

# Sliding Mode Adaptive Output Feedback Control of Nonlinear Systems Using Neural Networks

Feipeng Da, Shumin Fei, and Xianzhong Dai

**Abstract** — An adaptive output feedback control scheme is proposed for the output tracking of a class of nonlinear systems represented by input-output models. By augmenting a series of integrators at the input side, the system is represented by the input, the output and their derivatives. Neural networks are used to adaptively compensate for the nonlinearities. Sliding mode and high order filters are presented in the adaptive controller design. By using Lyapunov's stability theory, the global stability of the system is proven. Simulation results show the effectiveness of the proposed method.

## I. INTRODUCTION

THE sliding mode control (SMC) approach is based on switching functions of the state variables which are used to create a "sliding surface". When this surface is attained, the switching functions keep the trajectory on the surface, thus yielding desired system dynamics [1][2]. But one difficulty in applying the sliding mode control is the need for the knowledge of the full state vector. A well-known control scheme based on only input and output measurements is presented in [3]. It needs to augment two 'state' filters in the course of the controller design. Since the introduction of this idea, it has been used in a number of studies [4]-[7]. However, this methodology has some drawbacks such as needing persistence of excitation and having little robustness. A kind of forgetting factor (FF), the  $\sigma$ -modification, is proposed for improving the robustness of continuous-time with respect to unmodeled dynamics [4] and bounded disturbances [5]. However, the introduction of the FF can appear sudden intermittent output error "bursts". Liu and Costa [6] show that bursts can indeed occur in such systems solely due to the  $\sigma$ -modification. All the works above are for linear systems. For nonlinear systems, it is naturally thinking to design an observer to estimate the state of the system from

its output [8][9]. Due to nonlinearity, one should not expect that the observer design can be carried out independent of the state feedback design. For a class of feedback linearizable systems, a certain degree of separation can be achieved by designing high-gain observers. But, high-gain observers exhibit a peaking phenomenon in their transient behavior [10]. Khalil [11] considers a class of single-input-single-output nonlinear systems with unknown parameters, gives a semiglobal adaptive output controller. Seshagiri and Khalil [12] use neural networks in the nonlinear output feedback control. In the control scheme, neural networks are used to approximate the nonlinear function and use the same high-gain observers of [11] to fulfill the output feedback control. Other related research works can be found in [13]-[16].

In this paper, we consider a class of single-input-single-output nonlinear systems with unknown nonlinear functions and bounded disturbances. The 'state' of the system can be expressed as a function of the output, and their derivatives up to a certain order. Neural networks are used to approximate the nonlinear functions. Firstly the state adaptive controller is designed by using sliding mode control method. By using Lyapunov stability theory, the online adaptive tuning rules of neural networks weights are given and the whole system's stability is proved. Then high order filters are proposed to 'change' the state variables to output variables. It is proved that high order filter has no influence on the adaptive controller design, but the proposed filters make the state variables measurable practically. Simulation results are given to show that the sliding mode output feedback controller can obtain better tracking results and has more robustness than other controllers.

## II. PROBLEM STATEMENT

Consider a single-input-single-output system represented globally by the  $n$ th-order differential equation

$$y^{(n)} = f_0(\cdot) + \sum_{i=1}^p f_i(\cdot)\theta_i + (g_0 + \sum_{i=1}^p g_i\theta_i)u^{(m)} + d(\cdot) \quad (1)$$

where  $u$  is the control input,  $y$  is the measured output,  $y^{(i)}$  and  $u^{(i)}$  denotes the  $i$ th derivative of  $y$  and  $u$  respectively, and  $m < n$ . The functions  $f_i$  and  $g_i$ ,

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$i = 0, 1, \dots, p$ , are unknown smooth nonlinearities but bounded, which could depend on  $y, y^{(1)}, \dots, y^{(n-1)}, u, u^{(1)}, \dots, u^{(m-1)}$ , e.g.  $f_i(\cdot) = f_i(y, y^{(1)}, \dots, y^{(n-1)}, u, u^{(1)}, \dots, u^{(m-1)})$ ,  $g_i(\cdot) = g_i(y, y^{(1)}, \dots, y^{(n-1)}, u, u^{(1)}, \dots, u^{(m-1)})$ . The constant parameters  $\theta_1$  to  $\theta_p$  are unknown, but the vector  $\theta = [\theta_1, \dots, \theta_p]^T$  belongs to  $\Omega$ , a known compact convex subset of  $R^p$ .  $d(\cdot)$  denotes the bounded unknown disturbances.

