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Proportional plus integral control and disturbance rejection for differential linear repetitive processes

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Abstract— **Repetitive processes are a distinct class of two-dimensional systems (i.e. information propagates in two independent directions) of both systems theoretic and applications interest. They cannot be controlled by direct extension of existing techniques from either standard (termed 1D in the associated literature) or two-dimensional (2D) systems theory. Most of the currently available results for them focus on fundamental properties such as stability, controllability etc. Recently, however, there has been a move (prompted by the progress in this earlier research) towards the development of a control theory, and associated design algorithms, for the sub-classes of so-called differential and discrete linear repetitive processes which arise in applications such as iterative learning control. In this paper we continue this theme by investigating the role of proportional plus integral action in the differential case.**

I. Introduction

Repetitive processes are a distinct class of two-dimensional (2D) systems of both systems theoretic and applications interest. The essential unique characteristic of such a process is a series of sweeps, termed passes, through a set of dynamics defined over a fixed finite duration known as the pass length. On each pass an output, termed the pass profile, is produced which acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile. This, in turn, leads to the unique control problem for these processes in that the output sequence of pass profiles generated can contain oscillations that increase in amplitude in the pass-to-pass direction.

To introduce a formal definition, let $\alpha < +\infty$ denote the pass length (assumed constant). Then in a repetitive process the pass profile $y_k(t)$, $0 \leq t \leq \alpha$, generated on pass k acts as a forcing function on, and hence contributes to, the dynamics of the next pass profile $y_{k+1}(t), 0 \leq t \leq \alpha, k \geq 0$.

Physical examples of repetitive processes include long-wall coal cutting and metal rolling operations (see, for example, [1]). Also in recent years applications have arisen where adopting a repetitive process setting for analysis has distinct advantages over alternatives. Examples of these so-called algorithmic applications include classes of iterative learning control (ILC) schemes [2] and iterative algorithms for solving nonlinear dynamic optimal control problems based on the maximum principle [3]. In the case of ILC for the linear dynamics case, the stability theory for differential and discrete linear repetitive processes is the essential basis for a rigorous stability/convergence analysis of a powerful class of such algorithms.

Attempts to control these processes using standard (or 1D) systems theory/algorithms fail (except in a few very restrictive special cases) precisely because such an approach ignores their inherent 2D systems structure, i.e. information propagation occurs from pass-to-pass and along a given pass. In seeking a rigorous foundation on which to develop a control theory for these processes, it is natural to attempt to exploit structural links which exist between these processes and other classes of 2D linear systems.

The case of 2D discrete linear systems recursive in the positive quadrant $(i, j) : i \geq 0, j \geq 0$ (where i and j denote the directions of information propagation) has been the subject of much research effort over the years using, in the main, the well known Roesser [4] and Fornasini Marchesini [5] state space models. A key distinguishing feature of repetitive processes is that information propagation in one of the independent directions, along the pass, only occurs over a finite duration — the pass length. Moreover, in this paper the subject is so-called differential linear repetitive processes where the dynamics along the pass are governed by a linear matrix differential equation. This means that results for 2D discrete linear systems are not applicable.

The structure of linear repetitive processes means that there is a natural way to write down control laws for them which can be based on current pass state or output (pass profile) feedback control and feedforward control from the previous pass profile. For example, in the ILC application, one such family of control laws is composed of output feedback control action on the current pass combined with information 'feedforward' from the previous pass (or trial in the ILC context) which, of course, has already been generated and is therefore available for use.

The requirement to provide control laws for repetitive processes which achieve stability and/or performance, plus the progress in answering basic systems theoretic questions such as what is meant by controllability etc, has recently been the subject of an increasing level of research. One aspect of this has seen the emergence of LMI based methods, e.g. [6], as the only currently available method which allows control laws to be designed for stability and/or performance as opposed to just obtaining conditions for

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stability along the pass under control action.

