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CONNECTIVITY AND CONVERGENCE OF FORMATIONS

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Abstract— **In this work we consider the stability of vehicle formations in the case of a varying communication topology. We use a decentralized control law approach and explore the challenges and issues that arise in this framework. The vehicles considered are homogeneous, with discrete-time dynamics, and the communication between them is defined based on a pre-defined proximity rule. The resulting closed loop system is a switched dynamical system and in this paper we describe sufficient conditions that will lead to the stability of the vehicle formations.**

I. INTRODUCTION

The theory of decentralized control for coordinating large numbers of vehicles has recently become a topic of large interest. Vicsek et al. in [14] consider self-propelled particles using a neighbor-averaging law that leads to a common heading. In [4], [5] conditions based on a Nyquist criterion are developed that allow us to predict when the dynamical system describing a vehicle formation will be stable. Glavaski et al. in [7], [8] explore the convergence to formation in the case where the transmission connections between vehicles break randomly. In [10], [11] the convergence to formation is looked at when the communication graph depends on the relative physical position of agents.

One of the applications of interest to us is being able to analyze the stability of formations of uninhabited autonomous vehicles (UAVs) by considering the communication among them while performing a given mission. Using graph-theoretic tools, we focus on the general task of designing a formation control law that is decentralized and uses inter-vehicle sensed or communicated information, and it guarantees convergence to a stable vehicle formation. Necessary and sufficient conditions are developed in [12] for the convergence to and the stability of a vehicle formation in the case of a fixed vehicle communication infrastructure. In [15] the stability of multiple vehicle formations is analyzed, and an inter-formation communication infrastructure is proposed between subformations that guarantees the stability of the entire formation. A recent study at Honeywell Labs, Minneapolis MN, shows that this control law can be implemented to models of organic air vehicles. Given formations of such vehicles, and using a distributed control law, a stable hierarchical formation of OAVs is achieved [3]. Other possible additional requirements in this decentralized control framework are state-dependent communication links, for example proximity dependent communication, collision avoidance between vehicles that are converging to a formation, and obstacle avoidance.

A first step in this analysis is to consider the issues that arise in the formation stability analysis in the case of time or state-dependent communication structure. In this paper we consider homogeneous vehicles with discrete-time dynamics that exchange information based on some proximity rule, in order to achieve a required geometric formation. The formation required is pre-specified and although all vehicles have to agree on a global reference frame, this feedback law has the advantage that each vehicle computes its control based on local information. When in formation all vehicles move with the same velocity, and the vehicles can be positioned at desired relative distances and angles from each other. The latter property distinguishes this analysis from the vehicle flocking theory, where the control law does not determine where the vehicles will be relative to each other. The objective is to define and analyze the stability issues that arise in this distributed coordinated control framework.

II. ALGEBRAIC GRAPH THEORY

The information exchange between vehicles can be modelled by a directed graph, where the arrows show the direction from which position and velocity information is received by a vehicle from its "neighbor". In this paper, we refer to vehicles as being neighbors in the physical sense. A proximity radius ϵ_i is defined for each vehicle i, where vehicle j is a neighbor of vehicle i if the distance between vehicles i and j is less or equal to ϵ_i.

Definition 2.1: A directed graph G consists of a vertex set $V(G)$ and an edge set $E(G) \subseteq V(G) \times V(G)$. For an edge $e = (u, v) \in E(G)$, u is called the head vertex of e and v is called the tail vertex of e.

If $(u, v) \in E(G)$ for all $(v, u) \in E(G)$, then we call the graph undirected. We call the graph simple if there are no edges of the form (u, u) for $u \in V(G)$. Let G be a graph representing the communication links between a set of vehicles. The properties of such a communication graph are captured by its adjacency matrix $A_d(G)$ defined by:

Definition 2.2: The adjacency matrix of a graph G, denoted $A_d(G)$, is a square matrix of size $|V(G)| \times |V(G)|$ defined as follows:

$$A_d(G)_{ij} = \begin{cases} 1 & \text{if } (u_i, u_j) \in E(G) \\ 0 & \text{otherwise.} \end{cases}$$

where $u_i, u_j \in V(G)$.

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Definition 2.3: The indegree of a vertex u is the number of edges that have u as their head vertex. The indegree matrix of a graph G, denoted $D(G)$, is a diagonal matrix of size $|V(G)| \times |V(G)|$ defined as follows:

$$D(G)_{ii} = \text{indegree}(u_i)$$

where $u_i \in V(G)$.