It is indicated by [17] that any observable input-output linearable system has an input-output model that fits the form (1) except the term  $d(\cdot)$ . In the paper, we consider the more general case as

$$y^{(n)} = F(\cdot) + G(\cdot)u^{(m)} + d(\cdot), \quad (2)$$

where  $F(\cdot) = F(y, y^{(1)}, \dots, y^{(n-1)}, u, u^{(1)}, \dots, u^{(m-1)})$  and  $G(\cdot) = G(y, y^{(1)}, \dots, y^{(n-1)}, u, u^{(1)}, \dots, u^{(m-1)})$  are unknown smooth nonlinear functions but bounded.

Define  $y_r(t)$  the given reference signal,

$$Y(t) \stackrel{\Delta}{=} [y(t), y^{(1)}(t), \dots, y^{(n-1)}(t)]^T \quad \text{and}$$

$$Y_r(t) \stackrel{\Delta}{=} [y_r(t), y_r^{(1)}(t), \dots, y_r^{(n-1)}(t)]^T, \quad \text{and } Y, Y_r$$

be any given compact subsets of  $R^n$ . The objective of this paper is to design an output feedback controller such that for all  $Y(0) \in Y$ , for all  $Y_r(t) \in Y_r$ , all variables of the closed-loop system are bounded for all  $t \geq 0$  and  $\lim_{t \rightarrow \infty} |y(t) - y_r(t)| = 0$ .

The following assumptions are made on the system (2):

*Assumption 1:* There exists  $k$  such that  $|G(\cdot)| \geq k > 0$ , where  $k$  is a positive number. Without loss of generality, in this paper we assume  $G(\cdot) \geq k > 0$ .

*Assumption 2:*  $|d(\cdot)| \leq D(\cdot)$ , where  $D(\cdot)$  is a known bounded positive function.

*Assumption 3:*  $y_r(t)$  is bounded and has bounded derivatives up to  $n$ th order, and  $y_r^{(n)}(t)$  is piecewise continuous.

Now we take the same method used in [11] to add a series of  $m$  integrators at the input side of the system and denote the state of these integrators by  $z_1 = u, z_2 = u^{(1)}, \dots, z_m = u^{(m-1)}$ , and set  $v = u^{(m)}$  as the control input of the augmented system. By taking  $x_1 = y, x_2 = y^{(1)}, \dots, x_n = y^{(n-1)}$ , the extended system can be represented as

$$\begin{cases} \dot{x}_i = x_{i+1}, 1 \leq i \leq n-1 \\ \dot{x}_n = F(\cdot) + G(\cdot)v + d(\cdot) \\ \dot{z}_i = z_{i+1}, 1 \leq i \leq m-1 \\ \dot{z}_m = v \\ y = x_1 \end{cases},$$

where  $X = [x_1, \dots, x_n]^T$ ,  $z = [z_1, \dots, z_m]^T$ .

Take

$$\begin{cases} e_1 = y - y_r = x_1 - y_r \\ e_2 = \dot{y} - \dot{y}_r = x_2 - \dot{y}_r \\ \vdots \\ e_n = y^{(n-1)} - y_r^{(n-1)} = x_n - y_r^{(n-1)} \\ e = [e_1, e_2, \dots, e_n]^T \end{cases}, \quad (3)$$

Then (2) can be written as

$$\dot{e} = Ae + b\{F(\cdot) + G(\cdot)v + d(\cdot) - y_r^{(n)}\}, \quad (4a)$$

$$\dot{z} = A_z z + b_z v, \quad (4b)$$

where  $(A, b)$  and  $(A_z, b_z)$  are controllable canonical pairs of the form

$$A = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots \\ 0 & 0 & 0 & \cdots & 1 \\ 0 & 0 & 0 & \cdots & 0 \end{bmatrix}, b = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \\ 1 \end{bmatrix}. \quad (5)$$

### III. DESIGN OF THE ADAPTIVE OUTPUT FEEDBACK CONTROLLER

In this section, neural network approximation is firstly presented. Secondly the design of state feedback controller is discussed, and then a high order filter is introduced to estimate the error vector which is used in the output feedback controller. Stability analysis and simulation results are given to show that this controller can achieve accurate tracking, good transient behavior if the bound of the disturbances is known.