In this paper, we continue the development of simple structure control laws for differential linear repetitive processes and, in particular, the use of proportional plus integral control action, where this problem has been considered previously [7] for discrete linear repetitive processes. We begin in the next section with a summary of the relevant background. Throughout this paper, the null matrix and the identity matrix with the required dimensions are denoted by 0 and I, respectively. Moreover, $M > 0$ (< 0) denotes a real symmetric positive (negative) definite matrix.

II. Background

Following [8] the state space model of a differential linear repetitive process has the following form over $0 \leq t \leq \alpha,\ k \geq 0$

$$\begin{aligned}\dot{x}_{k+1}(t) &= Ax_{k+1}(t)+B_0y_k(t)+Bu_{k+1}(t)+Ew(t) \\ y_{k+1}(t) &= Cx_{k+1}(t)+D_0y_k(t)+Du_{k+1}(t)+Fw(t)\end{aligned} \quad (1)$$

where $x \in \mathbb{R}^n$ is the state vector, $y \in \mathbb{R}^m$ is the pass profile (the output vector) and $u \in \mathbb{R}^r$ is the input vector. The function $w(t)$ denotes a known disturbance and without the loss of the generality it is assumed that it is given by a known equation. It is also assumed that the disturbance is constant from pass-to-pass (i.e in the k direction) but can evolve dynamically along the pass.

To complete the process description, it is necessary to specify the boundary conditions, i.e. the state initial vector on each pass and the initial pass profile. Here no loss of generality arises from assuming $x_{k+1}(0) = d_{k+1},\ k \geq 0$, and $y_0(t) = f(t)$, where the $n \times 1$ vector d_{k+1} has known constant entries and $f(t)$ is an $m \times 1$ vector whose entries are known functions of t.

The stability theory [8] for linear constant pass length repetitive processes is based on the following abstract model of the underlying dynamics where E_α is a suitably chosen Banach space with norm $||.||$ and W_α is a linear subspace of E_α

$$y_{k+1} = L_\alpha\, y_k + b_{k+1},\ k \geq 0 \quad (2)$$

In this model $y_k \in E_\alpha$ is the pass profile on pass k, $b_{k+1} \in W_\alpha$, and L_α is a bounded linear operator mapping E_α into itself. The term $L_\alpha y_k$ represents the contribution from pass k to pass $k+1$ and b_{k+1} represents known initial conditions, disturbances and control input effects.

The linear repetitive process (2) is said to be asymptotically stable if $\exists$ a real scalar $\delta > 0$ such that, given any initial profile y_0 and any disturbance sequence $\{b_k\}_{k\geq 1} \in W_\alpha$ bounded in norm (i.e. $||b_k|| \leq c_1$ for some constant $c_1 \geq 0$ and $\forall\, k \geq 1$), the output sequence generated by the perturbed process

$$y_{k+1} = (L_\alpha + \gamma)y_k + b_{k+1},\ k \geq 0 \quad (3)$$

is bounded in norm whenever $||\gamma|| \leq \delta$. This definition is easily shown to be equivalent to the requirement that $\exists$ finite real scalars $M_\alpha > 0$ and $\lambda_\alpha \in (0, 1)$ such that

$$||L_\alpha^k|| \leq M_\alpha \lambda_\alpha^k,\ k \geq 0 \quad (4)$$

(where $||.||$ is also used to denote the induced operator norm). Necessary and sufficient conditions for this last condition are that

$$r(L_\alpha) < 1 \quad (5)$$

where $r(\cdot)$ denotes the spectral radius of its argument.

In the case of processes described by (1), stability can be determined in the absence of the disturbance terms and it can be shown that asymptotic stability holds if, and only if, $r(D_0) < 1$. Also if this property holds and the control input sequence applied $\{u_k\}_k$ converges strongly to u_∞ as $k \to \infty$ then the resulting output pass profile sequence $\{y_k\}_k$ converges strongly to y_∞ — the so-called limit profile defined (with $D = 0$, $E = 0$ and $F = 0$ for ease of presentation) over $0 \leq t \leq \alpha$ by

$$\begin{aligned}\dot{x}_\infty(t) &= (A + B_0(I_m - D_0)^{-1}C)x_\infty(t) + Bu_\infty(t) \\ y_\infty(t) &= (I_m - D_0)^{-1}Cx_\infty(t) \\ x_\infty(0) &= d_\infty\end{aligned} \quad (6)$$

where d_∞ is the strong limit of the sequence $\{d_k\}_k$.

