2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Special decentralized control problems and effectiveness of parameter-dependent Lyapunov function method

Zhisheng Duan, Jin-Zhi Wang and Lin Huang

Abstract— **This paper is devoted to studying decentralized control problems from a special viewpoint and testing the effectiveness of parameter-dependent Lyapunov function method. First it is pointed out that in order to stabilize some given interconnected systems, some subsystems should be assigned to be unstable in some special cases. Then a special kind of decentralized control problem is studied. This kind of problem can be viewed as harmonic control among independent subsystems. Research results show that two unstable systems can generate a stable system through some effective cooperations. Linear matrix inequality(LMI)-based decentralized controller design method is also given for the special problems studied here by using parameter-dependent Lyapunov function method developed for robust stability.**

I. INTRODUCTION

The domain of robust analysis and robust control synthesis has been thoroughly investigated in the last two decades. Since [14], Oliveira, Bernussou and Geromel have opened a new horizon for LMI-based robust stability study of systems with parametric uncertainty. The result in [14] has been generalized dramatically in recent years, see [1], [13], [?], [15], [16], [17], [19], [23], [25] and references therein. By introducing a new instrumental matrix variable, LMI corresponding to stability of linear continuous-time or discrete-time systems is relaxed to a new LMI in which Lyapunov matrix is independent of state matrix of systems. With this new extra degree of freedom, a parameter-dependent Lyapunov function can be built for robust stability and performance analysis and control. Generally this new method is less conservative than the traditional Lyapunov function method. And it is also pointed out in [15] that this method has less conservativeness than diagonal blocked Lyapunov function method on decentralized control. This paper is devoted to testing the effectiveness of parameter-dependent Lyapunov function method on decentralized control from a special viewpoint. Examples show that this method is also suitable for the unstable subsystem cases in interconnected systems.

Decentralized control of large scale systems has been studied extensively in the past four decades[3], [10], [11], [16], [20], [21], [22], [24]. The main difficulty of solving the decentralized control problem comes from the fact that the feedback gain is subject to structural constraints. Such constraints are of the same nature as the static output ones, which can be viewed as a full state feedback with structural constraints that select only the measured states. At the beginning study of large scale system theory, some people thought that a large scale system is decentrally stabilizable under controllability condition by strengthening the stability degree of subsystems. [24] showed that this idea is wrong by an example. And because of the existence of decentralized fixed modes, some large scale systems can not be decentrally stabilized at all. Generally, it is very conservative that closed-loop subsystems are all required to be stable. Under the stability of subsystems, the actions of interconnections are always ignored and even viewed as disadvantages. This kind of study is disadvantageous for the study of the actions of interconnections. Recently in [22], LMI method for decentralized control of nonlinear systems was presented and no stability assumptions were made for subsystems. Along the development of society, interconnections play more and more important roles in social systems, economic systems, power systems, etc. But the study of the effects of interconnections in large scale systems is still very little to the authors' knowledge. Recently, some applications of small gain theorem was given in [5] to strengthen robust stability of interconnected systems. In fact, small gain theorem in decentralized control was first introduced and used in 1982, see Section 5.1 of [21]. And in [21], an example(Example 2.18) was given to show that in order to stabilize the whole system, some subsystems must be unstable. The effects of nonlinear input and output coupling was studied in [6], [7], [8]. We study decentralized control problems from a special viewpoint in this paper.

LMI methods have played leading roles during the last twenty years in linear systems theory[2], [9], [12]. We establish LMI-based decentralized controller design method for the special problems studied here by using parameter-dependent Lyapunov function method. This paper mainly focuses on interconnected systems composed of two subsystems. The results can be generalized to multiple subsystem cases. The rest of this paper is organized as follows. In section 2, by studying the structure of interconnections we point out that it is impossible to stabilize all subsystems and the whole system simultaneously by using decentralized controllers in some special cases, that is, in order to stabilize the whole system, some subsystems should be assigned to be unstable. This result shows that the stability of interconnected systems is not only dependent on the stability degree of subsystems in some cases, but is closely dependent

This work is supported by the National Science Foundation of China under grant 60204007, 60334030, 10472001.

