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From Neural State-space Description of Marine Powerplants to Reduced-order Volterra Models

Nikolaos I. Xiros and Vasilios P. Tsourapas

Abstract— **The problem of reduced nonlinear input-output models for marine propulsion powerplants is visited by starting from the neural state-space description of the system, the neural networks of which can be trained by using steady-state data of the powerplant collected either from measurements or derived by use of conventional thermodynamic models. The analysis proposed is based on the Taylor expansion of the logistic sigmoid function, appearing in the neural torque approximators of the neural plant description, yielding a finite polynomial Volterra expression, approximating the dynamic behavior of the plant with desired accuracy. The reduced nonlinear model obtains finally the form of a sum of homogenous Volterra operators, and can be used for frequency domain characterization of the system and the design of nonlinear feedforward controllers.**

I. Introduction

Propulsion of modern merchant ships (containerships, VLCCs, etc.) utilizes the marine Diesel engine as prime mover. Typical marine propulsion plants include a single, long-stroke, slow-speed, turbocharged, two-stroke Diesel engine directly coupled with the vessel's fixed-pitch screw propeller [1]. This configuration has been adopted because it is characterized by quite large propulsion power outputs (typically more than 30-40 MW from a single unit) and simplicity. Marine Diesel engines are designed for steady-state operation and selected during the ship design stage, with MCR (Maximum Continuous Rating) at an rpm value that does not exceed 100-120 rpm [1]-[3].

There is a variety of alternative approaches to formulate a marine propulsion powerplant control model [1]-[4]. This problem has been tackled in [5] by the development, from physical principles, of a nonlinear state-space model depicting the engine-turbocharger interaction, which can be used for controller synthesis, in a way bearing significant similarity to the method in [6], which, however, results to a state-space model for the propeller thrust generation dynamics only.

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N. I. Xiros is with the National Technical University of Athens, School of Naval Architecture and Marine Engineering, Iroon Polytechniou 9, 15710 Zografos, Greece (tel.: +30-210-772-1102; fax: +30-210-772-1117; e-mail: nxiros@naval.ntua.gr).

V. P. Tsourapas is with the University Of Michigan, Naval Architecture and Marine Eng., NA&ME Building, 2600 Draper Road, Ann Arbor, Michigan 48109-2145 USA (e-mail: djvas@umich.edu)

The general form of the state-space equations of a marine powerplant is considered

$$\dot{\mathbf{x}}(t) = \mathbf{f}(\mathbf{x}(t), \mathbf{u}(t)) \tag{1}$$

However, due to the complexity of the underlying thermochemical processes, it is required to use detailed physical simulation models [3]-[5]. The mathematical interdependence between the various mechanical (e.g. torques) and thermodynamic (e.g. pressures, temperatures and flow rates) variables of the powerplant is neither explicit nor direct. As has been shown, these variables appear in a set of nonlinear and perplexed algebraic relations; thus, an explicit mathematical expression for the mapping **f** is impossible. On the other hand, the numerical iterative solution of physical models provides a grid of points for which the value of **f** is known. Based on the generated maps, suitably dimensioned neural nets are trained in order to approximate **f** with desired accuracy, but in explicit, analytical, closed form.

In this work, the above neural state-space model is employed to produce a nonlinear control Volterra-type model for the plant. Such a model is required for the design of a nonlinear supervisory (feedforward) controller. The positive effect of such a pre-filter is demonstrated in [5], where a propulsion plant control system architecture is proposed, including a feedforward part that acts as a pre-filter for abrupt setpoint changes and a feedback part that robustly compensates the plant against major sources of uncertainty and disturbance.

II. State Equations of the Marine Powerplant

A. Derivation of Neural Torque Approximators

Quasi-steady, cycle-mean-value models [3]-[5] have been extensively used in marine engineering practice and literature for performance prediction of large marine Diesel engines. These models are derived by extending the relations between thermodynamic variables (pressures, temperatures) holding in equilibrium for cycle mean estimation of thermodynamic and mechanical engine variables. An engine cycle, for large marine Diesel engines, coincides with the period of crankshaft rotation. Due to the cycle-mean approximation employed, combined with the

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inelastic (rigid) engine-propeller shaft assumption the dynamical (differential with respect to time) equations are restricted to the ones depicting the engine–propeller and turbocharger shaft equilibrium. In specific, for crankshaft (N_E) or propeller (N) speed (commonly referred to as rpm in literature) the dynamical equation is as follows

$$\dot{N}_E(t) = \dot{N}(t) = \frac{M(t) - L(t)}{J_{tot}} \tag{2}$$

In the above M is the cycle-mean torque delivery of the engine, L is the cycle-mean torque demand (load) imposed on the powerplant's shafting system and J_{tot} is the total shafting system inertia (engine plus propeller plus entrained water) averaged over a cycle. Note the assumption of engine–propeller direct coupling (no gearbox or clutch is intermitted) and, therefore, $N_E = N$.

