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Lyapunov-Based Tracking Control in the Presence of Uncertain Nonlinear Parameterizable Friction

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Abstract— **Modeling and compensation for friction effects has been a topic of considerable mainstream interest in motion control research. This interest is spawned from the fact that modeling nonlinear friction effects is a theoretically challenging problem, and compensating for the effects of friction in a controller has practical ramifications. If the friction effects in the system can be accurately modeled, there is an improved potential to design controllers that can cancel the effects; whereas, excessive steady-state tracking errors, oscillations, and limit cycles can result from controllers that do not accurately compensate for friction. A tracking controller is developed in this paper for a general Euler-Lagrange system that contains a new continuously differentiable friction model with uncertain nonlinear parameterizable terms. To achieve the semi-global asymptotic tracking result, a recently developed integral feedback compensation strategy is used to identify the friction effects on-line, assuming exact model knowledge of the remaining dynamics. A Lyapunov-based stability analysis is provided to conclude the tracking and friction identification results. On-going efforts are being directed at the development of an experimental testbed to illustrate the tracking and friction identification performance of the developed controller.**

I. INTRODUCTION

The modeling and compensation for friction effects has been a topic of considerable mainstream interest in motion control research. This interest is spawned from the fact that modeling nonlinear friction effects is a theoretically challenging problem, and compensating for the effects of friction in a controller has practical ramifications. If the friction effects in the system can be accurately modeled, there is an improved potential to design controllers that can cancel the effects (e.g., model-based controllers); whereas, excessive steady-state tracking errors, oscillations, and limit cycles can result from controllers that do not accurately compensate for friction. Friction is exaggerated at low velocities, which are present in high-precision and high-performance motion control systems; unfortunately, a general model for friction which describes the effects at low velocity has not been universally accepted. Many models of friction have been proposed to deal with the various regimes of friction, each with their own merits and limitations. See [1], [3], [9], [11]-[13], [16], [24], and [27] for a survey of friction modeling and control results. Given the difficulty in accurately modeling and compensating for friction effects, researchers have proposed a variety of (typically offline) friction estimation schemes with the objective of identifying the friction effects. For example, in [8], an offline maximum likelihood, frequency-based approach (differential binary excitation) is proposed to estimate Coulomb friction effects. Another frequency-based offline friction identification approach was proposed in [19]. Specifically, the approach in [19] uses a kind of frequency-domain linear regression model derived from Fourier analysis of the periodic steady-state oscillations of the system. The approach in [19] requires a periodic excitation input with sufficiently large amplitude and/or frequency content. A new offline friction identification tool is proposed in [20] where the static-friction models are not required to be linear parameterizable. However the offline optimization result in [20] is limited to single degree-of-freedom systems where the initial and final velocity are equal. Another frequency domain identification strategy developed to identify dynamic model parameters for presliding behavior is given in [14]. Additional identification methods include least-squares [5] and Kalman filtering [15].

In addition to friction identification schemes, researchers have developed adaptive, robust, and learning controllers to achieve a control objective while accommodating for the friction effects, but not necessarily identifying the friction model. For example, given a desired trajectory that is periodic and not constant over some interval of time, the development in [9] provides a learning control approach to damp out periodic steady-state oscillations due to friction. As stated in [9], a periodic signal is applied to the system and when the system reaches a steady-state oscillation, the learning update law is applied. In [22], a discontinuous linearizing controller was proposed along with an adaptive estimator to achieve an exponentially stable tracking result that estimates the unknown Coulomb friction coefficient. However, [29] describes a technical error in the result presented in [22] that invalidates the result. Additional development is provided in [29] that modifies the result in [22] to achieve asymptotic Coulomb friction coefficient estimation provided a persistence of excitation condition is satisfied. In [25], Tomei proposed a robust adaptive controller where only instantaneous friction is taken into account (dynamic friction effects are not included).