The authors are with the State Key Laboratory for Turbulence and Complex Systems and Department of Mechanics and Engineering Science, Peking University, Beijing 100871, P. R. China. Corresponding author: duanzs@pku.edu.cn

on the interconnections. In addition, for sake of studying the effects of interconnections, we study a special kind of decentralized control problem which can be viewed as harmonic stability problem among independent subsystems. The results show that two unstable subsystems can generate a stable interconnected system. In section 3, we present LMI-based decentralized controller design method. Some examples are given to illustrate the results in section 4. Examples show that parameter-dependent Lyapunov method is suitable for the cases of unstable subsystems. The last section concludes the paper.

Throughout this paper, $det(.)$ denotes the determinant of the corresponding matrix. Stability of matrix and polynomial means Hurwitz stability. The superscript T means transpose for real matrices.

II. SPECIAL DECENTRALIZED CONTROL PROBLEMS

In this paper, we mainly consider the following interconnected system composed of two subsystems,

$$\begin{cases} \dot{x}_1 = A_1x_1 + A_{12}x_2 + B_1u_1 \\ \dot{x}_2 = A_2x_2 + A_{21}x_1 + B_2u_2 \end{cases} \tag{1}$$

$u_1 = K_1x_1, u_2 = K_2x_2$, A_{12}, A_{21} are matrices with compatible dimensions. We say system (1) is decentrally stabilizable, i.e., there exist K_1, K_2 such that the state matrix of the closed-loop system

$$A_{cl} = \begin{bmatrix} A_1 + B_1K_1 & A_{12} \\ A_{21} & A_2 + B_2K_2 \end{bmatrix}$$

is Hurwitz stable.

First, we consider a simple example. In system (1) if

$$A_1 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, A_{12} = \begin{pmatrix} 0 & \alpha \\ 0 & 0 \end{pmatrix}, B_1 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$$A_{21} = \begin{pmatrix} 0 & \beta \\ 0 & 0 \end{pmatrix}, A_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}, B_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix},$$

$K_1 = -(k_1 \quad k_2)$, $K_2 = -(k_3 \quad k_4)$, then

$$A_{cl} = \begin{pmatrix} 0 & 1 & 0 & \alpha \\ -k_1 & -k_2 & 0 & 0 \\ 0 & \beta & 0 & 1 \\ 0 & 0 & -k_3 & -k_4 \end{pmatrix}.$$

Obviously at this time,

$$det(sI - A_{cl}) = (s^2 + k_2s + k_1)(s^2 + k_4s + k_3) - \alpha\beta k_1k_3.$$

For this simple case, one can get the following results easily:

(i) when $\alpha\beta = 1$, 0 is a fixed mode.

(ii) when $\alpha\beta > 1$, for any $k_i > 0, i = 1, 2, 3, 4$, i.e., $A_1 + B_1K_1, A_2 + B_2K_2$ are stable, A_{cl} can not be Hurwitz stable(the constant term of its characteristic polynomial is less than 0 at this time).

(iii) when $\alpha\beta < 1$, it is possible that the interconnected system and two subsystems can be stabilized simultaneously.

In [21], an example(Example 2.18) was also given to show that in order to stabilize the whole system, some subsystems must be unstable. From the example given above, we can give a simple structural property for such systems.

Theorem 1 If the interconnected system in (1) satisfies that

1) there exists A'_{12} such that $A_{12} = A'_{12}A_2$, and $A'_{12}B_2 = 0$,

2) there exists A'_{21} such that $A_{21} = A'_{21}A_1$, and $A'_{21}B_1 = 0$,

then there are not K_1, K_2 such that $A_1+B_1K_1, A_2+B_2K_2$ and A_{cl} are Hurwitz stable simultaneously when $det(I - A'_{21}A'_{12}) < 0$.