For the calculation of the load torque in (2), propeller-law loading [6]-[7] will be considered, i.e.

$$L = K_P \cdot N^2 \tag{3}$$

In the above, K_P, is the propeller torque coefficient that is either provided by the propeller manufacturer or can be calculated by use of the tables for various propeller series (e.g. the Wageningen B-screw series) [6]-[7].

The engine torque delivery, M, is calculated as a function of the thermodynamic variables. Specifically, engine torque is directly proportional to Brake Mean Effective Pressure (BMEP) of the engine, which in turn is determined as output of the cycle-mean thermodynamic model. The basic modeling assumption for the cycle-mean thermodynamic powerplant description is that air and fuel flow through the engine is continuous. The inlet ports and exhaust valves are modeled as orifices restricting air and exhaust gas flows. The turbocharger turbine and compressor operation is modeled using the maps of the manufacturer, whilst heat and mechanical work generation are modeled by empirical or semi-empirical relations. The parameters of the thermodynamic model are either known from physical laws (e.g. ideal gas constants) or determined experimentally and provided by engine and turbocharger manufacturers (e.g. compressor map). A second dynamical equation, appearing in cycle-mean thermodynamic engine models, is the one concerning turbocharger shaft speed (T/C rpm), N_{TC}, given below

$$\dot{N}_{TC}(t) = \frac{Q_T(t) - Q_C(t)}{J_{TC}} = \frac{Q(t)}{J_{TC}}\,. \tag{4}$$

In the above Q_T is the cycle-mean torque delivery of the turbocharger turbine, Q_C is the cycle-mean torque demand of the turbocharger compressor and J_{TC} is the total turbocharger shaft, compressor wheel and turbine rotor inertia reduced to the turbocharger shaft. Both Q_T and Q_C are functions of thermodynamic variables of the engine (e.g. scavenging and exhaust pressures and temperatures) and therefore are calculated as output of the thermodynamic model. For the dynamical equation (3) however their difference is of importance and therefore the total (balance) turbocharger torque Q is introduced.

In conventional marine powerplants the control action is the commonly rated (or dimensionless) fuel index position, Y, which determines the quantity of fuel injected in the engine cylinders per thermodynamic cycle. Indeed, if the fuel index position is given and the values of engine-propeller shaft and turbocharger shaft speeds are known, then the torque variables M, L and Q can be, at least conceptually, calculated as solutions of a nonlinear, perplexed algebraic set of equations. Therefore, from the system-theoretic point of view, it is clear that for cycle-mean models of marine powerplant operation, the state vector of the system is considered to be engine-propeller shaft and turbocharger shaft rpm (speed) variables, whilst the control action is the fuel index position.

However, a major mathematical obstacle arises when one attempts to solve the algebraic set of equations which determines the values of the torque variables M and Q for a given value for the triad of (N, N_{TC}, Y) in contrast to the case of the propeller torque, L, as shown in (3). This is due to the fact that the algebraic system determining functions M and Q is nonlinear and perplexed as demonstrated in [5].

On the other hand, explicit expressions of functions $M(N,N_{TC},Y)$ and $Q(N,N_{TC},Y)$, or, at least, approximants with arbitrarily small approximation error, are needed in order to complete the cycle-mean state-space description of the typical marine powerplant, introduced by dynamical equations (2) and (4). In this end, the following procedure is proposed in [5]: a) Solve the involved algebraic system by a numerical method, e.g. Newton method, for an adequately dense, pre-selected grid of values for the triad (N, N_{TC}, Y) in order to provide a numerical map for each one of the unknown torque functions $M(N,N_{TC},Y)$ and $Q(N,N_{TC},Y)$, and, b) Train appropriately sized neural nets that approximate the maps derived in (1).

