studies for pad conditioning were comparatively not so much, see, for example, [3, 7, 8]. Since in the CMP process, the amount of pad wear experienced at a point is proportional to the conditioning density at that point [3]. In [3, 7] only some simple kinematic analysis for the wear rate were given. Therefore in this paper we focus on the analysis of conditioning density.

The structure of this paper is organized as follows: In section II, we start from providing the kinematic model for the polishing trajectory caused by one diamond grain. Then in section III, we assemble the trajectories generated by the entire grains on diamond dresser, and provide a detail analysis for the conditioning density distribution under the assumptions that the diamond grains are uniformly distributed, and slowly sweeping motion during the dressing. Finally, numerical examples are given to verify the proposed analytical expression of conditioning density, and some observations are also drawn through those examples.

## II. The Polishing Trajectories Generated by One Grain

In this paper, we consider only the annular type of diamond grains distribution. Fig. 2 shows a typical geometry of an annular-type diamond dresser, in which $R_o$ and $R_i$ are the outer and inner radius respectively.

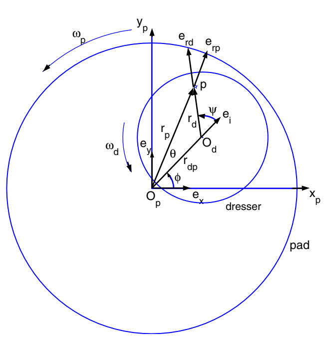


Fig. 3. Definitions in pad and dresser schematic

Fig. 3 demonstrates the variables used in this paper. Consider a typical contact point $p$ (or a diamond grain), we define

- $\vec{r}_p$: the position vector of contact point $p$ relative to the center of pad, $O_p$,
- $\vec{r}_d$: the relative position vector between contact point $p$ and the center of dresser, $O_d$,
- $(x_p, y_p)$: the body fixed coordinate on pad,
- $(\vec{e}_x, \vec{e}_y)$: the unit vectors of coordinate system $(x_p, y_p)$,
- $\vec{e}_i$: an *inertially fixed* unit vector along $\overrightarrow{O_pO_d}$,
- $\vec{e}_{rp}$: a unit vector in $\vec{r}_p$,
- $\vec{e}_{rd}$: a unit vector in $\vec{r}_d$,
- $\omega_p$: the absolute angular velocity of pad,
- $\omega_d$: the absolute angular velocity of dresser,
- $\psi$: the angle between $\vec{e}_{rd}$ and space fixed $e_i$,
- $\theta$: the angle between $\vec{e}_{rp}$ and $\vec{e}_i$,
- $\phi$: the angle between $\vec{e}_i$ and $\vec{e}_x$.

Also, define $\psi_p$ as the angle between $\vec{e}_{rp}$ and $\vec{e}_x$, hence,

$$\psi_p \triangleq \phi + \theta, \tag{2.1}$$

and for convenience define

$$r_p \triangleq \|\vec{r}_p\|,\ r_d \triangleq \|\vec{r}_d\|,\ r_c \triangleq \|\vec{r}_{dp}\|. \tag{2.2}$$

It is easy to see that the two unit vectors $\vec{e}_i$ and $\vec{e}_{rd}$ can also be written as

$$\vec{e}_i = \cos(\phi)\vec{e}_x + \sin(\phi)\vec{e}_y, \tag{2.3}$$

$$\vec{e}_{rd} = \cos(\psi + \phi)\vec{e}_x + \sin(\psi + \phi)\vec{e}_y. \tag{2.4}$$

Since the polishing pad and diamond dresser are rotating with constant angular velocities $\omega_p$ and $\omega_d$ respectively, and without loss of generality, let the pad start at the initial position that $\vec{e}_x$ coincides with inertially fixed unit vector $\vec{i}$. Hence, we have

$$\phi = -\omega_p t, \psi = \omega_d t + \psi_d,$$

where $\psi_d$ is the initial angular position of a typical contact point $p$ (or diamond grain) on dresser. In addition, the angle $\theta$ can be obtained from trigonometric relationship

$$\theta = \text{atan2}(r_d \sin\psi, r_c + r_d \cos\psi), \tag{2.5}$$

where atan2 is the four quadrant inverse tangent. Note that $\text{atan2}(y, x) = \tan^{-1}\left(\frac{y}{x}\right)$, if $x > 0$. Furthermore, assume that the dresser undergoes a sinusoidal sweeping motion

$$r_c(t) = d + d_s \sin(\psi_s), \tag{2.6}$$

where $\psi_s \triangleq \omega_s t$, and

- $d$: the mean distance between the centers of pad and dresser,
- $\omega_s$: the sweeping angular velocity,
- $d_s$: the stroke of sweeping motion, $d_s < d$.