Motivated by the desire to include dynamic friction models in the control design, numerous researchers have embraced the LuGre friction model proposed in [7]. For example, the result in [25] was extended in [26] to include the LuGre friction model [7], resulting in an asymptotic tracking result for square integrable disturbances. Robust adaptive controllers were also proposed in [17] and [23] to account for the LuGre model. Canudas et al. investigated the development of observer-based approaches for the LuGre model in [7]. In [4], Canudas and Lichinsky proposed an adaptive friction compensation method, and in [6] Canudas and Kelly proposed a passivity-based friction compensation term to achieve global asymptotic tracking using the LuGre model. In [2], Barabanov and Ortega developed necessary and sufficient conditions for the passivity of the LuGre model. In [27], three observer-based control schemes were proposed assuming exact model knowledge of the system dynamics. The results in [27] were later extended to include two adaptive observers to account for selected uncertainty in the model. The observer-based design in [27] was further extended in [12]. Specifically, in [12], a partial-state feedback exact model knowledge controller was developed to achieve global exponential link position tracking of a robot manipulator. Two adaptive, partial-state feedback global asymptotic controllers were also proposed in [12] that compensate for selected uncertainty in the system model. In addition, a new adaptive control technique was proposed in [12] to compensate for the nonlinear parameterizable Stribeck effect, where the average square integral of the position tracking errors were forced to an arbitrarily small value.

In this paper, a tracking controller is developed for a general Euler-Lagrange system that contains a new continuously differentiable friction model with uncertain nonlinear parameterizable terms. Friction models are often based on the assumption that the friction coefficient is constant with sliding speed and have a singularity at the onset of slip. Such models typically include a signum function of the velocity to assign the direction of friction force (e.g., [21], [24]), and many other models are only piecewise continuous (e.g., the LuGre model in [7]). The model proposed in this paper captures a number of essential aspects of friction without involving discontinuous or piecewise continuous functions. This simple continuously differentiable model represents a foundation that captures the major effects reported and discussed in friction modeling and experimentation. The proposed model is generic enough that other subtleties such as frictional anisotropy with sliding direction can be addressed by mathematically distorting this model without compromising the continuous differentiability. Based on the fact that the model is continuously differentiable, a new integral feedback compensation term originally proposed in [28] is exploited to enable a semi-global tracking result while identifying the friction on-line, assuming exact model knowledge of the remaining dynamics. A Lyapunov-based stability analysis is provided to conclude the tracking and friction identification results.

II. Dynamic Model and Properties

The class of nonlinear dynamic systems considered in this paper are assumed to be modeled by the following general Euler-Lagrange formulation:

$$M(q)\ddot{q} + V_m(q,\dot{q})\dot{q} + G(q) + f(\dot{q}) = \tau(t). \qquad (1)$$

In (1), $M(q) \in \mathbb{R}^{n\times n}$ denotes the inertia matrix, $V_m(q,\dot{q}) \in \mathbb{R}^{n\times n}$ denotes the centripetal-Coriolis matrix, $G(q) \in \mathbb{R}^n$ denotes the gravity vector, $f(\dot{q}) \in \mathbb{R}^n$ denotes a friction vector, $\tau(t) \in \mathbb{R}^n$ represents the torque input control vector, and $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathbb{R}^n$ denote the link position, velocity, and acceleration vectors, respectively. The friction term $f(\dot{q})$ in (1) is assumed to have the following form:

$$\begin{aligned} f(\dot{q}) \;=\;& \gamma_1(\tanh(\gamma_2\dot{q}) - \tanh(\gamma_3\dot{q})) \\ &+\gamma_4\tanh(\gamma_5\dot{q}) + \gamma_6\dot{q} \end{aligned} \qquad (2)$$