Proof Computing the determinant of A_{cl}, one can get

$$det(A_{cl}) = det(A_1 + B_1K_1)det(A_2 + B_2K_2 - A'_{21}A_1(A_1 + B_1K_1)^{-1}A'_{12}A_2).$$

Noticing conditions 1), 2),

$$det(A_{cl}) = det(A_1 + B_1K_1)det(A_2 + B_2K_2 - A'_{21}(A_1 + B_1K_1)(A_1 + B_1K_1)^{-1}A'_{12}(A_2 + B_2K_2)),$$

that is,

$$det(A_{cl}) = det(A_1 + B_1K_1)det(A_2 + B_2K_2)det(I - A'_{21}A'_{12}).$$

When $A_1 + B_1K_1, A_2 + B_2K_2$ are Hurwitz stable and $det(I - A'_{21}A'_{12}) < 0$, one gets that the constant term of the characteristic polynomial of A_{cl} is less than zero. Therefore, A_{cl} is unstable.

Remark 1 Obviously, under the conditions in Theorem 1, when $det(I - A'_{21}A'_{12}) = 0$, 0 is a fixed mode; when $det(I - A'_{21}A'_{12}) > 0$, it is possible that there exist K_1, K_2 such that $A_1 + B_1K_1, A_2 + B_2K_2, A_{cl}$ are Hurwitz stable simultaneously. And if A_{cl} is stable, there must be one of $A_1 + B_1K_1, A_2 + B_2K_2$ is unstable when $det(I - A'_{21}A'_{12}) < 0$. For the study of the effects of interconnections in large scale systems, it is important to design decentralized controllers when some subsystems must be unstable. At this time, the interconnections play real roles for the stability of large scale systems.

Corollary 1 For any interconnected matrix $A = \begin{pmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{pmatrix}$, if A_{12} and A_{21} can be written as $A_{12} = A'_{12}A_2, A_{21} = A'_{21}A_1$, and $det(I - A'_{21}A'_{12}) < 0$, then A_1, A_2 and A can not be stable simultaneously.

Obviously, the results above can be generalized to cases of multiple subsystems. For example, for an interconnected system composed of three subsystems, its closed-loop system matrix is given by

$$A_{cl} = \begin{bmatrix} A_1 + B_1K_1 & A_{12} & A_{13} \\ A_{21} & A_2 + B_2K_2 & A_{23} \\ A_{31} & A_{32} & A_3 + B_3K_3 \end{bmatrix}$$

Let $\overline{A}_1 = \begin{bmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{bmatrix}$, $\overline{B}_1 = diag(B_1, B_2)$, $\overline{A}_{13} = \begin{bmatrix} A_{13} \\ A_{23} \end{bmatrix}$, $\overline{A}_{31} = [A_{31} \quad A_{32}]$. If the following conditions are satisfied

1) there exists A'_{13} such that $\overline{A}_{13} = A'_{13}A_3$, and $A'_{13}B_3 = 0$,

2) there exists A'_{31} such that $\overline{A}_{31} = A'_{31}\overline{A}_1$, and $A'_{31}\overline{B}_1 = 0$.

then there are not K_1, K_2, K_3 such that $\overline{A}_1 +$

$\overline{B}_1 diag(K_1, K_2)$, $A_3 + B_3K_3$ and A_{cl} are stable simultaneously when $det(I - A'_{31}A'_{13}) < 0$.

The above results show that in some cases the stability of interconnected systems is closely dependent on the interconnections. In order to study the actions of the interconnections between subsystems further, we study a special kind of decentralized control problem which can be viewed as harmonic stability problem of subsystems. In this paper we study the following simple cases.

Consider the following interconnected system

$$\begin{aligned} \dot{x}_1 &= A_1x_1 + b_{12}u_{12}, \\ \dot{x}_2 &= A_2x_2 + b_{21}u_{21}, \end{aligned} \tag{2}$$

where $u_{12} = k_{12}x_2$, $u_{21} = k_{21}x_1$, b_{12}, b_{21} are given real vectors. k_{12}, k_{21} are real row vectors to be determined. There is information interchange between two subsystems. It means that two systems are cooperating. For this special decentralized control problem, one can get some simple result for its stabilizability with the following lemmas.