Hence, the loci of contact point $p$ can be expressed in the rotating coordinate $(x_p, y_p)$ as follows

$$\begin{aligned} r_p\vec{e}_{rp} = \vec{r}_p = \vec{r}_d + \vec{r}_{dp} &= r_d\vec{e}_{rd} + r_c\vec{e}_i, \\ &= x_p\vec{e}_x + y_p\vec{e}_y, \end{aligned} \tag{2.7}$$

where

$$\begin{aligned} x_p(t) &= r_d\cos(\psi + \phi) + r_c\cos(\phi), \\ y_p(t) &= r_d\sin(\psi + \phi) + r_c\sin(\phi), \end{aligned} \tag{2.8}$$

or in a more constructive matrix form

$$\begin{aligned} \begin{bmatrix} x_p(t) \\ y_p(t) \end{bmatrix} &= \begin{bmatrix} r_p\cos(\psi_p) \\ r_p\sin(\psi_p) \end{bmatrix} \\ &= R(-\omega_p t)R(\omega_d t)\begin{bmatrix} r_d\cos(\psi_d) \\ r_d\sin(\psi_d) \end{bmatrix} + R(-\omega_p t)\begin{bmatrix} r_c(t) \\ 0 \end{bmatrix}, \end{aligned} \tag{2.9}$$

where the rotational transformation matrix, $R(\cdot)$, is defined as

$$R(\cdot) \triangleq \begin{bmatrix} \cos(\cdot) & -\sin(\cdot) \\ \sin(\cdot) & \cos(\cdot) \end{bmatrix}. \tag{2.10}$$

Equation ( 2.8) implies that the polishing trajectories $(x_p, y_p)$ are a family of cycloids, and can also be realized as moving circles centered at the contact point $p$, with slowly varying radius $r_c$, that is,

$$[x_p - r_d\cos(\phi+\psi)]^2 + [y_p - r_d\sin(\phi+\psi)]^2 = r_c^2. \tag{2.11}$$

Hence, if $r_d + d_s < d$ then there exists unconditioned region in the central area of pad.

To reduce the number of variables, (2.9) can be further written in the following dimensionless form

$$\begin{aligned}\begin{bmatrix}\bar{x}_p(\tau)\\ \bar{y}_p(\tau)\end{bmatrix} &= \begin{bmatrix}\rho_p\cos(\psi_p)\\ \rho_p\sin(\psi_p)\end{bmatrix}\\ &= R(-\tau)R(\omega_{dn}\tau)\begin{bmatrix}\rho_d\cos(\psi_d)\\ \rho_d\sin(\psi_d)\end{bmatrix} + R(-\tau)\begin{bmatrix}\rho_c(\tau)\\ 0\end{bmatrix},\end{aligned} \tag{2.12}$$

where the dimensionless variables

$$\bar{x}_p \triangleq \frac{x_p}{d}, \bar{y}_p \triangleq \frac{y_p}{d}, \rho_p \triangleq \frac{r_p}{d}, \rho_d \triangleq \frac{r_d}{d}, \delta_s \triangleq \frac{d_s}{d}, \tag{2.13}$$

and

$$\tau \triangleq \omega_p t,\ \omega_{dn} \triangleq \frac{\omega_d}{\omega_p},\ \omega_{sn} \triangleq \frac{\omega_s}{\omega_p}, \tag{2.14}$$

$$\rho_c(\tau) \triangleq 1 + \delta_s\sin(\omega_{sn}\tau). \tag{2.15}$$

Taking the inner product of both sides of (2.7) yields

$$r_p^2 = r_d^2 + r_c^2 + 2r_d r_c\cos(\omega_d t + \psi_d), \tag{2.16}$$

or in dimensionless form

$$\rho_p^2 = \rho_d^2 + \rho_c^2 + 2\rho_d\rho_c\cos(\omega_{dn}\tau + \psi_d). \tag{2.17}$$

