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Modeling of Uncertainty and Applications in Monitoring and Control of Power Electronics

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***Abstract*—This paper describes the application of Polynomial Chaos Theory to the modeling, simulation, and control of power electronics systems. The result of this work is a circuit simulation method that is able to quantitatively account for the uncertainty of component parameters, and thereby reveal the effects of those uncertainties, during the design process This paper introduces the mathematical background and then three different applications: automatic system modeling under uncertainty, uncertainty-based control, uncertainty-based monitoring and diagnostics.**

I. INTRODUCTION

The issue of uncertainty is not new in the modeling and control field. There are interesting examples of research that try to apply Artificial Intelligence to provide a simplified or qualitative definition of the uncertainty on physics modeling. Interesting reviews of this topic can be found [1-6]. Similar issues have been considered in the electrical engineering field to overcome specific design problems [7, 8].

In the design of complex systems like an All Electric Ship, we need a modeling approach that supports analysis of conceptual designs from the very beginning—well before a system is fully defined. The usual approach to simulation requires that a system be well- and fully-defined before any calculations can be performed. Yet the design spiral requires that design iterations be based on rational decisions supported by preliminary assessments of system performance.

It has been common practice in engineering to analyze systems based on deterministic mathematical models with precisely defined input data. However, since such ideal situations are rarely encountered in practice, the need to address uncertainties is now clearly recognized, and there has been a growing interest in the application of probabilistic methods.

Similar considerations can be introduced for control applications. The control design for high-cost, safety critical applications such as military ships, ships in general and aircraft must take uncertainty into account. This is not a new topic: Robust Control theory has been introduced since a while, but the effort here proposed is aimed to analyze how a new modeling approach could effectively impact the design process simplifying, at least in part, the uncertainty management. Finally, whenever we identify a control approach, this can always find its dual in an observer approach and then in a possible monitoring approach.

This paper introduces preliminary results in these directions, trying to identify a common approach to modeling, control design and monitoring that could significantly enhance the design process in applications affected by a significant level of uncertainty due to their complexity. The All Electric Ship and its tight interactions of power electronics equipments on a power-limited electric scenario are significant examples such systems.

The Polynomial Chaos Theory (PCT) recently emerged as a successful approach for solving this issue, for example in the analysis of fluid-dynamic problems. We analyze here how the methodology can support modeling in general and control problems more specifically.

Ghanem & Spanos pioneered a polynomial chaos expansion method [9, 10]. It is based on the homogeneous chaos theory of Wiener [11] and is essentially a spectral expansion of the random variables. The expansion employs Hermite orthogonal polynomials in terms of Gaussian random variables. Cameron & Martin have proven that this expansion converges to any L2 functionals in random space in the L2 sense [12]. Combined with Karhunen-Loeve decomposition [13] of the inputs, polynomial chaos results in computationally tractable algorithms for large engineered systems. More recently, a more general framework, called generalized polynomial chaos or Askey chaos, has been proposed [14]. This expansion technique utilizes more orthogonal polynomials in the Askey scheme [15], and is more efficient at representing general non-Gaussian processes. Applications to ODE, PDE, Navier-Stokes equations and flow-structure interactions have been reported and convergence has been demonstrated for model problems [16, 17].

II. THE GENERALIZED POLYNOMIAL CHAOS: ASKEY-CHAOS

We give here a very short introduction to the mathematical approach. The key activity of the chaos expansion is to approximate the random process by a complete and orthogonal polynomial basis in terms of certain random variables. A second-order random process can be represented as:

$$X(\omega) = \sum_{j=0}^{\infty} a_j \Phi_j \left(\xi(\omega)\right) \quad (1)$$

where $\Phi_n(\xi)$ denotes the generalized polynomial chaos of order n in terms of the multidimensional random variables $\xi = (\xi_1, ..., \xi_n, ...)$. The expansion bases $\{\Phi_n\}$ are multidimensional hypergeometric polynomials defined as tensor-products of the corresponding one dimensional polynomial bases, and possess the orthogonality property

$$\langle \Phi_i(\xi), \Phi_j(\xi) \rangle = \langle \Phi_i^2(\xi) \rangle \delta_{ij} \quad (2)$$

where δ_{ij} is the Kronecker delta and $\langle .,. \rangle$ denotes the ensemble average which is the inner product in the Hilbert space of the variable ξ

$$\langle f(\xi) g(\xi) \rangle = \int f(\xi) g(\xi) w(\xi) d\xi. \quad (3)$$

Here $w(\xi)$ is the weighting function. In the case of discrete random variables, the integral in the above expression will be replaced by summation.