Lemma 1 Given a real monic polynomial $f(\lambda)$ with degree n, $f(\lambda)$ has no real root if and only if $f(x) > 0, \forall x \in \mathbf{R}$.

One can prove this lemma easily by writing $f(\lambda)$ as

$$f(\lambda) = (\lambda - \lambda_1)(\lambda - \lambda_2)\cdots(\lambda - \lambda_n)$$

where $\lambda_1, \lambda_2, \cdots, \lambda_n$ are roots of $f(\lambda)$.

Lemma 2 Given a real monic polynomial

$$f(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + a_{n-2}\lambda^{n-2} + \cdots + a_1\lambda + a_0$$

with $a_{n-1} > 0$, there exists a real stable polynomial

$$g(\lambda) = \lambda^n + a_{n-1}\lambda^{n-1} + b_{n-2}\lambda^{n-2} + \cdots + b_1\lambda + b_0$$

such that $g(\lambda) - f(\lambda)$ has at least one real root.

Proof When the degree of $f(\lambda)$ n is odd, it holds obviously. When n is even, we state that there are a sable polynomial $g(\lambda)$ given as above and two points $x_1, x_2 \in (-a_{n-1}, 0)$ such that

$$g(x_1) - f(x_1) < 0, \quad g(x_2) - f(x_2) > 0. \tag{3}$$

In fact, obviously there are a sable polynomial $g(\lambda)$ given as above and two points $x_1, x_2 \in (-a_{n-1}, 0)$ such that $g(x_1) < 0$ and $g(x_2) > 0$. At this time, $f(x_1)$ and $f(x_2)$ are two fixed numbers. Then $|g(x_i)|, i = 1, 2$ can be enlarged arbitrarily by enlarging the imaginary parts of the roots of $g(\lambda)$, so (3) can be guaranteed easily. By Lemma 1, we know that $g(\lambda) - f(\lambda)$ has at least one real root with such a $g(\lambda)$.

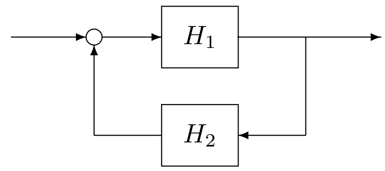

Fig. 1 feedback system

Theorem 2 If $(A_1, b_{12}), (A_2, b_{21})$ are controllable, and $a = tr(A_1) + tr(A_2) < 0$, where $tr(.)$ denotes the trace of the corresponding matrix, then there are k_{12} and k_{21} such that $A_{cl} = \begin{pmatrix} A_1 & b_{12}k_{12} \\ b_{21}k_{21} & A_2 \end{pmatrix}$ is Hurwitz stable.