By using this methodology the following approximant expressions are obtained for $M(N,N_{TC},Y)$ and $Q(N,N_{TC},Y)$, which are referred to as neural torque approximators, because they are evidently inspired directly from the feedforward neural net structure.

$$\begin{aligned} M(N, N_{TC}, Y) &= M_0 \cdot \{w_0 + \\ &+ \sum_{i=1}^{7} w_i \cdot F(w_{i0} + w_{i1} \cdot N + w_{i2} \cdot N_{TC} + w_{i3} \cdot Y)\} \end{aligned} \tag{5}$$

$$\begin{aligned} Q(N, N_{TC}, Y) &= Q_0 \cdot \{v_0 + \\ &+ \sum_{i=1}^{7} v_i \cdot F(v_{i0} + v_{i1} \cdot N + v_{i2} \cdot N_{TC} + v_{i3} \cdot Y)\} - Q_1 \end{aligned} \tag{6}$$

In the above, the weights w_j, v_j, $0 \le i \le 7$ and w_{ij}, v_{ij}, $1 \le i \le 7$, $0 \le j \le 3$, are determined by the neural approximator training process. For the marine engine modeling application, standard backpropagation has been reported in [5] to perform well. Constants in M_0, Q_0 and Q_1 have been introduced in order to scale the output of the neural nets (the bracketed expressions in (5) and (6)), which is in the range [0,1] back to their original value. An important remark is that the number of neurons in the hidden layer is seven (as can be seen by the range of i), according to the fundamental results of Kolmogorov and Hecht-Nielsen [8]-[9].

The activation function, $F(r)$, of the hidden layer neurons may be any monotonically increasing function that is everywhere differentiable. In this text, it will be the logistic sigmoid, i.e.

$$F(r)=\frac{1}{1+e^{-r}}=\frac{e^{r}}{1+e^{r}},r\in\mathbb{R} \tag{7}$$

Effectively, by combining the cycle-mean thermodynamic modeling approach with the neural torque approximators the following state equations are obtained for the typical marine powerplant.

$$\dot{N}=\frac{M_0}{J_{tot}}\left\{w_0+\sum_{i=1}^{7}w_iF\left(w_{i0}+w_{i1}N+w_{i2}N_{TC}+w_{i3}Y\right)\right\}-\frac{K_PN^2}{J_{tot}} \tag{8}$$

$$\dot{N}_{TC}=\frac{Q_0}{J_{TC}}\left\{v_0+\sum_{i=1}^{7}v_iF\left(v_{i0}+v_{i1}N+v_{i2}N_{TC}+v_{i3}Y\right)\right\}-\frac{Q_1}{J_{TC}} \tag{9}$$

The equilibrium (or steady-state) points of the plant are defined as the triads (N, N_{TC}, Y) for which the following holds.

$$\left.\begin{aligned}\dot{N}&=0\\ \dot{N}_{TC}&=0\end{aligned}\right\}\Leftrightarrow\left\{\begin{aligned}&M(N,N_{TC},Y)=K_PN^2\\ &Q(N,N_{TC},Y)=0\end{aligned}\right. \tag{10}$$

The steady-state points of the plant are the ones for which the plant is designed for continuous, long-term operation and are usually known from the engine selection stage of a new building. For a given steady-state point, determined by a triad (N_0, N_{TC0}, Y_0), the following changes of variables are introduced, which will be used in the sequel.

$$N=N_0+n,N_{TC}=N_{TC0}+n_{TC},Y=Y_0+y \tag{11}$$

A steady-state point of importance for any practical operating marine powerplant is MCR which is determined as the point at which $Y = 100\%$ (rated value) and the engine power delivery ($P = MN$) is the value required for the ship to attain service speed under calm sea conditions. Note that, by use of change of variables (11), propeller torque may be rewritten as follows.

$$L=K_PN^2=K_PN_0^2+2K_PN_0n+K_Pn^2 \tag{12}$$

Some useful remarks concerning the sensitivity of the torque variables, M and Q, to the values of N, N_{TC} and Y at operating points close to steady-state ones, and especially to MCR, are now presented. Mainly because of their close-to-steady-state operation and the fact that they are two-stroke, in order to maximize efficiency, the following conditions hold in a large vicinity around any steady-state operating point.

$$\left.\frac{\partial M}{\partial N},\frac{\partial M}{\partial N_{TC}}\right|_{(N_0,N_{TC0},Y_0)}\simeq 0,\quad \left.\frac{\partial Q}{\partial N}\right|_{(N_0,N_{TC0},Y_0)}\simeq 0 \tag{13}$$

These facts are rather useful for the analysis presented in the sequel.