The outdegree of a vertex u for a graph G is defined analogously, as the number of edges having u as the tail vertex.

Definition 2.4: Given a graph G, the Laplacian matrix associated with it is given by

$$L_G = D(G) - A_d(G)$$

where $D(G)$ is the indegree matrix and $A_d(G)$ is the adjacency matrix associated with the graph.

The diagonal entry d_{ii} of the Laplacian matrix is then the indegree of vertex i and the negative entries in row i correspond to the neighbors of vertex i. For any communication graph considered, the row sums of the corresponding Laplacian matrix are always zero and hence zero is always an eigenvalue. In the case of undirected graphs, the Laplacian matrix is symmetric and all its eigenvalues are non-negative [2], [9]; the smallest eigenvalue of the Laplacian, λ_1, is zero; and its multiplicity equals the number of connected components of the graph. The second eigenvalue, λ_2, is directly related to the connectivity of the graph [6]. This is also the case for directed graphs. The properties of the eigenvalues of the Laplacian matrices in the case of directed graphs are explored in [12].

Definition 2.5: Given a directed graph G, $P = (u_1, \ldots, u_k)$ is a directed path in G if for every $1 \le i < k$ there exists edge $(u_i, u_{i+1}) \in E(G)$.

Definition 2.6: Given a directed graph G, we call T a rooted directed spanning tree of G, if T is a subgraph of G, it has no cycles and there exists a (directed) path from at least one vertex, the root, to every other vertex in T.

Definition 2.7: Given a directed graph G, we call G strongly connected if there exists a directed path between any two vertices in $V(G)$.

All graphs in this paper are directed graphs, unless mentioned otherwise. One property of the eigenvalues for the Laplacian matrix of such a communication graph can be derived from Gershgorin's Theorem. Since all diagonal entries of the Laplacian are the indegrees of the corresponding vertex, it follows that all its eigenvalues will be located in the disc centered at $d = max_i(indegree(i))$ of radius d, so for any eigenvalue λ of the Laplacian

$$|\lambda| \le 2d$$

Further properties of the eigenvalues of such a graph are explored in [13].

III. Problem Formulation

We assume given N homogeneous vehicles with the following discrete-time dynamics:

$$x_i(k+1) = A_{veh}x_i(k) + B_{veh}u_i(k)$$

where $i = 1, 2, ..., N$ and the entries of x_i represent the n configuration variables for vehicle i, referred to as position-like variables, and their derivatives, referred to as velocity-like variables, and u_i is the control input for vehicle i.

For each vehicle we use the error signal $z_i(k)$ for coordination:

$$z_i(k) = \sum_{j \in J_i(k)} (x_i(k) - x_j(k)) - (h_i - h_j)$$

where $J_i(k)$ is the set of neighbors of vehicle i at time k.

Definition 3.1: [12] A moving formation of N vehicles is given by a vector $h = h_p \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \in R^{2nN}$. The N vehicles are said to be in formation at time k if there exist R^n valued vectors q and w such that $(x_p)_i(k) - (h_p)_i = q$ and $(x_v)_i(k) = w$, for $i = 1, 2, \ldots, N$, where the subscript p refers to the position components of x_i and the subscript v refers to the corresponding velocities. The vehicles converge to formation h if there exist real valued functions $q(\cdot)$ and $w(\cdot)$ such that $(x_p)_i(k) - (h_p)_i - q(k) \to 0$ and $(x_v)_i(k) - w(k) \to 0$, as $k \to \infty$ for $i = 1, 2, \ldots, N$.
The figure below illustrates the interpretation of vectors in the definition.

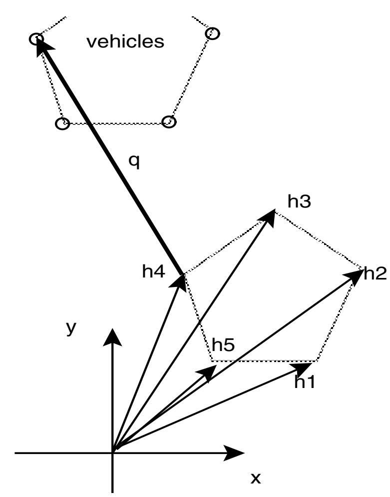

Fig. 1. Example of a pentagon formation with the corresponding offset vectors

At any time k, the information exchange between vehicles is modelled by a graph, $G(k)$, where the N vertices represent the vehicles, and the edges represent the communication links. In the previous section we explained when two vehicles are considered to be neighbors. It is important to note that if a vehicle is a neighbor of vehicle i at time k, it is not guaranteed that the same vehicle will be a neighbor of i at time $k+1$. In fact, depending on the formation requirements, vehicles may change neighbors often, while converging to formation.