Proof: Suppose $(A_1, b_{12}), (A_2, b_{21})$ are with the standard controllable model. Let the orders of A_1 and A_2 be n and m, respectively. Set $H_1(s) = k_{21}(sI - A_1)^{-1}b_{12}$, $H_2(s) = k_{12}(sI - A_2)^{-1}b_{21}$, $k_{12} = (\beta_0, \beta_1, \cdots, \beta_{m-1})$, $k_{21} = (\alpha_0, \alpha_1, \cdots, \alpha_{n-1})$, $d_1(s) = det(sI - A_1)$, $d_2(s) = det(sI - A_2)$, $k_1(s) = \alpha_0 + \alpha_1 s + \cdots + \alpha_{n-1}s^{n-1}$, $k_2(s) = \beta_0 + \beta_1 s + \cdots + \beta_{m-1}s^{m-1}$, then A_{cl} is stable if and only if the feedback system shown in Fig. 1 is stable, i.e., the polynomial $d_{cl}(s) = d_1(s)d_2(s) - k_1(s)k_2(s)$ is stable. Obviously, $d_{cl}(s)$ is a monic polynomial and the coefficient of s^{n+m-1} in $d_{cl}(s)$ is $-a = -(tr(A_1) + tr(A_2)) > 0$. Let $d_1(s)d_2(s) = s^{n+m} - as^{n+m-1} + d_0(s)$, then the stability of $d_{cl}(s)$ is completely determined by $d(s) = d_0(s) - k_1(s)k_2(s)$. When at least one of n, m is odd, one can choose $d(s)$ arbitrarily such that $d_{cl}(s)$ is stable, and decompose $d(s) - d_0(s)$ into the product of real polynomials $k_1(s)$ and $k_2(s)$. This means that we find real vectors k_{12}, k_{21} such that A_{cl} is stable. When all n, m are even, the degrees of $k_1(s)$ and $k_2(s)$ are odd, but the degree of $k_1(s)k_2(s)$ is even. At this time, one needs choose $d(s)$ such that d_{cl} is stable and $d(s) - d_0(s)$ has a real root in order to decompose $d(s) - d_0(s)$ into the product of two real polynomials. Obviously, this can be completed using Lemma 2. By Lemma 2, there is a stable polynomial $d_{cl}(x)$ with the sum of all its roots equal to a, and $d_1(s)d_2(s) - d_{cl}(s)$ has at least one real root. This implies that $d_1(s)d_2(s) - d_{cl}(s)$ can be decomposed into the product of two real polynomials $k_1(s)$ and $k_2(s)$. This completes the proof.

Remark 2 From the proof of the theorem, we know that the eigenvalues of A_{cl} can be assigned arbitrarily with the only constraint $a = \lambda_1 + \cdots + \lambda_{n+m}$ when one of n and m is odd. When n and m are even simultaneously, the eigenvalues of A_{cl} can also be assigned properly with the constraints $a = \lambda_1 + \cdots + \lambda_{n+m}$ and k_{12}, k_{21} being real vectors.

Remark 3 System (2) can be viewed as cooperative behavior between two subsystems. Two subsystems can be disabled themselves(they can be unstable), but they can realize a stable system through intercrossed feedback. Subsystem does not use the information itself, but it use the other subsystem's information. That is, they can help with each other to attain some target. Of course, there may be self-feedback in subsystems themselves. We can imagine that under cooperations subsystems need not to be controllable or stabilizable themselves. See the following system.

If there is self-feedback in subsystems, system (2) can be stated as follows.

$$\begin{aligned} \dot{x}_1 &= A_1x_1 + b_1u_1 + b_{12}u_{12}, \\ \dot{x}_2 &= A_2x_2 + b_2u_2 + b_{21}u_{21}, \end{aligned} \tag{4}$$

where $u_{12}, u_{21}, b_{12}, b_{21}$ are given as in system (2), b_1, b_2 are real vectors with compatible dimensions, $u_1 = k_1x_1, u_2 = k_2x_2$. By using Theorem 2, one can get the following result easily.

Theorem 3 If $(A_1, [b_1\ b_{12}])$, $(A_2, [b_2,\ b_{21}])$ are controllable and b_1, b_2 are not zero vectors simultaneously, then there are real vectors $k_1,\ k_{12},\ k_2,\ k_{21}$ such that system (4) is stable, i.e., $A_{cl} = \begin{pmatrix} A_1 + b_1k_1 & b_{12}k_{12} \\ b_{21}k_{21} & A_2 + b_2k_2 \end{pmatrix}$ is Hurwitz stable.

Remark 4 One can see clearly in Theorem 3, (A_1, b_1) and (A_2, b_2) need not to be controllable or stabilizable. Theorem 3 shows that two subsystems with effective control can cooperate easily for sake of stability. The actions of interconnections are shown here to some degree. And obviously, the framework in Theorems 1, 2, 3 can be generalized to multi-subsystem cases. In addition, refer to [4] for the case of b_1, b_2, b_{12}, b_{21} being matrices in which it is a little tedious for constructing intercrossed feedback.