B. Test Case Presentation

The theoretical procedure has been applied to a marine propulsion powerplant with the MAN-B&W 6L60 marine engine [10]. Some plant details are provided in Table I.

TABLE I
TEST CASE POWERPLANT DETAILS

MAN-B&W 6L60 engine specifics

MCR = 9177kW @ 114.6rpm

z_c	Number of engine cylinders	6	
K_P	Propeller torque coefficient	58.838	Nm/rpm^2
J_{tot}	Engine-propeller shaft inertia	59800.0	kg m^2
J_{TC}	Turbocharger shaft inertia	4.83	kg m^2
p_a, T_a	Ambient (atmospheric) conditions	10^5, 290.0	Pa, $^{\circ}$K

The trained neural torque approximators for this test case are given in [5]. In Fig. 1 a sample of the validation set of the approximators is provided. In this figure the dependence of turbocharger torque, Q, and engine torque delivery, M, is given with respect to turbocharger speed for three different settings of shaft speed and fuel index. Note the accordance of the results for this specific case to remark (13).

III. A REDUCED VOLTERRA MODEL FOR SHAFT SPEED

A. Theoretical Development

If condition (13) is taken into account, then the dynamical equation (8) for the engine-propeller shaft speed becomes as follows.

$$\dot{N}=\frac{M_0}{J_{tot}}\cdot\left\{w_0+\sum_{i=1}^{7}w_iF\left(w_{i0}+w_{i1}N_0+w_{i2}N_{TC0}+w_{i3}Y\right)\right\}-\frac{K_PN^2}{J_{tot}} \tag{14}$$

Note that the above is true only within a vicinity of a steady-state operating point (N_0, N_{TC0}, Y_0). However, this vicinity, in practice, is significantly wider than the region at which a fully linearized (first order of nonlinearity) plant model is valid as the one used e.g. in [5]; therefore, the nonlinear model derived in the sequel has correspondingly significantly extended validity. On the other hand, a Volterra nonlinear model is much more insightful than the neural state-space model; indeed, note that Volterra models can be seen as a rather broad generalization of the describing function methodology, which is quite popular in practical nonlinear control design. For example, a Volterra plant model can be used for establishing the powerplant frequency response to both control action and disturbance signals. This analysis may in turn allow for the synthesis of norm-minimizing (H_2 or H_∞) robust controllers as done in [5] for the linearized models. Note that such controllers will be inherently nonlinear; in effect, the uncertainty margins, are expected to be much narrower than the ones applied for linear robust controller synthesis, leading to significantly less conservative designs.

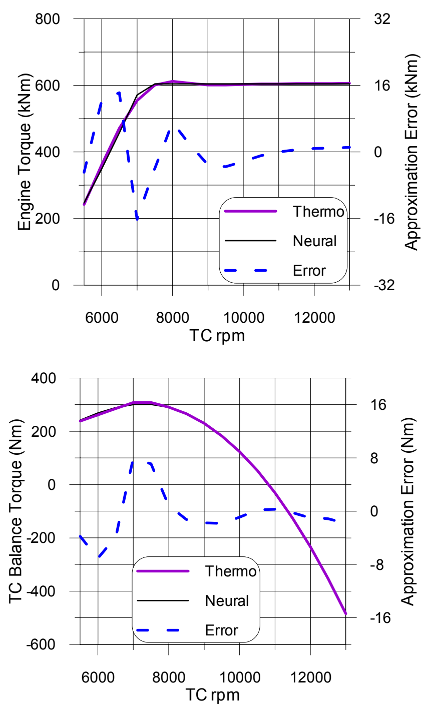

Fig. 1. Neural Torque Approximator validation for N = 110 rpm and Y = 80%.