For simplicity of the analysis, we consider all vehicles to be homogeneous and we look for a feedback matrix F_{veh} for each vehicle system. The system of N-homogeneous vehicles is given by:

$$\begin{aligned} x(k+1) &= Ax(k) + Bu(k) \\ z(k) &= L(k)(x(k) - h) \end{aligned}$$

where x is the augmented state vector, $A = I_N \otimes A_{veh}$, $B = I_N \otimes B_{veh}$, and $L(k) = L_G(k) \otimes I_{2n}$. The coordinated control problem is to find a decentralized feedback control matrix $F = diag[F_{veh}]$ such that if:

$$u(k) = Fz(k)$$

then the vehicles converge to formation. We remark here that this feedback matrix F should be the same regardless of the communication graph between vehicles.

The closed loop system becomes:

$$x(k+1) = Ax(k) + BFL(k)(x(k) - h)$$

In the analysis that follows, we must analyze the stability of a dynamical system that is of the form:

$$x(k+1) = (A + BFL(k))x(k)$$

We note that the Laplacian matrix $L_G(k)$ is defined based on the proximity radius for each vehicle. The matrix is therefore state-dependent, so $L(k) = L(x(k))$ and it is a constant matrix that belongs to the finite set of Laplacian matrices of graphs on N vertices. Therefore, the system above is a switched dynamical system. Its stability depends in part on the stability of the individual systems:

$$x(k+1) = (A + BFL_s)x(k)$$

but it is not guaranteed by that, where L_s is a Laplacian matrix for a graph on N vertices.

Proposition 3.2: Let $M = \{C_1, ..., C_m\}$ and consider the switched linear system

$$x(k+1) = C_{i(k)}x(k), C_{i(k)} \in M$$

If all matrices in M have a spectral radius less than 1 and the Lie algebra associated to M is solvable, then the system has a common quadratic Lyapunov function.

Proof: [1] ■

Proposition 3.3: The vehicles are in formation if and only if there exists a communication graph G with Laplacian matrix L such that for communication graph Laplacian matrices $L_G(k)$ we have the following conditions:

$$L_G(k) \rightarrow L$$

$$(L \otimes I_{2n})(x - h) = 0$$

Proof: If such an L exists, then the distances between vehicles must be fixed. On the other hand, the vehicles are in a moving formation if the distances between them are fixed and therefore the communication graph becomes independent of time. Let L be the Laplacian matrix for this graph. Also, as the vehicles are in formation, there exist $q(\cdot)$ and $w(\cdot)$ such that $(x_p)_i(k) - (h_p)_i = q(k)$ and $(x_v)_i(k) = w(k)$, $i = 1, 2, \ldots, N$. Then

$$\begin{aligned} (L \otimes I_{2n})(x(k) - h) &= \\ (L \otimes I_{2n}) \left(1_N \otimes \left((x_p)_i \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + (x_v)_i \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} \right) \right) &- \\ (L \otimes I_{2n}) \left(h_p \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) &= \\ (L \otimes I_{2n}) \left(1_N \otimes \left(q(k) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} + w(k) \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) \right) &= \\ (L \otimes I_{2n})(1_N \otimes \alpha(k)) &= \\ 0 & \end{aligned}$$

■

We will now will provide a sufficient for convergence to formation in the case of time-dependent communication graphs. First we establish a lemma.

Lemma 3.4: Consider a finite collection of matrices $M = \{P(k): k = 1, \ldots, m\}$, each of the form $P(k) = \begin{pmatrix} P_1(k) & P_2(k) \\ 0 & P_3(k) \end{pmatrix}$. Let A be the Lie algebra generated by M and let B be the Lie algebra generated by the matrices $P_3(k)$, for $k = 1, \ldots, m$. Then B is isomorphic to a quotient Lie algebra of A. In particular, if A is solvable, then so is B.