It is worthy to note that $r_p$ (hence $\rho_p$) is independent of $\omega_p$. Hereafter, without loss of generality, let us assume that

$$\rho_d \le 1, \delta_s < 1. \tag{2.18}$$

For the case $\omega_{sn} = 0, \omega_{dn} \neq 0$, the trajectory reaches maximum $\rho_p$ in every dimensionless time interval $\Delta\tau$,

$$\Delta\tau = \frac{2\pi}{\omega_{dn}}. \tag{2.19}$$

This is obtained by taking the derivative of both sides of (2.17) with respect to $\tau$,

$$\begin{aligned}\dot{\rho}_p = \frac{1}{\rho_p}&\left[(\rho_c + \rho_d\cos\psi)\,\delta_s\omega_{sn}\cos(\omega_{sn}\tau)\right.\\ &\left. -\ \rho_c\rho_d\omega_{dn}\sin\psi\right],\end{aligned} \tag{2.20}$$

and letting ( 2.20) equal zero then solve the resultant equation. From ( 2.20), we know that $\dot{\rho}_p$ is determined by $\omega_{dn}$ and $\omega_{sn}$. In the mean time, it can be shown that the angular displacement, $\psi_p$, advances

$$\Delta\psi_p = -\Delta\tau = -\frac{2\pi}{\omega_{dn}}, \tag{2.21}$$

for every $\Delta\tau$ in the $\tau$ domain.

Furthermore, after some algebraic manipulations, it can be shown that

$$\begin{aligned}\rho_p\dot{\psi}_p &= \rho_p\frac{d}{d\tau}\tan^{-1}\left(\frac{y_p}{x_p}\right),\\ &= -\rho_p - \delta_s\omega_{sn}\sin\theta\cos(\psi_s) + \frac{\omega_{dn}}{2\rho_p}\left(\rho_p^2 + \rho_d^2 - \rho_c^2\right).\end{aligned} \tag{2.22}$$

Hence, if $\omega_{sn} = 0, \rho_d \le 1$, and $0 < \omega_{dn} < 2$ then we have

$$\dot{\psi}_p = -1 + \frac{1}{2}\omega_{dn} + \frac{\omega_{dn}}{2\rho_p^2}\left(\rho_d^2 - 1\right) < 0, \tag{2.23}$$

which means the trajectories on the polishing pad will proceed backwardly.

It is easy to rewrite the normalized translational vector as

$$\begin{aligned}R(-\tau)\begin{bmatrix}\rho_c(\tau)\\ 0\end{bmatrix} &= \begin{bmatrix}\cos(-\tau)\\ \sin(-\tau)\end{bmatrix}\\ &+ \frac{\delta_s}{2}\begin{bmatrix}\sin(1+\omega_{sn})\tau - \sin(1-\omega_{sn})\tau\\ \cos(1+\omega_{sn})\tau - \cos(1-\omega_{sn})\tau\end{bmatrix}.\end{aligned} \tag{2.24}$$

This implies that, in general, the motion of the vector $\vec{r}_p$ is quasi periodic. The period (if exists) in the $\tau$ domain of $\vec{r}_p$ on the pad is the common period of frequencies, $(\omega_{dn} - 1, 1 - \omega_{sn}, 1 + \omega_{sn}, 1)$.

## III. THE CONDITIONING TRAJECTORIES GENERATED BY A DIAMOND DRESSER

It is evident from (2.11) that $\psi_d$ determines the "orientation" of the loci on the pad. This behavior is particularly obvious when $\omega_{dn}$ is an integer or a "simple" rational number, e.g. $1.2, \frac{1}{2}, \frac{4}{3}, \ldots$. This can be seen from Fig. 4, in which the polishing trajectories caused by one grain of conditioning dresser when $\omega_{dn} = 3$ with various initial angular position $\psi_d$. Furthermore, Fig. 5 illustrates the trajectories generated by one grain of dresser at different location of radial distance $\rho_{d_j}$. Hence, it is evident from Figures 4 and 5 that the distribution of diamond grains, or $(\rho_d, \psi_d)$ equivalently, plays the role of dispensing the trajectories on the pad when $\omega_p \neq 0$.