In effect, this result states that if a process is asymptotically stable then its repetitive dynamics can, after a 'sufficiently large' number of passes, be replaced by those of a 1D differential linear system. Note, however, that this property does not guarantee that the limit profile is stable in the normal sense, i.e. all eigenvalues of $(A + B_0(I_m - D_0)^{-1}C)$ have strictly negative real parts — a point which is easily illustrated by the case when $A = -1$, $B = 0$, $B_0 = 1 + \beta$, $C = 1$, $D = 0$, D_0 and $\beta > 0$ is a real scalar.

The reason why asymptotic stability does not guarantee a limit profile which is 'stable along the pass' is due to the finite pass length. In particular, asymptotic stability is easily shown to be bounded-input bounded-output (BIBO) stability with respect to the finite and fixed pass length. Also in cases where this feature is not acceptable, the stronger concept of stability along the pass must be used. In effect, for the abstract model (2), this requires that (4) holds uniformly with respect to the pass length α. One of several equivalent statements of this is the requirement that $\exists$ finite real scalars $M_\infty > 0$ and $\lambda_\infty \in (0, 1)$ independent of α and satisfy

$$||L_\alpha^k|| \leq M_\infty \lambda_\infty^k,\ \forall\, \alpha > 0,\ \forall\, k \geq 0 \quad (7)$$

Several equivalent sets of necessary and sufficient conditions for stability along the pass of processes of the form considered here are known [8] but here it is the following set which will be required.

Theorem 1: [8] Suppose that the pair $\{A, B_0\}$ is controllable and the pair $\{C, A\}$ is observable. Then the differential linear repetitive process generated by processes of the form (1) is stable along the pass if, and only if, (i) $r(D_0) < 1$, (ii) all eigenvalues of A have strictly negative real parts, and (iii) all eigenvalues of the transfer function matrix

$$G(s) = C(sI_n - A)^{-1}B_0 + D_0 \quad (8)$$

have modulus strictly less than unity $\forall\, s = \imath\omega,\ \omega \geq 0$.
The first condition here (i.e. $r(D_0) < 1$) is asymptotic stability and the second condition can be interpreted physically as the requirement that the first pass profile is uniformly bounded with respect to the pass length. Note, however, that these conditions are not strong enough for stability along the pass as the following simple example considered previously in this section, i.e. $A = -1,\ B = 0, B_0 = 1+\beta, C = 1, D = 0, D_0$, with $\beta > 0$. In particular, the limit profile (6) in this case is unstable as 1D linear system and $G(s) = \frac{1+\beta}{s+1}$. Hence stability along the pass requires that $\beta < 0$. In physical terms, this means that each frequency component of the initial profile must be attenuated from pass-to-pass.

Consider now the problem of control law design. Then previous work, e.g. [6], has shown that an LMI setting allows both to test for stability along the pass and design physically based control laws. The new results in this paper will also use this approach for which the following result is the basic starting point.

Theorem 2: [6] A differential linear repetitive process described by (1) is stable along the pass if $\exists$ matrices $Y > 0$ and $Z > 0$ satisfying the following LMI

$$\begin{bmatrix} YA^T + AY & B_0Z & YC^T \\ ZB_0^T & -Z & ZD_0^T \\ CY & D_0Z & -Z \end{bmatrix} < 0 \tag{9}$$

Previous work has shown that to control these processes based on the use of current pass information alone will only work in very restricted special cases. Instead, control laws must be based on current pass action augmented by information from the previous pass profile. One such control law is defined over $0 \leq p \leq \alpha,\ k \geq 0$ as

$$u_{k+1}(t) = K_1 x_{k+1}(t) + K_2 y_k(t) \tag{10}$$

where K_1 and K_2 are appropriately dimensioned matrices to be designed, and the following result (which makes use of Theorem 2) shows how to design for closed loop stability along the pass.