The orthogonal polynomials involved in generalized polynomial chaos are listed in Table 1, together with their corresponding underlying random variables. The correspondence between the type of polynomial Φ and its underlying random variables ξ is determined by the requirement that the weighting function of Φ takes the same form as the probability density function of ξ. For example, the classical polynomial chaos, as a subset of the generalized polynomial chaos, employs Hermite polynomials in terms of Gaussian random variables. The weighting function of Hermite polynomials is $w(\xi) = \exp(-\xi^2/2)$, and is the same as the probability density function of Gaussian random variables (except for the scaling factor).

III. STOCHASTIC ORDINARY DIFFERENTIAL EQUATION

We consider the ordinary differential equation

$$\frac{dy(t)}{dt} = -ky(t), \quad y(0) = y_0, \quad (4)$$

where the decay rate coeffcient *k* is a random variable with certain distribution and mean value $\bar{k}$. The deterministic solution is

$$y(t) = y_0 \exp(-\bar{k}t), \quad (5)$$

and the mean of the stochastic solution is

$$\bar{y}(t) = y_0 \int \exp(-kt) f(k) dk \quad (6)$$

where *f(k)* is the probability density function (PDF) of *k*. The integration is taken within the support defined by the corresponding distribution.

By applying the generalized polynomial chaos expansion to the solution *y(t)* and random input *k*

$$y(t) = \sum_{i=0}^{M} y_i(t) \Phi_i(\xi), \quad k = \sum_{i=0}^{M} k_i \Phi_i(\xi) \quad (7)$$

and substituting the expansion into the governing equation, we obtain

$$\sum_{i=0}^{M} \frac{dy_i(t)}{dt} \Phi_i(\xi) = -\sum_{i=0}^{M} \sum_{j=0}^{M} k_i y_j(t) \Phi_i(\xi) \Phi_j(\xi). \quad (8)$$

We then project the above equation onto the random space spanned by the orthogonal polynomial basis $\{\Phi_i(\xi)\}$ by taking the inner product of the equation with each basis. By taking $\langle \square, \Phi_k \rangle$ and utilizing the orthogonality condition, we obtain the following set of equations:

$$\frac{dy_k(t)}{dt} = -\frac{1}{\langle \Phi_k^2 \rangle} \sum_{i=0}^{M} \sum_{j=0}^{M} k_i y_j(t) e_{ijk}, \quad k = 0, \cdots, M, \quad (9)$$

where $e_{ijk} = \langle \Phi_i \Phi_j \Phi_k \rangle$. Note that the coefficients are smooth and thus any standard ordinary differential solver can be employed here.

IV. AUTOMATIC DEFINITION OF UNCERTAIN MODELS ON THE BASE OF POLYNOMIAL CHAOS THEORY

The resistive companion modeling method is defined as a standard ordering of equations represented by nodal analysis. This method is used in the computer simulation of dynamic systems. It allows for a general format for models that can easily automate model creation and system solution. The method requires the equations of models to be represented in a specific form called the resistive companion equation. All models must be represented in this format in order to combine them into a system representation. The resistive companion equation is expressed in (10).

$$I(t) = G(h) * V(t) - B(t-h) \quad (10)$$

where I(t) is the device current, V(t) the device voltage, while G(t) has the dimension of a conductance and represents the relation between samples at the same time instants, while B(t-h) includes all the information of state evolution (also called history vector)

A good overview of the resistive companion method is reported in [20]. Here we focus on extension of the method to include polynomial chaos theory and on the implementation for a specific simulation package: the Virtual Test Bed.