By employing, the change of variables in (11), (14) is transformed as follows.

$$\dot{n} = \frac{M_0}{J_{tot}}\left\{w_0 + \sum_{i=1}^{7} w_i F\left(r_{i0} + w_{i3}y\right)\right\} - \frac{K_P N_0^2}{J_{tot}} - k_1 n - k_2 n^2 \quad (15)$$

In the above, the following parameter definitions are implied.

$$r_{i0} = w_{i0} + w_{i1}N_0 + w_{i2}N_{TC0} + w_{i3}Y_0$$
$$k_1 = \frac{2K_P N_0}{J_{tot}}, k_2 = \frac{K_P}{J_{tot}} \quad (16)$$

If the Taylor expansion of the activation function $F(r)$ is considered (here it is reminded that $F(r)$ is assumed to be everywhere differentiable), then

$$F\left(r_{i0} + w_{i3}y\right) = \sum_{p=0}^{+\infty} F_{i,p} y^p, F_{i,p} = \frac{F^{(p)}\left(r_{i0}\right)}{p!} w_{i3}^p. \quad (17)$$

In effect, by substituting the above in (15)

$$\dot{n} = \frac{M_0}{J_{tot}}\left\{w_0 + \sum_{p=0}^{+\infty}\left(\sum_{i=1}^{7} w_i F_{i,p}\right) y^p\right\} - \frac{K_P N_0^2}{J_{tot}} - k_1 n - k_2 n^2. \quad (18)$$

Taking into advantage the fact that (N_0, N_{TC0}, Y_0) is assumed to be a steady-state point

$$M_0\left(w_0 + \sum_{i=1}^{7} w_i F_{i,0}\right) = M_0\left(w_0 + \sum_{i=1}^{7} w_i F\left(r_{i0}\right)\right) = K_P N_0^2. \quad (19)$$

Conclusively, the following infinite-order, scalar, Volterra model is obtained for shaft speed, which is valid over the entire operating range of the plant, as long as the Taylor expansion (17) and the assumption for engine torque in (13) are valid

$$\dot{n} = \sum_{p=1}^{+\infty} a_p y^p - k_1 n - k_2 n^2. \quad (20)$$

In the above, evidently,

$$a_p = \frac{M_0}{J_{tot} p!}\sum_{i=1}^{7} w_i w_{i3}^p F^{(p)}\left(r_{i0}\right). \quad (21)$$

In a subsequent phase, one observes that the practical range of y is limited according to

$$0\% \le Y_0 + y < 100\% \Leftrightarrow -Y_0 \le y < 100\% - Y_0. \quad (22)$$

For example, if Y_0 = 100% (i.e. the steady-state operating point is MCR) then y lies in the range [–100%,0%]. In any case, due to the fact that y lies in a limited, finite range, the power series in (17), and in consequence in (20) does not need to be infinite; it can be limited to a parsimonious value of p, $p_p < \infty$. Although, the actual value of p_p, may be any natural number, it will be assumed, without any loss of generality, that $p_p = 3$, because this matches with the results for the test case as demonstrated later on. The reader can then straightforwardly generalize the results to any other finite value of p_p. Therefore by rewriting (20)

$$\dot{n} = \sum_{p=1}^{3} a_p y^p - \sum_{p=1}^{2} k_p n^p. \quad (23)$$

The next step in the process of deriving a Volterra model for shaft speed is to obtain a closed loop expression, in the form of a Volterra operator, from fuel index y, to shaft speed n. To achieve this, at first, the standard Volterra form of the closed-loop operator, $\mathbf{C}$, is formulated

$$n = \mathbf{C}[y] = \sum_{p=1}^{\infty} \mathbf{C}_p[y]. \quad (24)$$

In the above $\mathbf{C}_p$ stands for the homogenous Volterra operator of the p-th order [11]-[12] which contributes to the construction of $\mathbf{C}$.

By substituting (24) in (23)

$$\mathbf{C}[y]=\frac{1}{\mathbf{D}_1}\left[\sum_{p=1}^{3}a_p y^p\right]-\frac{1}{\mathbf{D}_1}\left[\sum_{p=1}^{2}k_p\left(\mathbf{C}[y]\right)^p\right]. \tag{25}$$

In the above, $\frac{1}{\mathbf{D}_1}$ stands for the integration operator, which is evidently of first order, with impulse response the common unit step function.

By using the theoretical results provided in [11]-[12], for the interconnection of nonlinear systems in parallel additively or multiplicatively as well as in cascade, (23) yields the following.

$$C_1(s)=\frac{a_1}{s}-\frac{k_1}{s}C_1(s) \tag{26}$$

$$\begin{aligned} C_2(s_1,s_2) &= \frac{a_2}{s_1+s_2}- \\ &-\frac{1}{s_1+s_2}\left(k_1C_2(s_1,s_2)+k_2C_1(s_1)C_1(s_2)\right) \end{aligned} \tag{27}$$

$$\begin{aligned} C_3(s_1,s_2,s_3) &= \frac{a_3-k_1C_3(s_1,s_2,s_3)}{s_1+s_2+s_3}- \\ &-\frac{k_2}{s_1+s_2+s_3}\left(C_1(s_1)C_2(s_2,s_3)+C_2(s_1,s_2)C_1(s_3)\right) \end{aligned} \tag{28}$$

Note the non-accidental recursive structure of Eqs. (26) to (28) concerning the various homogenous terms of $\mathbf{C}$. Also, although the analysis given here is limited to order 3 the operator in general obtains infinite homogenous terms.