Theorem 3: [6] Suppose that a differential linear repetitive process described by (1) is subject to a control law of the form (10). Then the resulting closed loop process is stable along the pass if $\exists$ matrices $Y > 0,\ Z > 0,\ M$, and N such that the following LMI holds

$$\begin{bmatrix} YA^T + AY + N^TB^T + BN & B_0Z + BM \\ ZB_0^T + M^TB^T & -Z \\ CY + DN & D_0Z + DM \end{bmatrix.$$
$$\left.\begin{matrix} YC^T + N^TD^T \\ ZD_0^T + M^TD^T \\ -Z \end{matrix}\right] < 0 \tag{11}$$

If this condition holds then the control law matrices K_1 and K_2 are given by

$$K_1 = NY^{-1},\ \ K_2 = MZ^{-1} \tag{12}$$

Note that the solution for the LMI (11) in this last result provides only one member (a set of matrices) of the convex subspace which constitutes its feasibility set. This is enough to ensure stability along the pass closed loop in the absence of any other constraints. It is, of course, possible that adding additional constraints, such as minimization of the condition numbers of Y and/or Z (to guard against numerical problems which could appear in forming their inverses) could lead to an improved solution but this is not treated here.

III. 2D PROPORTIONAL + INTEGRAL CONTROL

The control law of the previous section requires measurement of the complete current pass state vector and if this is not possible then an observer would have to be used or else the current pass output vector used. Moreover, there remains the general question of how to design control laws to give desired performance. In which context, their basic dynamic structure immediately leads to the requirement that the process is controlled to meet the following objectives:

- The required pass profile, or reference vector, denoted here by $y_{ref}(t)$ is produced as the resulting limit profile.
- The influence of disturbances which are constant from pass-to-pass are rejected.
- The performance on intermediate passes is acceptable and, in particular, stable along the pass.

Control law design to meet these specifications form the new results in this paper where, noting the obvious requirement to avoid controller complexity, the use of proportional plus integral action as the control law is considered. It is also important to stress that the above performance objectives are by no means exhaustive and what is being undertaken here is an examination of the feasibility of designing one possible control law structure.

Define for pass k and 'position' $t \in [0, \alpha]$ along this pass the so-called total tracking error $\chi_k(t)$ as

$$\chi_k(t) = \sum_{j=0}^{k} (y_j(t) - y_{ref}(t)) \tag{13}$$

Then it follows immediately that

$$\chi_{k+1}(t) = \chi_k(t) + y_{k+1}(t) - y_{ref}(t) \tag{14}$$

or, on using (1),

$$\begin{aligned} \chi_{k+1}(t) = \chi_k(t) &+ Cx_{k+1}(t) + Du_{k+1}(t) \\ &+ D_0 y_k(t) + Fw(t) - y_{ref}(t) \end{aligned} \tag{15}$$

Also introduce the so-called extended pass profile vector as

$$z_k(t) = \begin{bmatrix} y_k(t) \\ \chi_k(t) \end{bmatrix} \tag{16}$$

Then use of the second equation of (1) together with (15) yields the following state space model of the so-called augmented linear repetitive process

$$\begin{aligned}\dot{x}_{k+1}(t) &= Ax_{k+1}(t)+[B_0\ \ 0\,]\,z_k(t) \\ &\quad +Bu_{k+1}(t)+Ew(t) \\ z_{k+1}(t) &= \left[\begin{array}{c} C \\ C \end{array}\right]x_{k+1}(t)+\left[\begin{array}{cc} D_0 & 0 \\ D_0 & I \end{array}\right]z_k(t) \\ &\quad +\left[\begin{array}{c} 0 \\ -I \end{array}\right]y_{ref}(t)+\left[\begin{array}{c} D \\ D \end{array}\right]u_{k+1}(t) \\ &\quad +\left[\begin{array}{c} F \\ F \end{array}\right]w(t)\end{aligned} \tag{17}$$