Proof: First notice that $\left[\begin{pmatrix} P_1 & P_2 \\ 0 & P_3 \end{pmatrix}, \begin{pmatrix} Q_1 & Q_2 \\ 0 & Q_3 \end{pmatrix} \right] = \begin{pmatrix} [P_1, Q_1] & * \\ 0 & [P_3, Q_3] \end{pmatrix}$. Consider the set H of matrices of the form $\begin{pmatrix} H_1 & H_2(k) \\ 0 & 0 \end{pmatrix}$ (where the dimensions correspond to those of the blocks in M). A direct calculation shows that $H \cap A$ is an ideal of the Lie algebra A. It is easy to see that $A/(H \cap A)$ is isomorphic to B. ■

Theorem 3.5: The vehicles converge to formation if at each time k, the communication graph $G(k)$ has a rooted directed spanning tree and the Lie algebra of matrices $A + BFL(k)$ is solvable.

Proof: We note that this is a sufficient condition, but not necessary. It was shown in [12] that a graph has a rooted directed spanning tree if and only if zero is an eigenvalue of multiplicity one of the corresponding Laplacian matrix. In analogy with [12] we consider the extended discrete-time system:

$$\begin{aligned} x(k+1) &= Ax(k) + BFL(k)x(k) \\ & \quad -BFL(k) \left(I_{nN} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \right) h_p \\ h_p(k+1) &= h_p(k) \end{aligned}$$

where $h_p(k) = h_p, \forall k$. Equivalently, the system becomes:

$$y(k+1) = M(k)y(k)$$

where $M(k) = \begin{pmatrix} A+BFL(k) & -BFL(k)\left(I_{nN} \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix}\right) \\ 0 & I_{nN} \end{pmatrix}$

The eigenvalues of $M(k)$ consist of those of $A + BFL(k)$ and those of I_{nN}. Consider now the subspace S of R^{3nN}:

$$S = \left\{ \begin{pmatrix} x \\ h_p \end{pmatrix} : L(k)(x-h) = 0 \right\}.$$

While the matrices $L(k)$ depend on k (and, in fact on x), the space $\mathcal{S}$ is independent of k since, for every k, the nullspace of $L(k)$ is spanned by the all-ones vector $\mathbf{1}_N$ (here we used our connectivity assumption on the graphs $G(k)$). Indeed, a basis of S is given by:

$$\begin{aligned} B &= \left\{ \begin{pmatrix} 1_N \otimes e_i \\ 0 \end{pmatrix} : e_i \in R^{2n}, i = 1, \ldots, 2n \right\} \\ &\bigcup \left\{ \begin{pmatrix} e_j \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ e_j \end{pmatrix} : e_j \in R^{nN}, j = 1, \ldots, nN \right\} \end{aligned}$$

In [12] it was shown that for a fixed L, the space S is M-invariant. Here we show that S is $M(k)$-invariant, regardless of k. Let $y(k) \in S$, for some k. Then,

$$y(k) = \begin{pmatrix} 1_N \otimes \alpha \\ 0 \end{pmatrix} + \begin{pmatrix} \beta \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \beta \end{pmatrix}$$

$$M(k)y(k) = \begin{pmatrix} 1_N \otimes A_{veh}\alpha \\ 0 \end{pmatrix} + \begin{pmatrix} \beta \otimes \begin{pmatrix} 1 \\ 0 \end{pmatrix} \\ \beta \end{pmatrix}$$

and hence, $M(k)y(k) \in S$. The last equation comes from basic rules of multiplication of Kronecker products and the specific form of A_{veh}. Therefore, for all k, S is $M(k)$-invariant. So, the linear transformation matrix induced by every $M(k)$ on S is constant of the form: $\begin{pmatrix} A & 0 \\ 0 & I_{nN} \end{pmatrix}$. Extending the above basis to a basis of R^{3nN} we can write all the matrices $M(k)$ in block upper triangular form

$$M(k) = \begin{pmatrix} \begin{pmatrix} A & 0 \\ 0 & I_{nN} \end{pmatrix} & * \\ 0 & Q(k) \end{pmatrix}.$$

The matrices $Q(k)$ become the induced matrices on the quotient space R^{2nN}/S. The eigenvalues of $Q(k)$ are those of $A + \lambda(k)BF$, for $\lambda(k)$ any nonzero eigenvalue of $L(k)$.