#### A. Neural Network Approximation

It has been proven theoretically that the two layers neural network with appropriate weights can approximate any sufficient smooth function in arbitrary accuracy on a compact set [18]. So we can use two layers feed-forward neural networks to approximate the unknown bounded smooth nonlinear functions  $F(\cdot)$  and  $G(\cdot)$  respectively, and obtain the following estimation as

$\hat{F} = \hat{W}_F^T \sigma_F$ ,  $\hat{G} = \hat{W}_G^T \sigma_G$ , where  $\hat{W}_F$  and  $\hat{W}_G$  are the neural networks weight vectors, and  $\sigma_F$  and  $\sigma_G$  are neural networks activation functions which can be taken as sigmoid function or radial basis function etc.

The adaptive tuning rules of  $\hat{W}_F$  and  $\hat{W}_G$  are given below by the Lyapunov stability theory.

*Assumption 4:* Assume the neural networks weight vector estimation  $\hat{W}_F$  and  $\hat{W}_G$  are bounded.

Since F and G are bounded, then there exist positive arbitrary small constants  $p_F$  and  $p_G$  such that

$$\|\tilde{F}\| = \|\tilde{W}_F^T \sigma_F\| = \|\hat{F} - F\| < p_F \quad \text{and} \\ \|\tilde{G}\| = \|\tilde{W}_G^T \sigma_G\| = \|\hat{G} - G\| < p_G \quad \text{hold.}$$

*Remark 1:* In order to prevent the neural networks parameters drifting phenomena, some methods such as projection method [19],  $\sigma$ -modification [20] or  $\mathcal{E}$ -modification [21] can be used.

### B. State Feedback Controller Design

Since the linear system  $\dot{z} = A_z z + b_z v$  is controllable but unstable, so we can choose a matrix  $K_z$  as the state feedback, let

$$A_m = A_z - bK_z = \begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \dots & \vdots \\ 0 & 0 & 0 & \dots & 1 \\ -k_1 & -k_2 & -k_3 & \dots & -k_m \end{bmatrix}, \quad (6)$$

where  $k_i > 0$  and the polynomial  $\lambda^m + k_1 \lambda^{(m-1)} + \dots + k_{m-1} \lambda + k_m$  is Hurwitz. We call it as the linear feedback controller (LFC).

In the paper we use sliding mode method to design the adaptive controller. Firstly define switching function  $s(e) = \sum_{i=1}^{n-1} c_i e_i + e_n$ , where the coefficients  $c_i$  are constants and satisfy Hurwitz polynomial  $\lambda^n + c_1 \lambda^{(n-1)} + \dots + c_{n-1} \lambda$ . Then the derivative of  $s(e)$  is

$$\dot{s}(e) = \sum_{i=1}^{n-1} c_i \dot{e}_{i+1} + \dot{e}_n \\ = \sum_{i=1}^{n-1} c_i \dot{e}_{i+1} + F(\cdot) + G(\cdot)v - y_r^{(n)} + d(\cdot), \quad (7)$$

Based on (7), in order to satisfy the sliding mode condition

$s \cdot \dot{s} \leq -\eta |s|$  [2], we take the control law as

$$v = \frac{1}{\hat{G}} \left( -\sum_{i=1}^{n-1} c_i e_{i+1} - \hat{F} + y_r^{(n)} - (D(\cdot) + \eta) \text{sgn}(s) \right), \quad (8)$$

where  $\eta$  is a positive constant and

$$\text{sgn}(s) = \begin{cases} 1, & s > 0 \\ 0, & s = 0 \\ -1, & s < 0 \end{cases}. \quad \text{Then the adaptive tuning}$$

rules of neural networks weight  $\hat{W}_F$  and  $\hat{W}_G$  are taken as

$$\dot{\hat{W}}_F = \Gamma_F s \sigma_F, \quad (9)$$

$$\dot{\hat{W}}_G = \Gamma_G \frac{s}{\hat{G}} \sigma_G (y_r^{(n)} - \hat{F}(\cdot) - \sum_{i=1}^{n-1} c_i e_{i+1} - D(\cdot)), \quad (10)$$

where  $\Gamma_F = \Gamma_F^T > 0$ ,  $\Gamma_G = \Gamma_G^T > 0$ .