The introduction of uncertainty into circuit analysis comes from applying the polynomial chaos theory to the resistive companion equation. Each circuit variable is expanded onto the polynomial basis:

$$I(t) = \sum_{i=0}^{P} I_i(t) \Psi_i \quad (11)$$

$$V(t) = \sum_{i=0}^{P} V_i(t) \Psi_i \quad (12)$$

$$G(h) = \sum_{i=0}^{P} G_i(h)\Psi_i \quad (13)$$

$$B(t-h) = \sum_{i=0}^{P} B_i(t-h)\Psi_i \quad (14)$$

Replacing the variables back into (10) yields the expanded resistive companion equation shown in (15).

$$\sum_{i=0}^{P} I_i(t)\Psi_i = \sum_{i=0}^{P}\sum_{j=0}^{P} G_i(h)V_j(t)\Psi_i\Psi_j - \sum_{i=0}^{P} B_i(t-h)\Psi_i \quad (15)$$

By applying a Galerkin projection onto the polynomial the system generates a set of deterministic equations that can be solved by a traditional Modified Nodal Analysis solver..

The structure of each equation is given in (16):

$$I_l(t) = \frac{1}{\left\langle \Psi_l^2 \right\rangle} \sum_{i=0}^{P}\sum_{j=0}^{P} G_i(h)V_j(t)\left\langle \Psi_i\Psi_j\Psi_l \right\rangle - B_l(t-h) \quad (16)$$

Let us now suppose to apply the procedure to an uncertain capacitor. The PCT expansion of the capacitance parameter will be as in (17):

$$C(\xi) = C_0\Psi_0 + C_1\Psi_1 + C_2\Psi_2 ... \quad (17)$$

The conductance term of the resistive companion equation can then be written as (18)

$$G(\xi,h) = \frac{2*C_0}{h}\Psi_0 + \frac{2*C_1}{h}\Psi_1 + \frac{2*C_2}{h}\Psi_2 ... \quad (18)$$

Substituting back in the generalized resistive companion form and projecting according to Galerkin we obtain (19a,b,c)

$$I_0(t) = (G_0V_0 + G_1V_1) - B_0(t-h) \quad (19a)$$

$$I_1(t) = (G_1V_0 + G_0V_1 + 2G_1V_2) - B_1(t-h) \quad (19b)$$

$$I_2(t) = (G_1V_1(t) + G_0V_2(t)) - B_2(t-h) \quad (19c)$$

Those three equations can be interpreted as corresponding to the three equivalent circuits shown in **Fig. 1**.

The procedure summarized here has been implemented in the Virtual Test Bed [21]. In the following we report a simple application example.

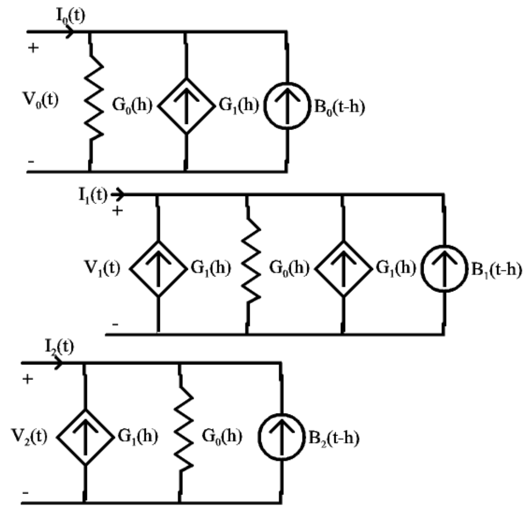

Fig. 1. Equivalent circuit representation of the extended resistive companion equation**.**

V. Modeling a simple power circuit with PCT

Let us consider the circuit in Fig. 2.

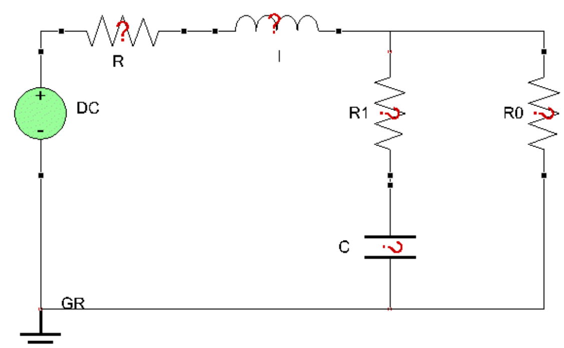

Fig. 2. An example of RLC circuit with uncertainty

The parameters of the circuit are as follows:

- Input Voltage DC: 10 V
- R = 0.5 Ω
- R1 = 0.5 Ω
- R0 = 20 Ω
- L = 1mH
- C = 10 μF

All the parameters apart from the input voltage are assumed to be uncertain but the probabilistic distributions of each of them are known.