We choose a feedback matrix F that stabilizes each matrix $A + \lambda(k)BF$. The construction of such a matrix is shown in the section that follows. From our assumptions and Lemma (3.4), the Lie algebra generated by the matrices $Q(k)$ is solvable. Therefore the quotient system above is asymptotically stable [1]. Moreover, stability in the quotient is equivalent to having $L(k)(x(k) - h) \to 0$, which in turn is equivalent to convergence to formation. ∎

IV. STABILITY

It remains to show that under the connectivity assumption about the communication graph, a stabilizing feedback matrix F exists. In the proof of the following proposition, we will show how such a matrix can be constructed.

Proposition 4.1: For the double integrator discrete-time vehicle model, a stabilizing feedback matrix F_{veh} exists.

Proof: We are looking at stabilizing a matrix of the form

$$\begin{pmatrix} 1 + \lambda f_1 \frac{dt^2}{2} & dt + \lambda f_2 \frac{dt^2}{2} \\ \lambda f_1 dt & 1 + \lambda f_2 dt \end{pmatrix}$$

The characteristic polynomial of this matrix is of the form $p(x) = x^2 - sx + p$, where s and p can be complex numbers. We are interested in deriving conditions that will ensure that the roots of this polynomial are in the interior of a circle of radius 1 centered at the origin. By using the transformation $x = \frac{1+w}{1-w}$ and tools developed in [12], we can derive the necessary and sufficient conditions. As these are rather lengthy inequalities, we illustrate here the sufficient conditions in the case when the communication graph is undirected:

$$\begin{aligned} f_1 &< 0 \\ f_2 &< 0 \\ f_2 - f_1 &< -\frac{4}{\lambda dt^2} \end{aligned}$$

for λ a nonzero eigenvalue of the communication graph. If we allow $f_1 = f_2$, then the last condition above becomes:

$$f_2 > -\frac{2}{\lambda dt}$$

In our case, the communication graph is state-dependent, so the nonzero eigenvalues of the communication graph will vary with k. Since the number of possible graphs on N vertices is finite, and in particular the set of nonzero eigenvalues is finite, we can choose negative f_1 and f_2 that satisfy the required inequalities. The resulting feedback matrix $F_{veh} = \begin{pmatrix} f_1 & f_2 \end{pmatrix}$ will stabilize all matrices of the form $A + BFL(k)$, as required in the previous section. Using similar methods, conditions can be deduced for the case of directed graphs. ∎

V. CONNECTIVITY CRITERIA

In the previous sections we showed that stability of the vehicle formation is guaranteed in the case when the communication graph contains a rooted directed spanning tree at all times k. The issue of graph connectivity has been considered in previous research, however determining criteria for the communication graph to remain connected in the case when the control law depends on the proximity of neighbors remains an open problem.

Consider again the system defined previously:

$$y(k+1) = M(k)y(k)$$

By selecting a matrix F that stabilizes each $M(k)$ in the quotient space R^2nN/S, we therefore have that the induced matrix on this space has eigenvalues in the interior of the circle of radius 1, centered at the origin, and therefore each $M(k)$ must be a contraction mapping on this space. The challenge that remains in this case is to find the connection of the relative distance between vehicles, and the function defined above. In particular, one would like to say that if at time k the communication graph has a rooted directed spanning tree, then one must exist at time $k+1$. For the case when $h = 0$, then the problem should be: if two vertices are adjacent at time k, then they must be adjacent at time $k+1$. This not only depends on the vehicle dynamics given, but also on the eigenvalues of the Laplacian matrices from the set considered. In the section that follows we illustrate this via some of the examples.

VI. Simulation Results

We consider N vehicles with dynamics given by the discrete-time double integrator vehicle model. The initial states are fixed or random. We choose a proximity radius that would guarantee that in the final formation required every vehicle has at least one neighbor. We illustrate the connection between the proximity radius, the required formation shape, the feedback control matrix, and the initial conditions: In Figure 2, we have six vehicles converging