The ensemble of the whole trajectories on the polishing pad is represented by the following expression,

$$\begin{aligned}\bigoplus_{j=1}^{N_d}\begin{bmatrix}\rho_{p_j}\cos(\psi_{p_j})\\ \rho_{p_j}\sin(\psi_{p_j})\end{bmatrix} &= R(-\tau)\begin{bmatrix}\rho_c(\tau)\\ 0\end{bmatrix}\\ &+ R(-\tau)R(\omega_{dn}\tau)\left\{\bigoplus_{j=1}^{N_d}\begin{bmatrix}\rho_{d_j}\cos(\psi_{d_j})\\ \rho_{d_j}\sin(\psi_{d_j})\end{bmatrix}\right\},\end{aligned}$$

where

- $(\rho_{p_j}, \psi_{p_j})$: is the $j$th polishing trajectory generated by the $j$th single diamond grain located at $(\rho_{d_j}, \psi_{d_j})$ on conditioning dresser,
- $N_d$: is the total number of diamond grains,
- $\bigoplus_{j=1}^{N_d}$: stands for the collection of the trajectories generated by the $N_d$ grains on diamond dresser.

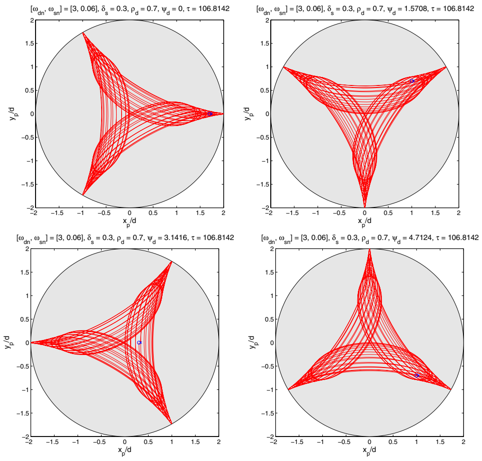

Fig. 4. Trajectories on polishing pad generated by one grain of dresser with different $\psi_{d_j}$.

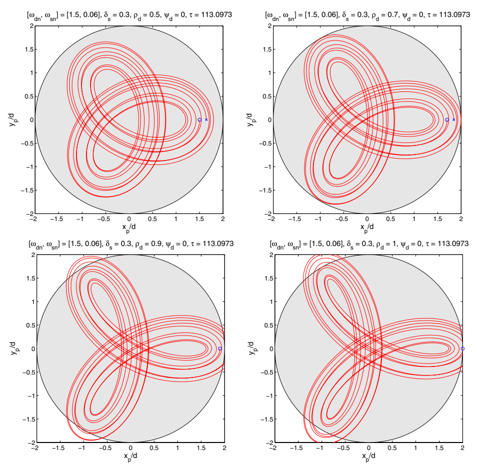

Fig. 5. Trajectories on polishing pad generated by one grain of dresser with various $\rho_{d_j}$.

Since we have known that the amount of wear experienced at a point is proportional to the conditioning density at that point [3, 7]. Therefore in this section, we are going to determine the effect of the distribution of $(\rho_d, \psi_d)$ on conditioning density. The definition of *conditioning density* (CD) is defined as the (time) average of total segment length per unit area in the radial direction,

$$CD(\rho_p) = \frac{1}{T}\int_0^T \lim_{d\rho_p \to 0} \frac{\sum_{j\in I(\rho_p)} \frac{dl_j}{d\tau} d\tau}{2\pi\rho_p d\rho_p}, \tag{3.1}$$

where

- $\rho_p$: is the assigned radius on polishing pad (See Fig. 6).
- $T$: is the elapsed time in $\tau$ domain, here we let $T$ be the period of the sweeping motion, i.e. $T = \frac{2\pi}{\omega_{sn}}$,
- $dl_j$: the length of trajectory segment caused by the grain "$j$" located at $(\rho_{d_j}, \psi_{d_j})$ on diamond dresser,

$$I(\rho_p) \triangleq \{j | \rho_p \leq \rho_{p_j} \leq \rho_p + d\rho_p, j = 1, \ldots, N_d\}.$$

Note that the set $I(\rho_p)$ is time varying in general.