Suppose that as $k\rightarrow\infty$, $x_k(t)\rightarrow x_\infty(t)$, $u_k(t)\rightarrow u_\infty(t)$ and $y_k(t)\rightarrow y_{ref}(t)$, $\chi_k(t)\rightarrow\chi_\infty(t)$, (hence $z_k(t)\rightarrow z_\infty(t)$). Then from (17) we obtain

$$\begin{aligned}\dot{x}_{\infty}(t) &= Ax_{\infty}(t)+[B_0\ \ 0\,]\,z_{\infty}(t) \\ &\quad +Bu_{\infty}(t)+Ew(t) \\ z_{\infty}(t) &= \left[\begin{array}{c} C \\ C \end{array}\right]x_{\infty}(t)+\left[\begin{array}{cc} D_0 & 0 \\ D_0 & I \end{array}\right]z_{\infty}(t) \\ &\quad +\left[\begin{array}{c} D \\ D \end{array}\right]u_{\infty}(t)+\left[\begin{array}{c} 0 \\ -I \end{array}\right]y_{ref}(t) \\ &\quad +\left[\begin{array}{c} F \\ F \end{array}\right]w(t)\end{aligned} \tag{18}$$

Now define the following incremental vectors

$$\begin{aligned}\hat{z}_k(t) &= z_k(t)-z_\infty(t) \\ \hat{u}_k(t) &= u_k(t)-u_\infty(t) \\ \hat{x}_k(t) &= x_k(t)-x_\infty(t)\end{aligned} \tag{19}$$

Then subtracting (18) from (17) and using (19) yields

$$\begin{aligned}\dot{\hat{x}}_{k+1}(t) &= A\hat{x}_{k+1}(t)+\hat{B}_0\hat{z}_k(t)+B\hat{u}_{k+1}(t) \\ \hat{z}_{k+1}(t) &= \hat{C}\hat{x}_{k+1}(t)+\hat{D}_0\hat{z}_k(t)+\hat{D}\hat{u}_{k+1}(t)\end{aligned} \tag{20}$$

where

$$\hat{B}_0=\left[\begin{array}{cc} B_0 & 0 \end{array}\right],\ \hat{C}=\left[\begin{array}{c} C \\ C \end{array}\right]$$

$$\hat{D}_0=\left[\begin{array}{cc} D_0 & 0 \\ D_0 & I \end{array}\right],\ \hat{D}=\left[\begin{array}{c} D \\ D \end{array}\right]$$

and hence the disturbance term $w(t)$ is completely decoupled from the process dynamics. The only problem in the above analysis is that (20) is asymptotically unstable (this property is determined by the eigenvalues of the matrix $\hat{D}_0$ and some of these are equal to unity) and hence unstable along the pass. Consequently the result is only achievable if we can find a control law to ensure this property. Here we consider the control law defined by

$$\begin{aligned}\hat{u}_{k+1}(t) &= K_x\hat{x}_{k+1}(t)+K_z\hat{z}_k(t) \\ &= K_x\hat{x}_{k+1}(t)+K_{z1}\hat{y}_k(t)+K_{z2}\hat{\chi}_k(t) \\ &= \left[\begin{array}{ccc} K_x & K_{z1} & K_{z2} \end{array}\right]\left[\begin{array}{c} \hat{x}_{k+1}(t) \\ \hat{y}_k(t) \\ \hat{\chi}_k(t) \end{array}\right]\end{aligned} \tag{21}$$

which is the differential linear repetitive process version of the classical proportional plus integral control action. (In the case of the integral action, this arises from the total tracking error contribution which is formed by summing across the passes.)

Now have the following result which shows how to design this control law to ensure that (20) is stable along the pass.