III. Parameter-dependent Lyapunov method

Although we analyzed some special decentralized control problems in the section above, it is still hard to design decentralized controllers. Fortunately, [14] presented parameter-dependent Lyapunov method. In what follows, by using this method we present LMI-based design method for the problems discussed above. First we introduce the following lemma to begin this section.

Lemma 3 Given a real matrix $A \in \mathbf{R}^{n\times n}$, A is Hurwitz stable if, and only if, there exist a matrix $P = P^T > 0$ and any matrix V such that

$$\begin{pmatrix} -V - V^T & V^TA^T + P & V^T \\ AV + P & -P & 0 \\ V & 0 & -P \end{pmatrix} < 0. \tag{5}$$

One can turn (5) into Lyapunov inequality $PA^T + AP < 0$ easily by using the well known projection lemma in LMI method. By introducing a new variable V, the products of PA and A^TP are relaxed to new products AV and V^TA^T. V needs not be symmetric and positive definite. In this way Lyapunov matrix P can be parameter-dependent for the study of robust stability and robust performances[14], [15], [23], [25]. The case of diagonal blocked matrix V for decentralized control of discrete-time systems was considered in [15]. Here we discuss upper trigonal constraint of V for system (1) as follows, lower trigonal constraint can be considered similarly. Sometimes, upper trigonal constraint is less conservative than diagonal constraint. Corresponding to system (1), we suppose

$$V = \begin{pmatrix} V_1 & V_1 \\ 0 & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & X_1 \\ 0 & X_2 \end{pmatrix}. \tag{6}$$

Remark 5 For simplicity, we assumed that V and X acquire the upper trigonal structure as in (6). In fact, it is only fit for the case of $order(A_1) = order(A_2)$. If $order(A_1) \neq order(A_2)$, for example $order(A_1) < order(A_2)$, we can add zero blocks in V and X as folows to meet such cases,

$$V = \begin{pmatrix} V_1 & V_{12} \\ 0 & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & X_{12} \\ 0 & X_2 \end{pmatrix},$$

where $V_{12} = (V_1 \quad 0), X_{12} = (X_1 \quad 0)$. When $order(A_1) > order(A_2)$, we assume V and X acquire the following lower trigonal structure

$$V = \begin{pmatrix} V_1 & 0 \\ V_{21} & V_2 \end{pmatrix}, \quad X = \begin{pmatrix} X_1 & 0 \\ X_{21} & X_2 \end{pmatrix},$$

where $V_{21} = (V_2 \quad 0), X_{21} = (X_2 \quad 0)$, then we can get the similar result as in Theorem 4.

Let

$$A = \begin{pmatrix} A_1 & A_{12} \\ A_{21} & A_2 \end{pmatrix}, B = \begin{pmatrix} B_1 & 0 \\ 0 & B_2 \end{pmatrix}, K = \begin{pmatrix} K_1 & 0 \\ 0 & K_2 \end{pmatrix} \tag{7}$$

in system (1). By Lemma 3, one can get the following result for stability of system (1).

Theorem 4 If there are $P = P^T$ and V, X with form (6) such that

$$\begin{pmatrix} -V - V^T & V^TA^T + X^TB^T + P & V^T \\ AV + BX + P & -P & 0 \\ V & 0 & -P \end{pmatrix} < 0,$$

then there exist diagonal blocked matrix K as in (7) such that $A_{cl} = A + BK$ is stable. At this time, decentralized controllers can be obtained as $K_1 = X_1V_1^{-1}, K_2 = X_2V_2^{-1}$.

Remark 6 From Theorem 4, one can see that P is not blocked, and V_1, V_2 are generally not symmetric, of course not positive definite. Intuitively, one can imagine that at this time, $A_1 + B_1K_1$ or $A_2 + B_2K_2$ can be unstable under stability of A_{cl}. One can also see this from the forthcoming examples. In addition, one can establish some similar results for systems (2) and (4).