From the definition of $\Delta\theta$ illustrated in Fig. 6, it is not hard to see that

$$\begin{aligned}\Delta\theta = &\cos^{-1}\left[\min\left(\frac{\rho_p^2+\rho_c^2-R_{on}^2}{2\rho_p\rho_c}, 1\right)\right] \\ &- \cos^{-1}\left[\min\left(\frac{\rho_p^2+\rho_c^2-R_{in}^2}{2\rho_p\rho_c}, 1\right)\right],\end{aligned} \tag{3.2}$$

where $R_{on} \triangleq \frac{R_o}{d}, R_{in} \triangleq \frac{R_i}{d}$. The length of each trajectory segment $dl_j$ can be obtained from

$$(dl_j)^2 = (d\rho_{p_j})^2 + (\rho_{p_j} d\psi_{p_j})^2, j = 1, \ldots, N_d. \tag{3.3}$$

Hence, for $\omega_{sn} \ll 1$, after implementing (2.20) and (2.22), it can be shown that

$$\begin{aligned}\frac{dl_j}{d\tau} &= \sqrt{\left(\frac{d\rho_{p_j}}{d\tau}\right)^2 + \left(\rho_{p_j}\frac{d\psi_{p_j}}{d\tau}\right)^2}, \\ &\approx \sqrt{\rho_{d_j}^2\omega_{dn}^2 - (\rho_{p_j}^2 + \rho_{d_j}^2 - \rho_c^2)\omega_{dn} + \rho_{p_j}^2}, \\ &= \begin{cases} \rho_{p_j}, & \omega_{dn} = 0, \\ \rho_c, & \omega_{dn} = 1, \\ \sqrt{|\omega_{dn}(\omega_{dn}-1)|}\sqrt{s(\rho_{d_j}^2 + \kappa)}, & \text{otherwise,} \end{cases}\end{aligned} \tag{3.4}$$

where

$$s \triangleq \text{sgn}(\omega_{dn}(\omega_{dn}-1)), \kappa \triangleq \frac{(1-\omega_{dn})\rho_{p_j}^2 + \omega_{dn}\rho_c^2}{\omega_{dn}(\omega_{dn}-1)}.$$

Note that $\kappa < 0$ if $0 < \omega_{dn} < 1$. Assume that inside the infinitesimal area $2\pi\rho_p d\rho_p$ the line segments produced by each diamond grain is approximately equal to the average value,

$$\begin{aligned}\left(\frac{dl_j}{d\tau}\right)_a &\triangleq \frac{1}{R_{on}-R_{in}}\int_{R_{in}}^{R_{on}} \frac{dl_j}{d\tau} d\rho_{d_j}, \\ &= \begin{cases} \rho_p, & \omega_{dn} = 0, \\ \rho_c, & \omega_{dn} = 1, \\ \frac{\sqrt{|\omega_{dn}(\omega_{dn}-1)|}}{R_{on}-R_{in}}\int_{R_{in}}^{R_{on}} \sqrt{s(\rho_{d_j}^2+\kappa)} d\rho_{d_j}, & \text{otherwise} \end{cases}\end{aligned} \tag{3.5}$$

for $j \in I(\rho_p)$, and the total number of line segments is

$$N(\rho_p) = 2\rho_p \Delta\theta d\rho_p D_d, \tag{3.6}$$

where $D_d \triangleq \frac{N_d}{\pi(R_{on}^2 - R_{in}^2)}$ is the density of diamond grains distribution on dresser. Consequently we have

$$\sum_{j\in I(\rho_p)} \frac{dl_j}{d\tau} = N(\rho_p)\left(\frac{dl_j}{d\tau}\right)_a, \tag{3.7}$$

which in turn yields

$$CD(\rho_p) = \frac{1}{T}\int_0^T \frac{\Delta\theta}{\pi} D_d \left(\frac{dl_j}{d\tau}\right)_a d\tau. \tag{3.8}$$