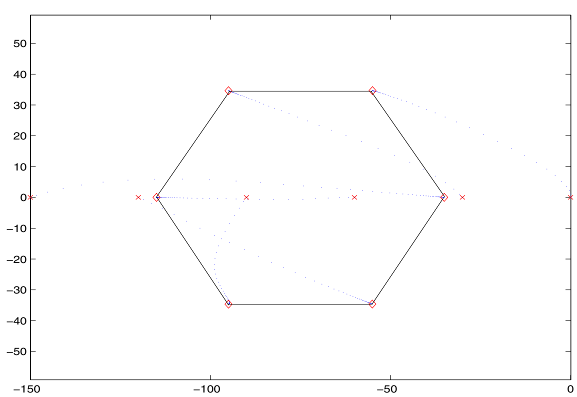

Fig. 2. Stationary Hexagonal Formation

to a stationary hexagonal formation, starting from given initial conditions. The communication between vehicles is neighbor dependent, in the sense that each vehicle receives position and velocity information only from the vehicles that are in its proximity. Both in Figure 2 and Figure 3 the communication graph is undirected. The values of the gains in this case are $f_1 = -3$ and $f_2 = -3$.

In Figure 4, we illustrate the six vehicles, with the same initial conditions as in the vehicles in Figure 2, but with a different feedback matrix values: $f_1 = -1$ and $f_2 = -1$.

Keeping the gain values and detection radius, we show in Figure 5 that other initial conditions can still lead to a convergent formation. In this case the initial conditions were chosen randomly from a given set.

In all of the above simulations we notice that for small enough initial conditions and h, if the graph is connected

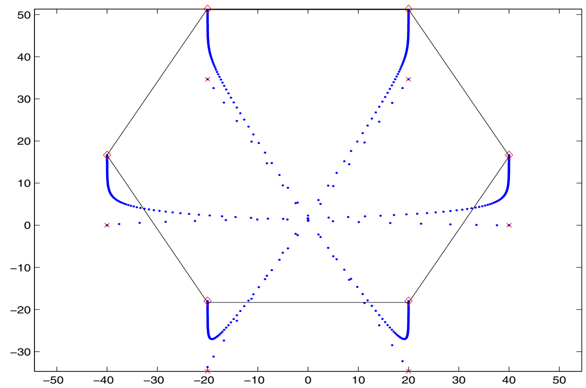

Fig. 3. A Moving Hexagonal Formation

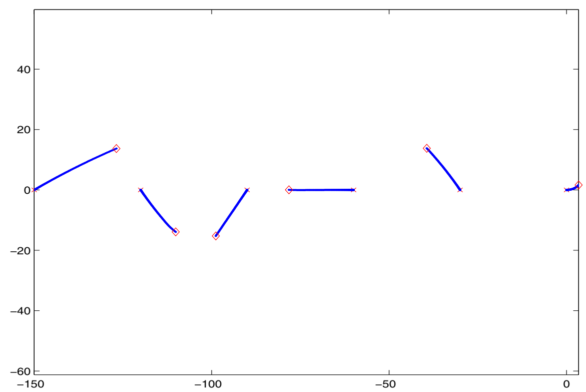

Fig. 4. Same proximity radius and initial conditions, but different feedback

at time k, it will remain connected at time $k + 1$.

VII. Conclusion

The goal in this paper was to illustrate some of the issues that arise in the analysis of the stability of vehicle formations in the case where the communication structure is switched based on a state, or time-dependent rule. We recognize some of the sufficient conditions for stability and propose necessary conditions.

The next goal of this study is to determine conditions under which stable formations can be achieved while ensuring vehicle collision avoidance and obstacle avoidance. Such control laws would also be based on a proximity rule, as it would be reasonable to assume that if a vehicle is in danger of collision, then it can identify the other vehicle as a neighbor, and adjust its relative distance to it, prior to collision.

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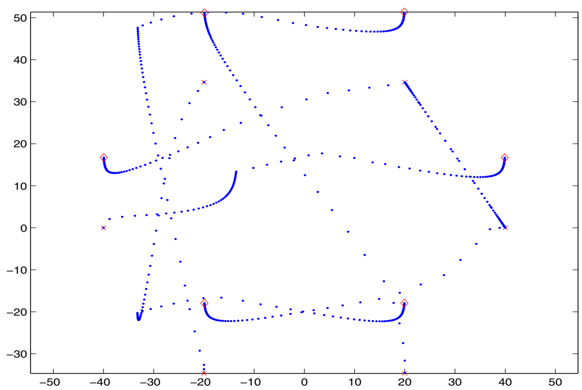

Fig. 5. Stationary Hexagonal Formation with random initial conditions

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