In this example we assume a uniform distribution of each parameter (50% for the capacitor, and 5% for all the other parameters) so that a Legendre Chaos is adopted.

Fig. 3 reports the results obtained by the simulation supposing to apply the DC voltage with zero initial conditions on the L and C. The different traces reported in the figure can be used to determine the properties of the probability distribution of the voltage as a function of time. For each instant in time 5 plots are available: the central trace represents the most likely value The two external traces represent the probabilistic minimum and maximum values at any instant and the two intermediate traces represent the voltages corresponding to 25% and 75% cumulative probability. The complete probability distribution is then obtained with a single run of the simulation allowing for very quick evaluation of the impact of the uncertainty on the system behavior.

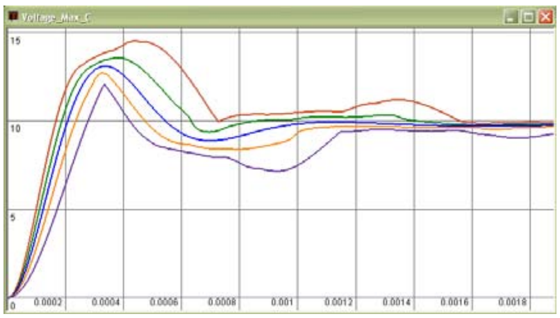

Fig. 3. Voltage transient as function of time under uncertainty.

VI. APPLICATION TO POWER ELECTRONICS: CONVERTER MODELING

Assuming that we have an analytical model of the uncertain system, we can use the extended model as a starting point for the control design process. The idea is illustrated in the following with reference to a power electronics application where the traditional linear optimal control problem is extended to include the PCT description.

Let us start by considering a DC/DC buck converter with a simple inductive/capacitive output filter. The averaged model can be described by the following state equations:

$$\frac{di_l}{dt} = -\frac{v_c}{L} + d\frac{V_{cc}}{L}$$
$$\frac{dv_c}{dt} = -\frac{v_c}{RC} + \frac{i_l}{C} \qquad (20)$$

where:

- L output filter inductance
- C output filter capacitance
- R load resistance
- v_c capacitor voltage
- i_l capacitor voltage
- d duty cycle
- Vcc input voltage

Let us suppose that the uncertainty on the input voltage and on the load resistance is such that:

$$V_{cc} = \overline{V}_c + \sigma_v \xi_1$$
$$G = \frac{1}{R} = \overline{G} + \sigma_g \xi_2 \qquad (21)$$

where ξ_1, ξ_2 are two stochastic variables with zero mean and unity variance.

We can apply the polynomial chaos expansion to the system to obtain a new system of differential equations. If we stop at the first term of the expansion for each stochastic variable we obtain a new set of equations with 6 independent states.

Let us assume to synthesize the feedback control loop as a feed-forward optimal control. The linear time-invariant state feedback is calculated by optimizing the cost-function:

$$J = x^T(\infty)Qx(\infty) + u^T(\infty)Ru(\infty) \qquad (22)$$

where x is the vector of state variable, u is the control output (i.e. the duty cycle) and R,Q two positive definite weight matrices.

The simulation reported in Fig. 4, illustrates the results of the time-domain simulation of the stochastic model under the uncertain condition described above. The intermediate curves describe the most likely value while the two other waveforms describe the variation of the variables in a range that includes the 60% of the cumulative probability.

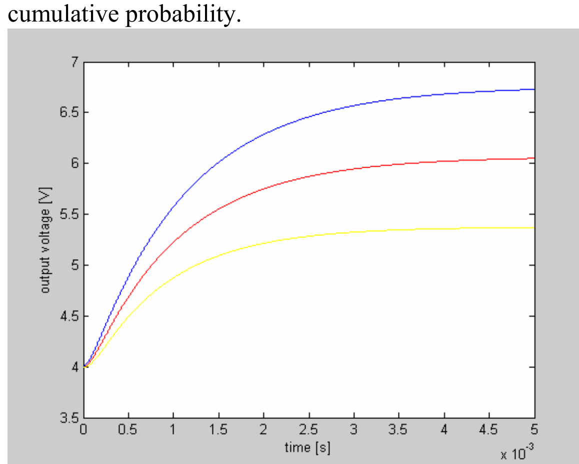

Fig. 4. capacitor voltage (voltage [V] vs time [s])