The integral appears in (3.5) can be obtained from integral table and is given in the following for convenience,

$$\int \sqrt{r^2+k}dr = \frac{1}{2}\left[r\sqrt{r^2+k}+k\ln\left(r+\sqrt{r^2+k}\right)\right],$$
$$\int \sqrt{-r^2+k}dr = \frac{1}{2}\left[r\sqrt{-r^2+k}+k\tan^{-1}\left(\frac{r}{\sqrt{-r^2+k}}\right)\right]$$

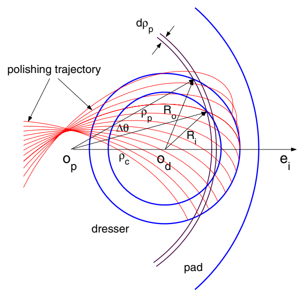


Fig. 6. Geometry of pad and annular-type dresser

Although from (3.2) and (3.5) we know that it is not easy to analytically determine the conditioning density as a function of $\rho_p$, we still are able to obtain simpler expressions for the following three cases. Also for convenience, we define the unit conditional density as

$$\overline{CD}(\rho_p) = \frac{1}{D_d}CD(\rho_p). \tag{3.9}$$

### A. Case I: $\omega_{dn} \approx 0$

For the case $\omega_{dn} \approx 0$, each trajectory caused by $j$th grain, (2.12) can be approximated by, for $j = 1, \ldots, N_d$,

$$\begin{bmatrix}\bar{x}_{p_j}(\tau)\\ \bar{y}_{p_j}(\tau)\end{bmatrix} \approx R(-\tau)\left\{\begin{bmatrix}\rho_{d_j}\cos(\psi_{d_j})\\ \rho_{d_j}\sin(\psi_{d_j})\end{bmatrix}+\begin{bmatrix}\rho_c(\tau)\\ 0\end{bmatrix}\right\}, \tag{3.10}$$

which implies that the trajectories $\bigoplus_{j=1}^{N_d}(\bar{x}_{p_j}, \bar{y}_{p_j})$ is approximated by a series of concentric circles given by the following equation,

$$\bar{x}_{p_j}^2(\tau)+\bar{y}_{p_j}^2(\tau) \approx \left[\rho_{d_j}\cos(\psi_{d_j})+\rho_c(\tau)\right]^2+\rho_{d_j}^2\sin^2(\psi_{d_j}).$$

As a result, the length of each trajectory segment inside the intersection area of $2\pi\rho_p d\rho_p$ and diamond dresser on the pad, $\rho_p\Delta\theta d\rho_p$, can be obtained from the following approximation,

$$dl_j \approx \rho_p d\psi_p = \rho_p d\tau, j \in I(\rho_p). \tag{3.11}$$

This agrees with the one given in (3.4). Hence, we have

$$\overline{CD}(\rho_p) \approx \frac{\omega_{sn}}{2\pi}\int_0^{\frac{2\pi}{\omega_{sn}}}\frac{\Delta\theta}{\pi}\rho_p d\tau. \tag{3.12}$$

### B. Case II: $\omega_{dn} \approx 1$

For this case, each trajectory on the polishing pad can be approximated by (for $j = 1, \ldots, N_d$,)

$$\begin{bmatrix}\bar{x}_{p_j}(\tau)\\ \bar{y}_{p_j}(\tau)\end{bmatrix}-\begin{bmatrix}\rho_{d_j}\cos(\psi_{d_j})\\ \rho_{d_j}\sin(\psi_{d_j})\end{bmatrix} \approx R(-\tau)\begin{bmatrix}\rho_c(\tau)\\ 0\end{bmatrix}, \tag{3.13}$$

which implies

$$\left[\bar{x}_{p_j}(\tau)-\rho_{d_j}\cos(\psi_{d_j})\right]^2+\left[\bar{y}_{p_j}(\tau)-\rho_{d_j}\sin(\psi_{d_j})\right]^2 = \rho_c^2(\tau).$$