where $\gamma_i \in \mathbb{R}\ \forall i = 1,2,...,6$ denote unknown positive constants. The friction model in (2) has the following properties: 1) it is symmetric about the origin, 2) it has a static coefficient of friction, 3) it exhibits the Stribeck effect where the friction coefficient decreases from the static coefficient friction with increasing slip velocity near the origin, 4) it includes a viscous dissipation term, and 5) it has a Coulombic friction coefficient in the absence of viscous dissipation. To a good approximation, the static friction coefficient is given by $\gamma_1 + \gamma_4$, and the Stribeck effect is captured by $\tanh(\gamma_2\dot{q}) - \tanh(\gamma_3\dot{q})$. The Coulombic friction coefficient is given by $\gamma_4\tanh(\gamma_5\dot{q})$, and the viscous dissipation is given by $\gamma_6\dot{q}$. Figures 1 and 2 illustrate the sum of the different effects in the friction model where $\gamma_1 = 0.25$, $\gamma_2 = 100$, $\gamma_3 = 1$, $\gamma_4 = 0.5$, $\gamma_5 = 100$, $\gamma_6 = 0.01$. Figure 3 shows the flexibility of such a model. For example, hydrodynamic lubrication in many operating regimes is viscous, lacking the other effects, which are easily set to zero in the model. Simple Coulombic friction models are often good for solid lubricant coatings at moderate sliding speeds. To capture this effect, the static and viscous terms can be set to zero. For some sticky or non-lubricous polymers, there exists an abrupt change from static to kinetic friction, which is captured by making the Stribeck decay portion very rapid.

The subsequent development is based on the assumption that $q(t)$ and $\dot{q}(t)$ are measurable and that $M(q)$, $V_m(q,\dot{q})$, $G(q)$ are known. Moreover, the following properties and assumptions will be exploited in the subsequent development:

Property 1: The inertia matrix $M(q)$ is symmetric, positive definite, and satisfies the following inequality $\forall\ y(t) \in \mathbb{R}^n$:

$$m_1\|y\|^2 \le y^T M(q) y \le \bar{m}(q)\|y\|^2 \qquad (3)$$

where $m_1 \in \mathbb{R}$ is a known positive constant, $\bar{m}(q) \in \mathbb{R}$ is a known positive function, and $\|\cdot\|$ denotes the standard Euclidean norm.

Property 2: If $q(t) \in \mathcal{L}_\infty$, then $\dfrac{\partial M(q)}{\partial q}$, and $\dfrac{\partial^2 M(q)}{\partial q^2}$ exist and are bounded. Moreover, if $q(t),\ \dot{q}(t) \in \mathcal{L}_\infty$ then $V_m(q,\dot{q})$ and $G(q)$ are bounded.

Property 3: Based on the structure of $f(\dot{q})$ given in (2), $f(\dot{q})$, $\dot{f}(\dot{q}, \ddot{q})$, and $\ddot{f}(\dot{q}, \ddot{q}, \dddot{q})$ exist and are bounded provided $q(t)$, $\dot{q}(t)$, $\ddot{q}(t)$, $\dddot{q}(t) \in \mathcal{L}_\infty$.

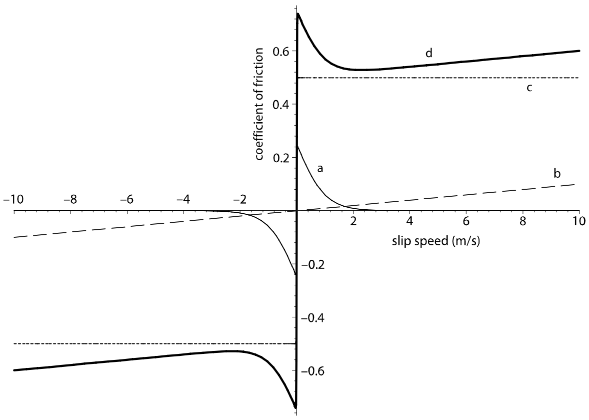

Fig. 1. Friction model as a composition of different effects including: a) Stribeck effect, b) viscous dissipation, c) Coulomb effect, and d) the combined model.