Theorem 4: Suppose that the model of (20) is subject to a control law of the form (21). Then the resulting closed loop process is stable along the pass if there exist matrices $\hat{Y}>0$, $\hat{Z}>0$, $\hat{M}$ and $\hat{N}$ such that the following LMI holds

$$\left[\begin{array}{ccc} \hat{Y}A^T+A\hat{Y}+\hat{N}^TB^T+B\hat{N} & \hat{B}_0\hat{Z}+B\hat{M} & \hat{Y}\hat{C}^T+\hat{N}^T\hat{D}^T \\ \hat{Z}\hat{B}_0^T+\hat{M}^TB^T & -\hat{Z} & \hat{Z}\hat{D}_0^T+\hat{M}^T\hat{D}^T \\ \hat{C}\hat{Y}+\hat{D}\hat{N} & \hat{D}_0\hat{Z}+\hat{D}\hat{M} & -\hat{Z} \end{array}\right]<0 \tag{22}$$

If this condition holds, the control law matrices K_x and K_z are given by

$$K_x=\hat{N}\hat{Y}^{-1},\ \ K_z=\hat{M}\hat{Z}^{-1} \tag{23}$$

Proof: Simply note that (20) is of the form (1) (with $w(t)=0$) and hence Theorem 3 can be applied to the closed loop process state space model. ■

To show how (21) can be actually employed, first note that

$$\begin{aligned}\hat{u}_{k+1}(t) &= u_{k+1}(t)-u_\infty(t) \\ &= K_x\hat{x}_{k+1}(t)+K_z\left[\begin{array}{c} y_k(t)-y_{ref}(t) \\ \chi_k(t)-\chi_\infty(t) \end{array}\right]\end{aligned} \tag{24}$$

or, using the original variables,

$$\begin{aligned}u_{k+1}(t) &= K_x\Big(x_{k+1}(t)-x_\infty(t)\Big) \\ &\quad +K_{z1}\Big(y_k(t)-y_{ref}(t)\Big) \\ &\quad +K_{z2}\Big(\chi_k(t)-\chi_\infty(t)\Big)+u_\infty(t)\end{aligned} \tag{25}$$

This control law can also be applied to the process in non-incremental form, i.e. as

$$u_{k+1}(t)=K_xx_{k+1}(t)+K_{z1}y_k(t)+K_{z2}\chi_k(t) \tag{26}$$

Then from (24) it is straightforward to see that

$$-K_xx_\infty(t)-K_{z1}y_{ref}(t)-K_{z2}\chi_\infty(t)+u_\infty(t)=0 \tag{27}$$

Consequently on any pass it is not required to know information which is generated on future passes, i.e. $\chi_\infty(t)$ and $u_\infty(t)$, which considerably simplifies the effort required to construct the control law output to be applied to the process since there is no need to pre-compute these two terms.

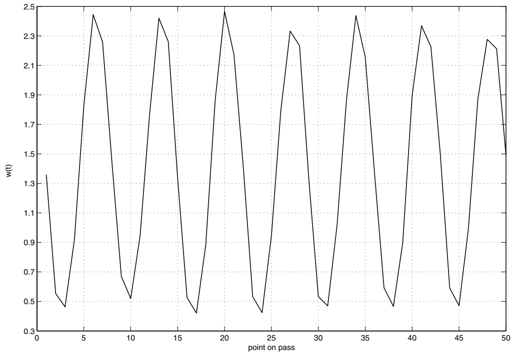

Fig. 1. The disturbance $w(t)$

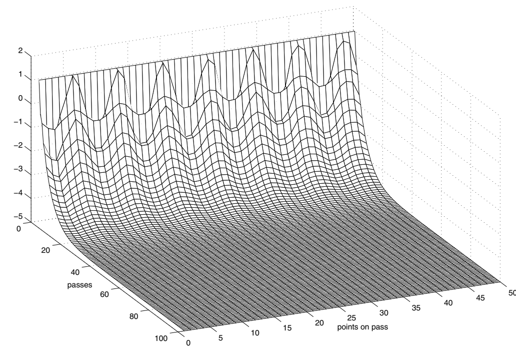

Fig. 2. The output of the closed loop repetitive process

Numerical example

To illustrate the new results developed in this paper, consider the special case of (1) defined by