This indicates that the trajectories $\bigoplus_{j=1}^{N_d}(\bar{x}_{p_j}, \bar{y}_{p_j})$ can be approximated by a series of circles with slowly varying radius $\rho_c$, centered at $\bigoplus_{j=1}^{N_d}(\rho_{d_j}\cos(\psi_{d_j}), \rho_{d_j}\sin(\psi_{d_j}))$. Hence, it is not hard to see that

$$\frac{dl_j}{d\tau} \approx \rho_c, j \in I(\rho_p). \tag{3.14}$$

This result also agrees with the one shown in (3.4). Hence, we have the conditioning density for each grain

$$\overline{CD}(\rho_p) \approx \frac{\omega_{sn}}{2\pi}\int_0^{\frac{2\pi}{\omega_{sn}}}\frac{\Delta\theta}{\pi}\rho_c d\tau. \tag{3.15}$$

### C. Case III: $\omega_{dn} \gg 1$

For the case $\omega_{dn} \gg 1$, we have for $j = 1, \ldots, N_d$,

$$\begin{bmatrix}\bar{x}_{p_j}(\tau)\\ \bar{y}_{p_j}(\tau)\end{bmatrix}-R(-\tau)\begin{bmatrix}\rho_c(\tau)\\ 0\end{bmatrix} \approx R(\omega_{dn}\tau)\begin{bmatrix}\rho_{d_j}\cos(\psi_{d_j})\\ \rho_{d_j}\sin(\psi_{d_j})\end{bmatrix},$$

which implies that

$$\left[\bar{x}_{p_j}(\tau)-\rho_c\cos(-\tau)\right]^2+\left[\bar{y}_{p_j}(\tau)-\rho_c\sin(-\tau)\right]^2 = \rho_{d_j}^2.$$

Obviously, the polishing trajectories $\bigoplus_{j=1}^{N_d}(\bar{x}_{p_j}, \bar{y}_{p_j})$ is a series of circles centered at comparatively slowly moving point, $(\rho_c\cos(-\tau), \rho_c\sin(-\tau))$, with radius $\rho_{d_j}$.

$$\frac{dl_j}{d\tau} \approx \rho_{d_j}\omega_{dn}, j \in I(\rho_p), \tag{3.16}$$

Equation (3.16) can also be obtained simply by letting $\omega_{dn} \gg 1$ in (3.4). For simplicity, we use the average value of $\frac{dl_j}{d\tau}$ over the range $R_{in} \le \rho_{d_j} \le R_{on}$,

$$\frac{1}{R_{on}-R_{in}}\int_{R_{in}}^{R_{on}}\frac{dl_j}{d\tau}d\rho_{d_j} = \frac{R_{on}+R_{in}}{2}\omega_{dn}, j \in I(\rho_p),$$

to represent the mean value of $\frac{dl_j}{d\tau}, j \in I(\rho_p)$. It follows that

$$\overline{CD}(\rho_p) \approx \frac{\omega_{sn}}{2\pi}\int_0^{\frac{2\pi}{\omega_{sn}}}\frac{\Delta\theta}{\pi}\frac{R_{on}+R_{in}}{2}\omega_{dn}d\tau. \tag{3.17}$$

## IV. Numerical Examples

*Example 4.1:* In this numerical experiment, we adopt the following data: $R_{on} = 0.4, R_{in} = 0.25, \delta_s = 0.6, \omega_{dn} = 0.5, \omega_{sn} = 0.01, N_d = 360$. The $\overline{CD}$ of three different kinds of diamond grain distributions (left hand side of figure) are shown in Fig 7. As we can see, the analytical expression of $\overline{CD}$ matches that obtained from numerical experiment very well. In addition, we also observed that the pattern of diamond grain distribution does not influence the conditioning density in a significant way, which agrees with the conclusion given in [5].