In the weight tuning process, the case  $\hat{G} \rightarrow 0$  may happen which can lead  $v \rightarrow \infty$  and may cause the unstable of the whole system. To avoid this case, we will use the follow modified tuning rules to make  $\hat{G}(\cdot) \geq k$  hold. If  $\hat{G}(\cdot) \geq k$ , use (10) as the tuning rule. If  $\hat{G}(\cdot) < k$ , using the tuning rule below

$$\hat{W}_G(t^+) = \hat{W}_G(t) + (k - \hat{G}) \|\sigma_G\|^{-2} \sigma_G, \quad (11)$$

where t denotes the time when  $\hat{G}(\cdot) < k$ , and  $t^+$  the next step time of t when the weight vector tuning rule (11) is being active. Since

$$\hat{G}(t^+) = \hat{W}_G(t^+)^T \sigma_G \\ = (\hat{W}_G(t) + (k - \hat{G}) \|\sigma_G\|^{-2} \sigma_G)^T \sigma_G = k, \quad (12)$$

then the weight tuning rule (12) could assure that

$\hat{G}(\cdot) \geq k$  always hold.

In the same case, since  $\hat{G}(\cdot) = k$ , the control law (8) should be also changed as

$$v(t^+) = \frac{1}{k} \left( -\sum_{i=1}^{n-1} c_i e_{i+1} - \hat{F} + y_r^{(n)} - (D(\cdot) + \eta') \cdot \text{sgn}(s) \right) \quad (13)$$

where  $\eta' = \eta + \left| s \sigma_G \left( -\sum_{i=1}^{n-1} c_i e_{i+1} - \hat{F} + y_r^{(n)} \right) \right|$ ,  $|\cdot|$  denotes absolute value.

### C. High Order Filters

In the design process above, the controller requires the knowledge of the output and its first (n-1) derivatives. It is well known that in the practical system it is difficult to get the high order differential term of the output  $y$ . In the paper we use the method adopted in [22]. Define Hurwitz polynomial  $H = p^n + a_1 p^{n-1} + \dots + a_{n-1} p + a_n$ ,

where  $p = \frac{d}{dt}$  is the differential operator. Using  $\frac{1}{H}$  as

the high order filter of state variable to obtain the error derivatives up to nth order. Therefore we can obtain the output vector after the filters as

$Y_f = [\frac{p^{n-1}}{H} y, \dots, \frac{1}{H} y]^T$ , the desired output vector

$Y_{rf} = [\frac{p^{n-1}}{H} y_r, \dots, \frac{1}{H} y_r]^T$ . Now we use the filtered

output variables to design the sliding mode controller.

$$\text{Let } \left\{ \begin{array}{l} \bar{e}_1 = \frac{y}{H} - \frac{y_r}{H} \\ \bar{e}_2 = \frac{\dot{y}}{H} - \frac{\dot{y}_r}{H} \\ \vdots \\ \bar{e}_n = \frac{y^{(n-1)}}{H} - \frac{y_r^{(n-1)}}{H} \\ \bar{e} = [\bar{e}_1, \bar{e}_2, \dots, \bar{e}_n]^T. \end{array} \right.$$

define sliding mode function as  $s(\bar{e}) = \sum_{i=1}^{n-1} c_i \bar{e}_i + \bar{e}_n$ ,

$$\begin{aligned} \text{then } \dot{s}(\bar{e}) &= \sum_{i=1}^{n-1} c_i \dot{\bar{e}}_{i+1} + \dot{\bar{e}}_n \\ &= \sum_{i=1}^{n-1} c_i \bar{e}_{i+1} + \frac{1}{H} (F(\cdot) + G(\cdot)v - y_r^{(n)} + d(\cdot)). \end{aligned}$$

Take the sliding mode control law as

$$v' = \frac{1}{\hat{G}/H} \left( -\sum_{i=1}^{n-1} c_i \bar{e}_{i+1} - \frac{1}{H} (F(\cdot) + y_r^{(n)}) - (D_1(\cdot) + \eta_1) \text{sgn}(s) \right). \quad (14)$$

If we choose  $D_1(\cdot) + \eta_1 = \frac{1}{H} (D(\cdot) + \eta)$ , comparing

(14) with (8), it can be seen that  $v' = v$ . This means that adding high order filter has no influence to the controller design. But by adding the filter, we could fulfill the output feedback control.