Let us now synthesize the optimal control feedback for the extended model. We have now 6 state variables so the Q matrix will be a 6x6 square matrix, instead of a simple 2x2 matrix. We have now a degree of freedom to force the solution that minimizes all the state variables other than the most likely values. This opportunity can be used to minimize the uncertainty level, i.e. the possible statistical variation of the controlled variable in steady state.

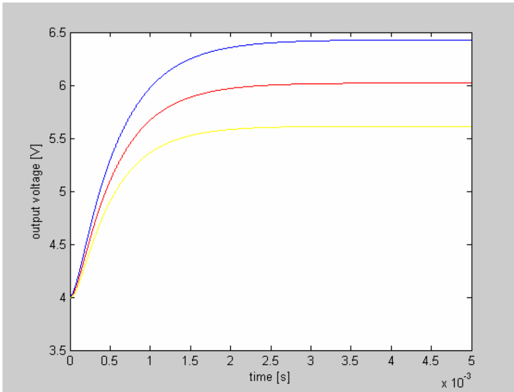

Fig. 5 capacitor output voltage with weight on the uncertainty equal to 100 (voltage [V] vs time [s])

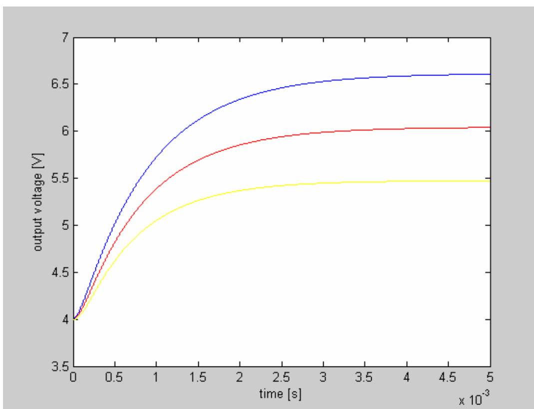

Fig. 6. capacitor output voltage with weight on the uncertainty equal to 10 (voltage [V] vs time[s])

The result is obtained by assigning significantly higher values to certain coefficients of the Q matrix. In the following we report the results obtained by synthesizing the control giving to the most likely value components weight equal to 10 (Fig. 5) and to 100 (Fig. 6) with respect to 1 in the Q matrix.

The simulation results clearly show that the control system is actually limiting the effect of the uncertainty with respect to the standard design of Figure 4. However, this result is obtained by allowing a higher dynamic and then at the expense of a higher picks current value (compare, for example, Fig. 7 and 8). As usual, a trade-off approach has to be applied.

VII. Application In Monitoring and Diagnostics

Finally, we describe a third application of the PCT in monitoring and diagnostics. Here an uncertainty-based model can be used to support a decision-making process.
Let us consider the same buck converter system described in the previous example. For diagnostic reasons it can be useful to evaluate if the temperature of the MOSFET exceeds some predefined value in order to predict possible failures. We will avoid the introduction of a temperature sensor by using only electrical measurements to estimate the junction temperature. The task can be quite challenging considering that the value of the MOSFET on-resistance can significantly vary from one component to another. If we suppose to have some statistical information on this value, however, we can define a model of the buck converter that includes the uncertainty in the on-resistance value.

As result of that, we will be able to integrate the PCT equation at real time and to compare the actual evolution of some electrical quantity such as the output voltage with the probability distribution of its evolution obtained by the PCT model.
Then a comparison between the two values can be performed with the help of an Artificial Intelligence based system such as a Fuzzy Logic system in order to diagnose the fault.
Fig 10 details how different values of on-state resistance highly affect the standard deviation of the output voltage. This quantity can be selected as an index for the fuzzy inference system as reported in Fig. 11 where any switching from 1 to 0 identifies the detection of a time evolution of the output voltage that indicates faulty conditions.