*Example 4.2:* Consider a data set taken from the experiment 1 of [7], $R_{on} = 0.357, R_{in} = 0, \delta_s = 0.682, \omega_{dn} =$

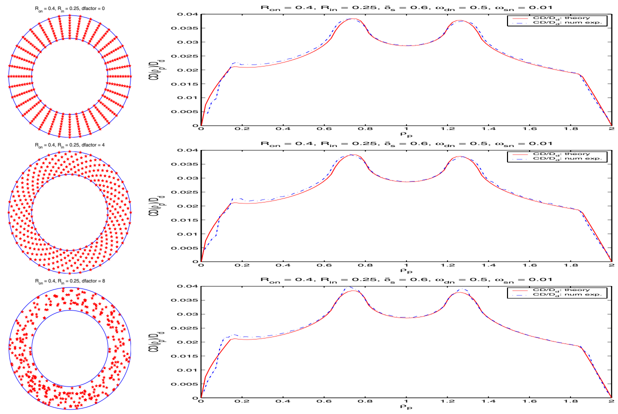


Fig. 7. $\overline{CD}$ of different kind of diamond grain distributions with the same $D_d$.

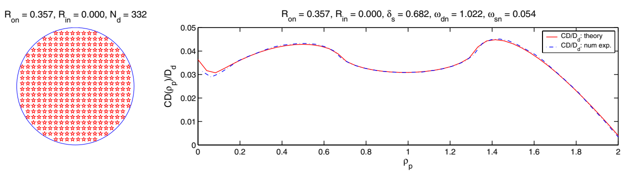


Fig. 8. $\overline{CD}$ of Example 4.2

$1.022, \omega_{sn} = 0.054$. The shape of $\overline{CD}$ shown in Fig. 8 is very similar to the wear thickness given in the paper.

*Example 4.3:* Three different sets of data are used for this example, $N_d = 360$ in all the following three cases:

1) $R_{on} = 0.5, R_{in} = 0.3, \delta_s = 0.5, \omega_{sn} = 0.01$.
2) $R_{on} = 0.3, R_{in} = 0.15, \delta_s = 0.7, \omega_{sn} = 0.01$.
3) $R_{on} = 0.5, R_{in} = 0.0, \delta_s = 0.5, \omega_{sn} = 0.001$.

The corresponding results are shown from the top to bottom in Fig. 9. Some interesting observations can be drawn:

1) $\omega_{dn} \approx 2$ provides a flatter distribution of conditioning density in the area around $\rho_p = 1$, although this needs to be verified analytically in the future.
2) In the mean time, it was also observed that to have flat distribution of pad wear rate we have to make the ratio of disk-radius-to-pad-radius as small as possible,
3) As $\omega_{dn}$ increases, the peak of pad wear rate moves toward the central area of pad ($\rho_p = 0$).

## V. Conclusions

In this paper, we present a precise CD function through a kinematic model describing the polishing trajectories generated by diamond dresser. The only assumptions used for deriving the CD are: the diamond grains are uniformly distributed, and slowly sweeping motion during the dressing. This CD function is verified through numerous numerical examples. In the mean time, it was also observed that to have flat distribution of pad wear rate we have to make the ratio of disk-radius-to-pad-radius as small as possible, and the effect of the pattern of grain distribution on conditioning density function is insignificant. In the future, lab experiments must be conducted to rigorously verify the relationship between pad wear and conditioning density function. Finally, a question we eagerly want to answer: Can a sweeping process achieve a uniform profile in the CD? The answer is no.

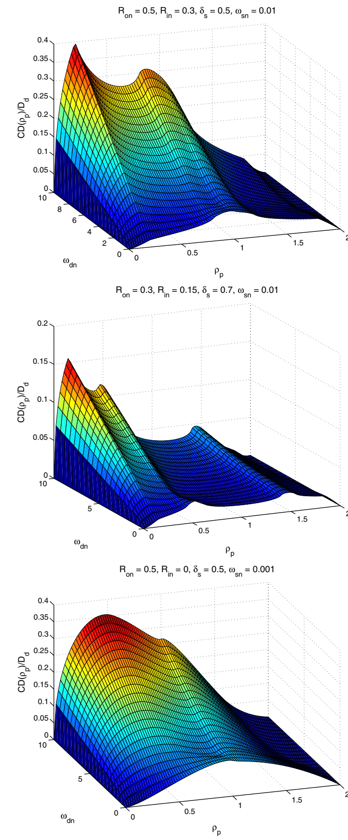


Fig. 9. $\overline{CD}(\rho_p)$ with $0 \le \omega_{dn} \le 10$.

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