### D. Stability Analysis

Theorem 1: For the system (2), take the control laws (8)(9)(10) or (8)(9)(11)(13), under the Assumptions 1-4,

then for any bounded initial conditions, the output  $y(t)$  is bounded and the tracking error can converge to zero.

Proof: Define Lyapunov function candidate

$$V = \frac{1}{2} s^2 + \frac{1}{2} \tilde{W}_F^T \Gamma_F^{-1} \tilde{W}_F + \frac{1}{2} \tilde{W}_G^T \Gamma_G^{-1} \tilde{W}_G. \quad (15)$$

Then  $\dot{V} = s\dot{s} + \tilde{W}_F^T \Gamma_F^{-1} \dot{\tilde{W}}_F + \tilde{W}_G^T \Gamma_G^{-1} \dot{\tilde{W}}_G$ . For

simplicity, let  $q = \sum_{i=1}^{n-1} c_i e_{i+1} - y_r^{(n)}$ , then rewrite (7) as

$\dot{s} = q + F + Gv + d$ . Since the sign term, there are two specific cases.

(1) If  $s > 0$ , then

$$\begin{aligned} \dot{s} &= q + F + d + \frac{G}{\hat{G}} (-q - \hat{F} - (D(\cdot) + \eta) \text{sgn}(s)) \\ &= \frac{1}{\hat{G}} (\hat{G}(q + F + d) - G(q + \hat{F} + (D(\cdot) + \eta))) \\ &= \frac{1}{\hat{G}} (\hat{G}q - Gq + \hat{G}F - G\hat{F} + \hat{G}d(\cdot) - G(D(\cdot) + \eta)) \\ &= \frac{1}{\hat{G}} (\tilde{G}q + \tilde{G}\hat{F} - \hat{G}\tilde{F} + \hat{G}(d - D) + \tilde{G}D - G\eta) \\ &= \frac{1}{\hat{G}} (\tilde{G}(q + \hat{F} + D) - \hat{G}\tilde{F} + \hat{G}(d - D) - G\eta) \\ &\leq \frac{1}{\hat{G}} (\tilde{G}(q + \hat{F} + D) - \hat{G}\tilde{F} - G\eta) \end{aligned}$$

so

$$\begin{aligned} s \cdot \dot{s} &\leq \frac{s}{\hat{G}} (\tilde{G}(q + \hat{F} + D) - \hat{G}\tilde{F} - G\eta) \\ &= \frac{1}{\hat{G}} (s \cdot \tilde{G}(q + \hat{F} + D) - s \cdot \hat{G}\tilde{F}) - \frac{G}{\hat{G}} \eta s \\ &= \frac{1}{\hat{G}} (s \cdot \tilde{W}_G \sigma_G(q + \hat{F} + D) - s \cdot \hat{G} \cdot \tilde{W}_F \sigma_F) - \frac{G}{\hat{G}} \eta |s| \end{aligned} \quad (16)$$

If  $\hat{G}(\cdot) \geq k$ , inserting (9)(10)(16) into (15). If  $\hat{G}(\cdot) < k$ , since  $\dot{\tilde{W}}_G = 0$ , then inserting (9)(11)(13)(16) into (15),

we can all get  $\dot{V} \leq -\frac{G}{\hat{G}} \eta |s| < 0$ .

(2) If  $s < 0$ , analogous to the analysis of (1), we could get the same conclusion.

From the analysis above, we have  $\dot{V} < 0$  for all  $t \geq 0$ . Since  $V$  is a decreasing monotonic function, and  $V$  has upper bound. Then  $s$  is bounded. From the definition of  $s$  and assumptions 1-4, we obtain that the output  $y$  is bounded. Because the continuous function must

be bounded in the bounded closed set,  $\dot{s}$  is bounded. Thus  $s$  is uniformly continuous,  $V$  and  $\dot{V}$  are uniformly continuous too. Because

$$\lim_{t \rightarrow \infty} \int_0^t \dot{V} dt = V(\infty) - V(0), \text{ and } V \text{ is bounded,}$$

$\lim_{t \rightarrow \infty} \int_0^t \dot{V} dt$  is finite and exists. Hence, by Barbalat's

lemma [11], we have  $\lim_{t \rightarrow \infty} \dot{V} = 0$ . So  $\lim_{t \rightarrow \infty} s = 0$ , this means the tracking error can converge to zero.