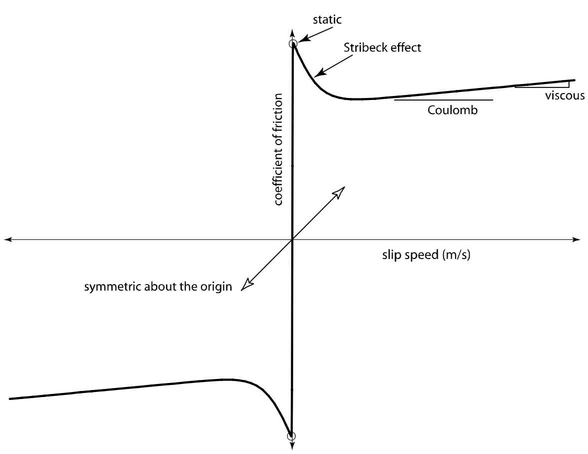

Fig. 2. Friction model effects.

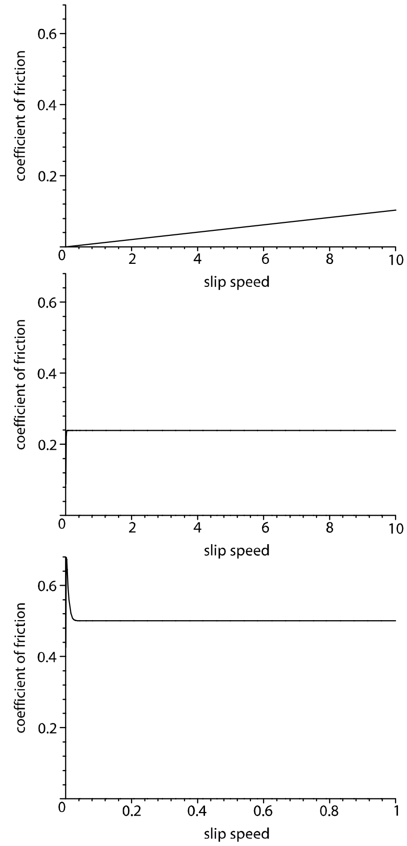

Fig. 3. Modular ability of the model to selectively model different friction regimes: top plot - viscous regime (e.g., hydrodynamic lubrication), middle plot - Coulombic friction regime (e.g., solid lubricant coatings at moderate sliding speeds), and bottom plot - abrupt change from static to kinetic friction (e.g., non-lubricous polymers).

III. Error System Development

The control objective is to ensure that the system tracks a desired trajectory, denoted by $q_d(t)$, that is assumed to be designed such that $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$, $\dddot{q}_d(t) \in \mathbb{R}^n$ exist and are bounded. A position tracking error, denoted by $e_1(t) \in \mathbb{R}^n$, is defined as follows to quantify the control objective:

$$e_1 \triangleq q_d - q. \tag{4}$$

The following filtered tracking errors, denoted by $e_2(t)$, $r(t) \in \mathbb{R}^n$, are defined to facilitate the subsequent design and analysis:

$$e_2 \triangleq \dot{e}_1 + \alpha_1 e_1 \tag{5}$$

$$r \triangleq \dot{e}_2 + \alpha_2 e_2 \tag{6}$$

where α_1, $\alpha_2 \in \mathbb{R}$ denote positive constants. The filtered tracking error $r(t)$ is not measurable since the expression in (6) depends on $\ddot{q}(t)$.

After premultiplying (6) by $M(q)$, the following expression can be obtained:

$$\begin{aligned} M(q)r &= M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) + f(\dot{q}) \\ &\quad -\tau(t) + M(q)\alpha_1\dot{e}_1 + M(q)\alpha_2 e_2 \end{aligned} \tag{7}$$

where (1), (4), and (5) were utilized. Based on the expression in (7) the control torque input is designed as follows:

$$\begin{aligned} \tau(t) &= M(q)\ddot{q}_d + V_m(q,\dot{q})\dot{q} + G(q) \\ &\quad +M(q)\alpha_1\dot{e}_1 + M(q)\alpha_2 e_2 + \mu(t) \end{aligned} \tag{8}$$

where $\mu(t) \in \mathbb{R}$ denotes a subsequently designed control term. By substituting (8) into (7), the following expression can be obtained:

$$M(q)r = f(\dot{q}) - \mu(t). \tag{9}$$

From (9), it is evident that if $r(t) \to 0$, then $\mu(t)$ will identify the friction dynamics; therefore, the objective is to design the control term $\mu(t)$ to ensure that $r(t) \to 0$. To facilitate the design of $\mu(t)$, we differentiate (9) as follows:

$$M(q)\dot{r} = \dot{f}(\dot{q}) - \dot{\mu}(t) - \dot{M}(q)r. \quad (10)$$

Based on (10) and the subsequent stability analysis, $\mu(t)$ is designed as follows:

$$\begin{aligned}\mu(t) \;&=\; (k_s+1)e_2(t) - (k_s+1)e_2(t_0) \quad (11)\\ &\quad + \int_{t_0}^{t} [(k_s+1)\alpha_2 e_2(\tau) + \beta sgn(e_2(\tau))]d\tau\end{aligned}$$

where k_s and β are positive constants. The expression in (11) for $\mu(t)$ does not depend on the unmeasurable filtered tracking error term $r(t)$. However, the time derivative of $\mu(t)$ (which is not implemented) can be expressed as a function of $r(t)$. The time derivative of (11) is given as

$$\dot{\mu}(t) = (k_s+1)r + \beta sgn(e_2). \quad (12)$$

After substituting (12) into (10), the following closed-loop error system can be obtained:

$$\begin{aligned}M(q)\dot{r} \;&=\; -\frac{1}{2}\dot{M}(q)r - (k_s+1)r \quad (13)\\ &\quad -e_2 - \beta sgn(e_2) + N(t)\end{aligned}$$

where $N(t) \in \mathbb{R}^n$ denotes the following unmeasurable auxiliary term:

$$N(q,\dot{q},t) \triangleq \dot{f}(\dot{q}) - \frac{1}{2}\dot{M}(q)r + e_2. \quad (14)$$

To facilitate the subsequent analysis, another unmeasurable auxiliary term $N_d(t) \in \mathbb{R}^n$is defined as follows:

$$\begin{aligned}N_d(t) \;&\triangleq\; \frac{\partial f(\dot{q}_d)}{\partial \dot{q}_d}\ddot{q}_d \quad (15)\\ &=\; \gamma_1\gamma_2\ddot{q}_d - \gamma_1\gamma_2\ddot{q}_d\left\|\tanh(\gamma_2\dot{q}_d)\right\|^2\\ &\quad -\gamma_1\gamma_3\ddot{q}_d + \gamma_1\gamma_3\ddot{q}_d\left\|\tanh(\gamma_3\dot{q}_d)\right\|^2\\ &\quad +\gamma_4\gamma_5\ddot{q}_d - \gamma_4\gamma_5\ddot{q}_d\left\|\tanh(\gamma_5\dot{q}_d)\right\|^2\\ &\quad +\gamma_6\ddot{q}_d.\end{aligned}$$

The time derivative of (15) is given as follows:

$$\begin{aligned}\dot{N}_d(t) \;&=\; \frac{\partial^2 f(\dot{q}_d)}{\partial \dot{q}_d^2}\ddot{q}_d^2 + \frac{\partial f(\dot{q}_d)}{\partial \dot{q}_d}\dddot{q}_d \quad (16)\\ &= \dddot{q}_d(\gamma_1\gamma_2 - \gamma_1\gamma_3 + \gamma_4\gamma_5 + \gamma_6)\\ &\quad -\gamma_1\gamma_2\dddot{q}_d\left\|\tanh(\gamma_2\dot{q}_d)\right\|^2\\ &\quad +\gamma_1\gamma_3\dddot{q}_d\left\|\tanh(\gamma_3\dot{q}_d)\right\|^2\\ &\quad -\gamma_4\gamma_5\dddot{q}_d\left\|\tanh(\gamma_5\dot{q}_d)\right\|^2\\ &\quad -2\gamma_1\gamma_2^2||\ddot{q}_d||^2\tanh(\gamma_2\dot{q}_d)[1-\left\|\tanh(\gamma_2\dot{q}_d)\right\|^2]\\ &\quad +2\gamma_1\gamma_3^2||\ddot{q}_d||^2\tanh(\gamma_3\dot{q}_d)[1-\left\|\tanh(\gamma_3\dot{q}_d)\right\|^2]\\ &\quad -2\gamma_4\gamma_5^2||\ddot{q}_d||^2\tanh(\gamma_5\dot{q}_d)[1-\left\|\tanh(\gamma_5\dot{q}_d)\right\|^2].\end{aligned}$$