$$A = \begin{bmatrix} 0.01 & 0 & -0.11 \\ 0 & 0.98 & -0.21 \\ 0.08 & 0.16 & -0.61 \end{bmatrix}, \; B_0 = \begin{bmatrix} 0.96 \\ -0.05 \\ -0.82 \end{bmatrix}$$

$$B = \begin{bmatrix} -1.0 & -1.91 \\ -0.07 & 2.59 \\ 0 & 0.88 \end{bmatrix}, \; E = \begin{bmatrix} 1.34 \\ -0.4 \\ -0.51 \end{bmatrix}$$

$$C = \begin{bmatrix} 1.28 & -0.5 & 0 \end{bmatrix}, \; D_0 = \begin{bmatrix} -1.04 \end{bmatrix}$$

$$D = \begin{bmatrix} -0.41 & 0.62 \end{bmatrix}, \; F = \begin{bmatrix} 1.09 \end{bmatrix}$$

with pass length $\alpha = 50$. This process is asymptotically unstable (and hence unstable along the pass) since $|D_0| = 1.04$. The disturbance has been generated using the following formula expressed in terms of the following MATLAB code

```
dww=14*pi/(alpha-1);
W = sin(-7*pi:dww:7*pi)
    + (rand(1,alpha)-0.5)/5 + 1.4;
```

and shown in Figure 1.

The boundary conditions are

$$d_{k+1} = \begin{bmatrix} 0.23 \\ 0.57 \\ -0.99 \end{bmatrix}, \; k \geq 0$$

$$f(t) = 1, \; 0 \leq t \leq \alpha$$

and the reference signal is $y_{ref}(t) = -5, \; 0 \leq t \leq \alpha$.

The LMI of Theorem 4 in this case is solved by

$$\hat{Y} = \begin{bmatrix} 1.3798 & 0.1134 & -0.3927 \\ 0.1134 & 0.6123 & 0.0709 \\ -0.3927 & 0.0709 & 0.9628 \end{bmatrix}$$

$$\hat{N} = \begin{bmatrix} 2.4415 & -1.4602 & -0.9755 \\ -0.9905 & -0.4808 & 0.1790 \end{bmatrix}$$

$$\hat{Z} = \begin{bmatrix} 0.3959 & 0.0793 \\ 0.0793 & 0.6243 \end{bmatrix}, \; \hat{M} = \begin{bmatrix} -0.2069 & 0.2094 \\ 0.1679 & -0.0257 \end{bmatrix}$$

and hence the control law matrices are given by

$$K_x = \begin{bmatrix} 1.9971 & -2.7551 & 0.0041 \\ -0.6758 & -0.6553 & -0.0415 \end{bmatrix}$$

$$K_{z1} = \begin{bmatrix} -0.6053 \\ 0.4436 \end{bmatrix}, \; K_{z2} = \begin{bmatrix} 0.4123 \\ -0.0974 \end{bmatrix}$$

Figure 2 shows the response of the resulting closed loop process. This confirms that the design objectives have been satisfied, i.e. close loop stability along the pass, the required limit profile is achieved and the influences of the disturbance have been decoupled.

IV. Conclusions

This paper has developed new results on the control of differential linear repetitive processes. These consist of the structure and design of proportional plus integral based control action which results in a closed loop stable process which can also reject disturbances which are constant from pass to pass. The importance of these results is that they show that previously known stabilization design based on an LMI setting do extend to allow the design of the control law to also meet performance specifications. Moreover, the control law itself only involves proportional plus integral action on available signals with consequent benefits in terms of actual implementation. These results are the first in the general area and there are many aspects to be addressed before their true potential can be established.

Note again that this paper only deals with disturbances which are constant from pass-to-pass and this reduces the general applicability of the new results developed. At present, it is not clear how (if at all) complete decoupling of disturbances which do not satisfy this assumption can be achieved. One practically relevant alternative is to seek to attenuate the effects of such disturbances to a prescribed degree using, for example, H_∞, H_2, or mixed H_2/H_∞ techniques. In which context some significant first results in this direction can be found in [9].

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