By solving recursively (26) to (28) the following expressions are obtained for the Laplace transforms of the first three homogenous terms of $\mathbf{C}$.

$$C_1(s)=\frac{a_1}{s+k_1} \tag{29}$$

$$C_2(s_1,s_2)=\frac{a_2\left(s_1+k_1\right)\left(s_2+k_1\right)-a_1^2k_2}{\left(s_1+s_2+k_1\right)\left(s_1+k_1\right)\left(s_2+k_1\right)} \tag{30}$$

$$\begin{aligned} C_3(s_1,s_2,s_3) &= \frac{a_3}{s_1+s_2+s_3+k_1}-\frac{k_2}{s_1+s_2+s_3+k_1}\cdot \\ &\cdot\left[\frac{a_1a_2\left(s_2+k_1\right)\left(s_3+k_1\right)-a_1^3k_2}{\left(s_2+s_3+k_1\right)\left(s_1+k_1\right)\left(s_2+k_1\right)\left(s_3+k_1\right)}+\right. \\ &\left.+\frac{a_1a_2\left(s_1+k_1\right)\left(s_2+k_1\right)-a_1^3k_2}{\left(s_1+s_2+k_1\right)\left(s_1+k_1\right)\left(s_2+k_1\right)\left(s_3+k_1\right)}\right] \end{aligned} \tag{31}$$

Alternatively, the above could be derived in an operator inversion framework. In this end, the following operators are introduced

$$\mathbf{I}[u]\triangleq u,\mathbf{A}[u]\triangleq\sum_{p=1}^{3}a_pu^p,\mathbf{K}[u]\triangleq\sum_{p=1}^{2}k_pu^p . \tag{32}$$

By use of these operators (24) (or equivalently (23)) is rewritten as follows

$$\begin{aligned} \mathbf{C}[y] &= \frac{1}{\mathbf{D}_1}\left[\mathbf{A}[y]\right]-\frac{1}{\mathbf{D}_1}[\mathbf{K}] \\ &\Updownarrow \\ \left[\mathbf{D}_1+\mathbf{K}\right]&\left[\mathbf{C}[y]\right]=\mathbf{A}[y] \end{aligned} . \tag{33}$$

In the above, $\mathbf{D}_1$ is the inverse of operator $\frac{1}{\mathbf{D}_1}$, in both notational and actual sense; therefore, is the differential (with respect to time) operator, which is of first order.

Finally, from (33) one obtains for $\mathbf{C}$, the following symbolic expression

$$\mathbf{C}=\left[\mathbf{D}_1+\mathbf{K}\right]^{-1}\left[\mathbf{A}\right]. \tag{34}$$

Despite the notational similarity of (34) to the relationship for linear systems, appearances are misleading, because as argued hereafter the calculation of $[\mathbf{D}_1+\mathbf{K}]^{-1}$, which in literature is referred to as the post-inverse operator of $[\mathbf{D}_1+\mathbf{K}]$, requires an order-by-order procedure similar to the one demonstrated with Eqs. (25)-(28). Indeed, one has to calculate, either in time or Laplace transform domain, an operator, which, when connected in tandem downstream to $[\mathbf{D}_1+\mathbf{K}]$, the overall nonlinear system obtains as impulse response Dirac's delta, $\delta(t)$, i.e. the identity operator $\mathbf{I}$ in (32). After $[\mathbf{D}_1+\mathbf{K}]^{-1}$ has been calculated, the various homogenous terms of $\mathbf{C}$ can be calculated as the tandem connection of $\mathbf{A}$ to $[\mathbf{D}_1+\mathbf{K}]^{-1}$, as shown in (34).