IV. Examples

Example 1 In the simple example studied in section 2, let $\alpha = \beta = 2$. By Theorem 4, one can get a $K =$

$\begin{pmatrix} -2.62 & -5.69 & 0 & 0 \\ 0 & 0 & 1.13 & -11.48 \end{pmatrix}$. Obviously $A_2+B_2K_2$ is not stable, but A_{cl} is stable.

Example 2 Consider system (2) defined by matrices

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -3 & -8 & -5 \end{pmatrix}, \; b_{12} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ -3 & -2 & -5 & 0 \end{pmatrix}, \; b_{21} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix}.$$

Obviously, the conditions in Theorem 2 are satisfied. A_2 is not stable here. Using the method in Theorem 4, one can get decentralized controllers

$$k_{21} = \begin{pmatrix} -1.7976 & -3.0015 & -5.4645 & -4.6861 \end{pmatrix},$$
$$k_{12} = \begin{pmatrix} 0.1507 & 1.0410 & 0.8293 & 1.3124 \end{pmatrix}$$

such that A_{cl} is stable in (2).

Example 3 Consider system (4) defined by matrices

$$A_1 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & -1 \end{pmatrix}, \; b_{12} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

$$A_2 = \begin{pmatrix} 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 \end{pmatrix}, \; b_{21} = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 1 \end{pmatrix},$$

and $b_1 = b_2 = \begin{pmatrix} 0 & 0 & 1 & 0 \end{pmatrix}^T$. Obviously, the conditions in Theorem 3 are satisfied. Using the method in Theorem 4, one can get decentralized controllers

$$k_1 = \begin{pmatrix} -2.7658 & -6.3430 & -6.2113 & -1.0951 \end{pmatrix},$$
$$k_2 = \begin{pmatrix} -3.4075 & -4.8896 & -4.6361 & -1.5468 \end{pmatrix}$$
$$k_{21} = \begin{pmatrix} -0.5882 & 0.7342 & -0.2848 & -1.1162 \end{pmatrix},$$
$$k_{12} = \begin{pmatrix} 1.4464 & 0.6239 & 0.0819 & 1.9946 \end{pmatrix}$$

such that A_{cl} is stable in (4).

From the examples and discussions above one can see that subsystems need not to be all stable, even in some special cases some subsystems must be unstable. This shows the special effects of interconnections.

V. Conclusions

This paper is devoted to studying decentralized control of some special interconnected systems. Some simple interconnected structures are established in which subsystems need not be stable. The results here can be generalized to cases of multiple subsystems. An LMI-based decentralized controller design method is also given by using parameter-dependent Lyapunov function method. Although this method is suitable for the problems here in some cases. It is still very conservative. It is an interesting topic to develop more effective decentralized controller design method for the problems here. We hope that these results can be helpful for understanding the actions of interconnections in large scale systems. Modern economic development with high-speed is always along with the bankruptcy of small enterprises. This means that the benefits of small enterprises are sacrificed for the privilege of high-speed development. For large scale systems, this paper indicates that sometimes the stability of subsystems should also be sacrificed for the stability of the whole system.

References

[1] D. Arzelier, D. Henrion and D. Peaucelle, Robust D stabilization of a polytope matrices, *International Journal of Control*, Vol. 75, No. 10, 2002, pp 744-752.

[2] S. Boyd, L. Elghaoui, E. Feron and V. Balakrishnan, *Linear Matrix Inequalities in Systems and Control Theory*, Philadelphia, PA: SIAM, 1994.

[3] E. J. Davison and U. Ozguner, Characterization of decentralized fixed modes for interconnected systems, *Automatica*, 19(2), 1983, pp 182-196.

[4] Z. S. Duan, L. Huang, J.Z. Wang and L. Wang, Harmonic control between two systems, *Acta Automatica Sinica*, 29(1), 2003, pp 14-22.

[5] Z. S. Duan, L. Huang, L. Wang and J.Z. Wang, Some applications of small gain theorem to interconnected systems, *Systems and Control Letters*, 52(3-4), 2004, 263-273.