### E. Simulation Example

Consider the nonlinear system

$$\begin{cases} \dot{\xi}_1 = \xi_2 + \xi_1^2 \\ \dot{\xi}_2 = \xi_1 u + \xi_3 \\ \dot{\xi}_3 = \xi_1 + d(\cdot) \\ y = \xi_1 \end{cases}$$

The system can be represented by the third-order differential equation

$$y^{(3)} = (\dot{y}u + y) + 2(\dot{y}^2 + y\dot{y}) + d(\cdot) + y\dot{u}$$

which takes the form of (1) with  $n=3, m=1$ . According to (2),  $F(\cdot) = (y\dot{u} + y) + 2(\dot{y}^2 + y\dot{y})$ ,  $G(\cdot) = y$ . We add an integrator at the input of the system, take  $z = u$  as the state of the integrator and set  $v = \dot{u}$  as the new control input of the augmented system. In the simulation, we choose  $y(0) = 0.45$ ,  $c_1 = 5$ ,  $c_2 = 5$ ,  $\eta = 0.1$ , and  $K_z = [5, 5]$ .  $\sigma_F$  and  $\sigma_G$  have the form of radial basis function and the centers are chosen randomly in  $[-1, +1]$ . The two RBF neural networks contain 49 nodes and 18 nodes respectively and the initial weight estimates are assumed to be 0. Take  $D(\cdot) = 5$  while  $d(\cdot) = 5 \sin(y\dot{y})$ . Simulation results are shown in Figs. 1 and 2.

*Remarks 2:* Note that this example is taken from [11] (example 3). However, the algorithm in [11] could only be applied when the input signal is persistence exciting. In this paper such condition is not needed and the proposed algorithm can result in perfect tracking performance even when the reference input is a step signal.

*Remark 3:* From the simulations, it can be seen that the proposed controller can achieve accurate tracking and has better transient behavior. But the control signal has serious chattering, this is the inherent property of SMC. Since in this paper our aim is to design adaptive output controller with SMC, the emphasis is not put on the chattering problem. It can be seen that the chattering is diminished through LFC because the LFC act as a low-pass filter.

## IV. CONCLUDING REMARKS

In this paper, we present the output feedback controller for nonlinear systems represented by input-output models. The controller can obtain better tracking results and better transient behavior. The global stability of the system is proven. Simulation results have shown the effectiveness of the control scheme.

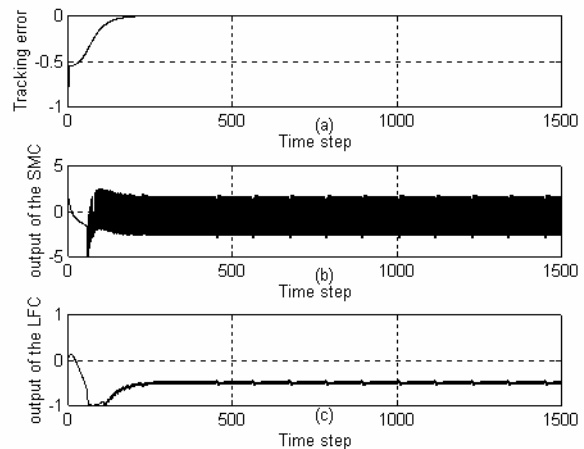


Fig. 1. Simulation results with  $y_r(t)=1(t)$ ,  $d(\cdot)=5\sin(y\dot{y})$ .

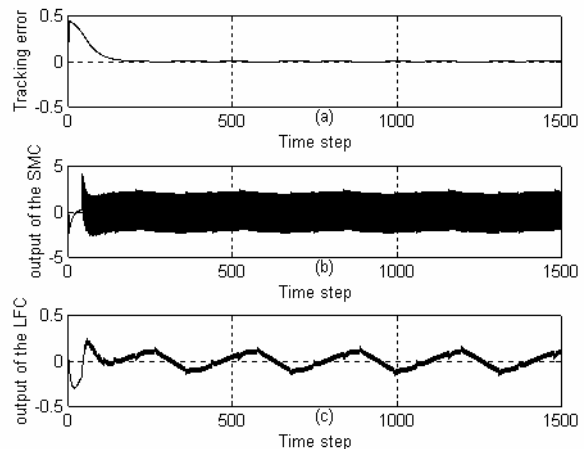


Fig. 2. Simulation results with  $y_r(t)=0.1\sin(t)$ ,  $d(\cdot)=5\sin(y\dot{y})$ .

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