After adding and subtracting (15), the closed-loop error system in (13) can be expressed as follows:

$$\begin{aligned}M(q)\dot{r} \;&=\; -\tfrac{1}{2}\dot{M}(q)r - (k_s+1)r - e_2 - \beta sgn(e_2) \quad (17)\\ &\quad +\tilde{N} + N\end{aligned}$$

where the unmeasurable auxiliary term $\tilde{N}(t) \in \mathbb{R}^n$ is defined as

$$\tilde{N}(t) \triangleq N(t) - N_d(t). \quad (18)$$

Based on the expressions in (15) and (16), the following inequalities can be developed:

$$\|N_d(t)\| \leq ||\ddot{q}_d|| \cdot |\gamma_1\gamma_2 + \gamma_4\gamma_5 + \gamma_6 - \gamma_1\gamma_3| \leq \zeta_{N_d} \quad (19)$$

$$\begin{aligned}\left\|\dot{N}_d(t)\right\| \;&\leq\; ||\dddot{q}_d|| \cdot |\gamma_1\gamma_2 + \gamma_4\gamma_5 + \gamma_6 - \gamma_1\gamma_3| \quad (20)\\ &\quad +||\ddot{q}_d||^2(2\gamma_1\gamma_2^2 + 2\gamma_1\gamma_3^2 + 2\gamma_4\gamma_5^2)\\ &\leq\; \zeta_{N_d2}\end{aligned}$$

where ζ_{N_d}, $\zeta_{N_d2} \in \mathbb{R}$ are known positive constants.

IV. Stability Analysis

Theorem 1: The controller given in (8) and (11) ensures that the position tracking error is regulated in the sense that

$$e_1(t) \to 0 \qquad as \qquad t \to \infty$$

provided β is selected according to the following sufficient condition:

$$\beta > \zeta_{N_d} + \frac{1}{\alpha_2}\zeta_{N_d2} \quad (21)$$

where ζ_{N_d} and ζ_{N_d2} are introduced in (19) and (20), respectively, and k_s is selected sufficiently large. The control system represented by (8) and (11) also ensures that all system signals are bounded under closed-loop operation and that the system friction can be identified in the sense that

$$f(\dot{q}) - \mu(t) \to 0 \qquad as \qquad t \to \infty.$$

Proof: Let $\mathcal{D} \subset \mathbb{R}^{3n+1}$ be a domain containing $y(t) = 0$, where $y(t) \in \mathbb{R}^{3n+1}$ is defined as

$$y(t) \triangleq [z^T(t) \quad \sqrt{P(t)}]^T \quad (22)$$

where $z(t) \in \mathbb{R}^{3n}$ is defined as

$$z(t) \triangleq [e_1^T \quad e_2^T \quad r^T]^T \quad (23)$$

and the auxiliary function $P(t) \in \mathbb{R}$ is defined as

$$P(t) \triangleq \beta\,||e_2(t_0)|| - e_2(t_0)^T N_d(t_0) - \int_{t_0}^{t} L(\tau)d\tau, \quad (24)$$

where β is a nonnegative design parameter to be determined later.