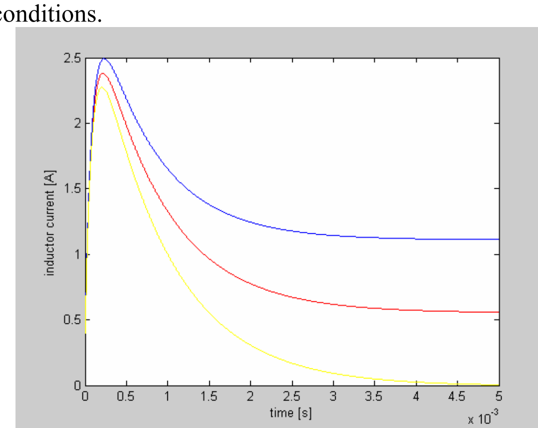

Fig. 7. inductor current with weight on the uncertainty equal to 10 (current [A] vs time [s])

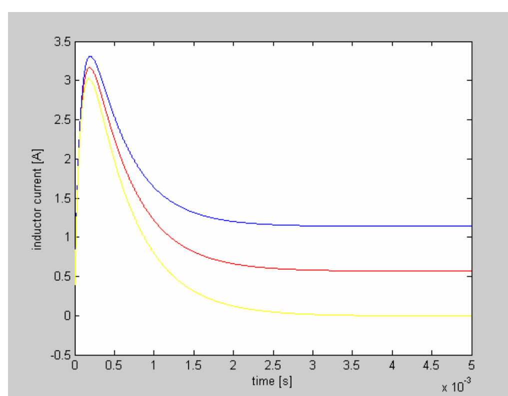

Fig. 8. inductor current with weight on the uncertainty equal to 100 (current [A] vs time [s])

VIII. Conclusion

This paper presented an overview of applications of the PCT to modeling and control problems.

First, an automatic procedure for system modeling was introduced. The procedure was implemented in a software package, the Virtual Test Bed, and it can now be used to analyze the behavior of the system in a way that is more efficient than the Monte-Carlo method.

Second, a new design procedure for power electronics control systems was presented. The methodology could be easily applied in other applications.

Finally an introduction to the application of PCT for modeling and diagnostic purposes was introduced.

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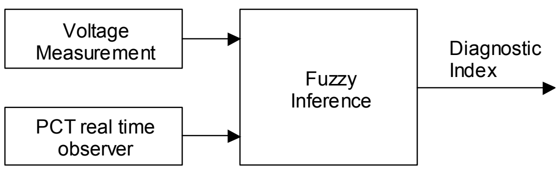

Fig. 9. The diagnostic System

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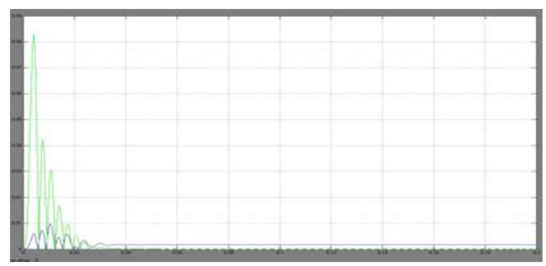

Fig. 10. Comparing two simulations with different values of on-state resistance (trend on standard deviation)

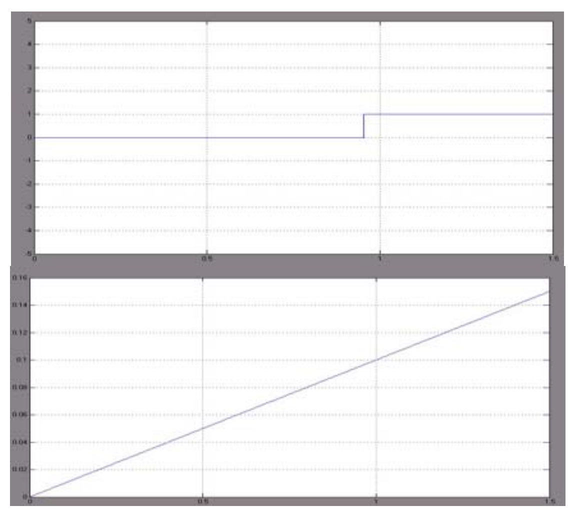

Fig. 11. Output of the fuzzy system (upper) and time variation of the on-state resistance (lower)