After the closed loop operator $\mathbf{C}$ homogenous terms, up to some order, have been calculated the shaft speed response of the marine powerplant, to a given fuel index profile can be determined. In the transform domain, the response of each term is calculated according to

$$N_p(s_1,\dots s_p)=C(s_1,\dots s_p)\cdot Y(s_1)\cdots Y(s_p) . \tag{35}$$

In the above, $Y(s)$ is the Laplace transform of the fuel index signal, and $N_p(s_1,\dots s_p)$ is the (p-th dimensional) Laplace transform of the p-th order homogenous term appearing in the plant response. The one-dimensional Laplace transform of the response is calculated by

$$N(s)=\sum_{p=1}^{\infty}N_{[p]}(s) . \tag{36}$$

The associated Laplace transform of the p-th order, $N_{[p]}(s)$, is calculated from the p-th dimensional transform of the p-th order homogenous term $N_p(s_1,\dots s_p)$, with a recursive procedure demonstrated hereafter for the initial three orders.

$$N_{[1]}(s_1) = N_1(s_1)$$

$$N_{[2]}(s_1) = \frac{1}{2\pi j}\int_{\sigma - j\infty}^{\sigma + j\infty} N_2\left(s_1 - s_2, s_2\right) ds_2 \qquad (37)$$

$$N_{[3]}(s_1) = \frac{1}{\left(2\pi j\right)^2}\int_{\sigma - j\infty}^{\sigma + j\infty}\int_{\sigma - j\infty}^{\sigma + j\infty} N_3\left(s_1 - s_2, s_2 - s_3, s_3\right)\cdot \cdot ds_2 ds_3$$

The above relationships are deduced from the inverse p-dimensional Laplace transform definition and the fact that in the time domain holds that

$$n_{[p]}(t) = n_p(t_1 = t, \ldots t_p = t)\,. \qquad (38)$$

B. Test Case Application

The afore mentioned theoretical results are now applied to the test case powerplant. The steady-state point to be considered is MCR for which

N_0 = 114.6 rpm, N_{TC0} = 14000 rpm, Y_0 = 100%.

By using the values for the engine torque approximator of the test case powerplant the points, r_{i0}, for the Taylor expansion (17) of the activation function $F(r)$ at each neuron are calculated as in Table II.

TABLE II
POINTS FOR TAYLOR EXPANSION OF ACTIVATION FUNCTION

i	r_{i0}
1	14.4526
2	0.9413
3	–1779
4	4.0273
5	16.5886
6	–41.089
7	3.0903

The quantity appearing in (21) for fixing the units is

$$\frac{M_0}{J_{tot}} = \frac{900\text{ kN m}}{59800\text{ kg m}^2} = 15.052\,\frac{\text{rad}}{\text{s}^2} = 143.72\,\frac{\text{rpm}}{\text{s}}\,.$$

The formulae for the calculation of the first three derivatives of the activation function $F(r)$ are also given below.

$$F^{(1)}\left(r\right) = \frac{e^{-r}}{\left(1 + e^{-r}\right)^2}$$

$$F^{(2)}\left(r\right) = 2\frac{e^{-2r}}{\left(1 + e^{-r}\right)^3} - \frac{e^{-r}}{\left(1 + e^{-r}\right)^2} \qquad .$$

$$F^{(3)}\left(r\right) = 6\frac{e^{-3r}}{\left(1 + e^{-r}\right)^4} - 6\frac{e^{-2r}}{\left(1 + e^{-r}\right)^3} + \frac{e^{-r}}{\left(1 + e^{-r}\right)^2}$$

By using Eq. (21) it is obtained that

$$a_1 = 0.7606\cdot\left(\frac{M_0}{J_{tot}}\right)$$

$$a_2 = -0.3202\cdot\left(\frac{M_0}{J_{tot}}\right)$$

$$a_3 = 0.3149\cdot\left(\frac{M_0}{J_{tot}}\right) .$$

Finally, the propeller related coefficients are given by (16)

$k_1 = 2.1535\text{ s}^{-1}, k_2 = 0.0094\text{ s}^{-1}\text{rpm}^{-1}$.

IV. CONCLUSION

A method, exploiting the neural state-space description of marine powerplant operation, for systematically deducing reduced-nonlinearity models has been presented. The resulting polynomial Volterrra representations obtain the form of a sum of multidimensional transfer functions, which correspond to homogenous Volterra kernels. These transfer functions may be used for frequency characterization of the plant response and, in extension, for designing nonlinear pre-filters and controllers for marine propulsion powerplants. The analysis methodology is exemplified to a typical case for assessing its potential. The validation of the methodology to other cases of ship propulsion plants remains a subject for future work as well as its utilization in controller synthesis and design.

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