In (24), the auxiliary function $L(t) \in \mathbb{R}$ is defined as

$$L(t) \triangleq r^T(N_d(t) - \beta sgn(e_2)). \quad (25)$$

Thus, the derivative $\dot{P}(t) \in \mathbb{R}$ can be expressed as

$$\dot{P}(t) = -L(t) = -r^T(N_d(t) - \beta sgn(e_2)). \quad (26)$$

Provided the sufficient condition introduced in (21) is satisfied, the following inequality can be obtained

$$\int_{t_0}^{t} L(\tau)d\tau \leq \beta\left|e_2(t_0)\right| - e_2(t_0)^T N_d(t_0). \quad (27)$$

Hence, (27) can be used to conclude that $P(t) \geq 0$. Let $V(y,t): \mathcal{D}\times[0,\infty)\to\mathbb{R}$ be a continuously differentiable positive definite function defined as

$$V(y,t) \triangleq e_1^T e_1 + \frac{1}{2}e_2^T e_2 + \frac{1}{2}r^T M(q) r + P \quad (28)$$

that can be bounded as

$$W_1(y) \leq V(y,t) \leq W_2(y) \quad (29)$$

provided the sufficient condition introduced in (21) is satisfied. In (29), the continuous positive definite functions $W_1(y)$, $W_2(y) \in \mathbb{R}$ are defined as

$$W_1(y) = \lambda_1\left\|y\right\|^2 \qquad W_2(y) = \lambda_2(q)\left\|y\right\|^2 \quad (30)$$

where λ_1, $\lambda_2(q) \in \mathbb{R}$ are defined as

$$\lambda_1 \triangleq \frac{1}{2}\min\{1, m_1\} \qquad \lambda_2(q) \triangleq \max\{\frac{1}{2}\bar{m}(q), 1\}$$

where m_1, $\bar{m}(q)$ are introduced in (3). After taking the time derivative of (28), $\dot{V}(y,t)$ can be expressed as

$$\begin{aligned}\dot{V}(y,t) &= r^T M(q)\dot{r} + \frac{1}{2}r^T \dot{M}(q) r \\ &\quad + e_2^T \dot{e}_2 + 2e_1^T \dot{e}_1 + \dot{P}.\end{aligned}$$

After utilizing (5), (6), (17), and (26) $\dot{V}(y,t)$ can simplified as follows:

$$\begin{aligned}\dot{V}(y,t) &= r^T \tilde{N}(t) - (k_s+1)\left\|r\right\|^2 \\ &\quad -\alpha_2\left\|e_2\right\|^2 - 2\alpha_1\left\|e_1\right\|^2 + 2e_2^T e_1.\end{aligned} \quad (31)$$

Because $e_2^T(t)e_1(t)$ can be upper bounded as

$$e_2^T e_1 \leq \frac{1}{2}\left\|e_1\right\|^2 + \frac{1}{2}\left\|e_2\right\|^2$$

$\dot{V}(y,t)$ can be upper bounded using the squares of the components of $z(t)$ as follows:

$$\begin{aligned}\dot{V}(y,t) &\leq r^T \tilde{N}(t) - (k_s+1)\left\|r\right\|^2 - \alpha_2\left\|e_2\right\|^2 \\ &\quad -2\alpha_1\left\|e_1\right\|^2 + \left\|e_1\right\|^2 + \left\|e_2\right\|^2.\end{aligned}$$

By using the fact $||\tilde{N}(t)|| \leq \rho(||z||)\left\|z\right\|$, the expression in (31) can be rewritten as follows:

$$\dot{V}(y,t) \leq -\lambda_3\left\|z\right\|^2 - k_s\left\|r\right\|^2 + \rho(||z||)\left\|r\right\|\left\|z)\right\| \quad (32)$$

where $\lambda_3 \triangleq \min\{2\alpha_1 - 1, \alpha_2 - 1, 1\}$ and the bounding function $\rho(||z||) \in \mathbb{R}$ is a positive globally invertible nondecreasing function; hence, α_1, α_2 must be chosen according to the following conditions:

$$\alpha_1 > \frac{1}{2}, \qquad \alpha_2 > 1.$$

The following can be obtained from (32) by combining the second and third terms

$$\dot{V}(y,t) \leq -\lambda_3\left\|z\right\|^2 + \frac{\rho^2(z)\left\|z\right\|^2}{4k_s}. \quad (33)$$