[6] Z. S. Duan, J.Z. Wang and L. Huang, Multi-input and multi-output nonlinear systems: Interconnected Chua's circuit, *International Journal of Bifurcation and Chaos,* 14(9), 2004, 3065-3081.

[7] Z. S. Duan, J.Z. Wang and L. Huang, Frequency-domain method for the property of dichotomy of modified Chua's equations, *International Journal of Bifurcation and Chaos,* 15(8), 2005, to appear.

[8] Z. S. Duan, J.Z. Wang and L. Huang, Input and output coupled nonlinear systems, *IEEE trans. on Circuits and Systems-I regular papers,* 52(3), 2005, In Press.

[9] P. Gahinet, A. Nemirovski and A. J. Laub, *LMI Control Toolbox,* The math Works, Inc. Natick, Mass, 1995.

[10] J. C. Geromel, J. Bernussou and P. L. D. Peres, Decentralized control through parameter space optimization, *Automatica,* Vol. 30, 1994, pp 1565-1578.

[11] Z. Gong and M. Aldeen, Stabilization of decentralized control systems, *J. of Math. Syst. Estim. and Contr.,* Vol. 7, No. 1, 1997.

[12] T. Iwasaki and R. E. Skelton, All controllers for the general H_∞ control problems: LMI existence conditions and state space formulas, *Automatica*, Vol. 30, No. 8, 1994, pp 1307-1317.

[13] Y. Jia, Alternative proofs for improve LMI representations for the analysis and the design of continuous-time systems with polytopic type uncertainty: A predictive approach, *IEEE Trans. on Automatic Control*, Vol. 48, No. 8, 2003, pp 1413-1416.

[14] M. C. de Oliveira, J. Bernussou and J. C. Geromel, A new discrete-time robust stability condition, *Systems and Control Letters*, Vol. 37, 1999, pp 261-265.

[15] M. C. de Oliveira, J. Bernussou and J. C. Geromel, Extended H_2 and H_∞ norm characterizations and controller parameterizations for discrete-time systems, *International Journal of Control*, Vol. 75, No. 9, 2002, 666-679.

[16] M. C. de Oliveira, J. C. Geromel and J. Bernussou, Design of dynamic output feedback decentralized controllers via a separation procedure, *Int. J. Control*, Vol. 73, No. 5, 2000, pp 371-381.

[17] P. J. de Oliveira, R. C. L. F. Oliveira, V. J. S. Leite, V. F. Montagner and P. L. D. Peres, H_∞ guaranteed cost computation by means of parameter-dependent Lyapunov functions, *Automatica*, Vol. 40, 2004, pp 1053-1061.

[18] D. Peaucelle, D. Arzelier, O. Bachelier, J. Bernussou(2000), A new robust D-stability condition for real convex polytopic uncertainty, *Systems and Control letters*, Vol. 40, 21-30.

[19] U. Shaked, Improved LMI representations for the analysis and the design of continuous-time systems with polytopic type uncertainty, *IEEE Trans. on Automatic Control*, Vol. 46, No. 4, 2001, pp 652-656.

[20] D. D. Siljak, *Large-Scale Dynamic Systems*, North-Holland and New York, 1978.

[21] D. D. Siljak, *Decentralized Control of Complex Systems*, New York: Academic, 1991.

[22] D. D. Siljak and D. M. Stipanovic, Robust stabilization of nonlinear systems: the LMI approach, *Math. Prob. Eng.,* vol. 6, 2000, pp 461-493.
[23] H. D. Tuan, P. Apkarian and T. Q. Nguyen, Robust and reduced-order filtering: new LMI-based characterizations and methods, *IEEE Trans. on Signal Processing*, Vol. 49, No. 12, 2001, pp 2975-2984.
[24] S. H. Wang, An example in decentralized control systems, *IEEE TAC,* 23, 1978, pp 938-938.
[25] L. Xie, L. Lu, D. Zhang and H. Zhang, Improved robust H_2 and H_∞ filtering for uncertain discrete-time systems, *Automatica*, Vol. 40, 2004, pp 873-880.