The following expression can then be obtained from (33):

$$\dot{V}(y,t) \leq -W(y) \quad (34)$$

where $W(y) = c\left\|z\right\|^2$, for some positive constant $c \in \mathbb{R}$, is a continuous positive semi-definite function that is defined on the following domain:

$$D \triangleq \{y \in \mathbb{R}^{3n+1} \mid \left\|y\right\| \leq \rho^{-1}(2\sqrt{\lambda_3 k_s})\}.$$

The inequalities in (29) and (34) can be used to show that $V(y,t) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, $e_1(t), e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $e_1(t)$, $e_2(t)$, and $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, standard linear analysis methods (e.g., Lemma 1.4 of [10]) can be used to prove that $\dot{e}_1(t)$, $\dot{e}_2(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$ from (5) and (6). Since $e_1(t)$, $e_2(t)$, $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the assumption that $q_d(t)$, $\dot{q}_d(t)$, $\ddot{q}_d(t)$ exist and are bounded can be used along with (4)-(6) to conclude that $q(t)$, $\dot{q}(t)$, $\ddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $q(t)$, $\dot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, Property 2 can be used to conclude that $M(q)$, $V_m(q,\dot{q})$, $G(q)$, and $f(\dot{q}) \in \mathcal{L}_\infty$ in $\mathcal{D}$. From (8) and (11), we can show that $\mu(t)$, $\tau(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $r(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, (12) can be used to show that $\dot{\mu}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Property 2 and Property 3 can be used to show that $\dot{f}(q)$ and $\dot{M}(q) \in \mathcal{L}_\infty$ in $\mathcal{D}$; hence, (10) can be used to show that $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Given that $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, then (4)-(6) can be used to conclude that $\dddot{q}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$. Since $\dot{e}_1(t)$, $\dot{e}_2(t)$, $\dot{r}(t) \in \mathcal{L}_\infty$ in $\mathcal{D}$, the definitions for $W(y)$ and $z(t)$ can be used to prove that $W(y)$ is uniformly continuous in $\mathcal{D}$.

Let $\mathcal{S} \subset \mathcal{D}$ denote a set defined as follows:

$$\mathcal{S} \triangleq \left\{y(t) \subset \mathcal{D} \mid W_2(y(t)) < \lambda_1\left(\rho^{-1}(2\sqrt{\lambda_3 k_s})\right)^2\right\}. \quad (35)$$

The region of attraction in (35) can be made arbitrarily large to include any initial conditions by increasing the control gain k_s (i.e., a semi-global type of stability result) [28]. Theorem 8.4 of [18] can now be invoked to state that

$$c\left\|z(t)\right\|^2 \to 0 \qquad as \qquad t\to\infty \qquad \forall y(t_0) \in \mathcal{S}. \quad (36)$$

Based on the definition of $z(t)$, (36) can be used to show that

$$r(t) \to 0 \qquad as \qquad t\to\infty \qquad \forall y(t_0) \in \mathcal{S}. \quad (37)$$

Hence, from (5) and (6), standard linear analysis methods (e.g., Lemma 1.6 of [10]) can be used to prove that

$$e_1(t) \to 0 \qquad as \qquad t\to\infty \qquad \forall y(t_0) \in \mathcal{S}.$$

The result in (37) can also be used to conclude from (9) that

$$\mu(t) - f(\dot{q}(t)) \to 0 \qquad as \qquad t\to\infty \qquad \forall y(t_0) \in \mathcal{S}.$$

V. CONCLUSION

In this paper, semi-global asymptotic tracking is proven in the presence of a proposed continuously differentiable friction model that contains uncertain nonlinear parameterizable terms. To achieve the tracking result, an integral feedback compensation term is used to identify the system friction effects. A Lyapunov-based stability analysis is provided to conclude the tracking and friction identification results. On-going efforts are being directed at the development of an experimental testbed to illustrate the tracking and friction identification performance of the developed controller. Specifically, future efforts will target experimental comparisons of the identified friction effects with the proposed model.

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