Robust fault diagnosis for linear time-delay systems with uncertainty

You Fuqiang, Tian Zuohua, Shi Songjiao

Abstract— **This paper deals with the problem of fault diagnosis problem for a class of linear systems with delayed state and uncertainty. The systems are transformed into two different subsystems. One is not affected by actuator faults, so the robust observer can be designed under certain condition. The other whose states can be measured is affected by the faults. The proposed observer is utilized in an analytical redundancy based approach for actuator and sensor fault detection and diagnosis in time-delay systems. Finally, the applicability and effectiveness of the proposed method is illustrated through numerical examples.**

Index Terms - **Fault detection and diagnosis; robust observer; linear systems; time delay; uncertainty**

I. INTRODUCTION

Owing to the increasing demand for high reliability in many industrial processes, much attention has been paid to the problem of fault detection and diagnosis (FDD) in dynamic systems over the past two decades. Fruitful results can be found in a survey paper [1] and books [2, 3].

It is well known that faults in a dynamic system can take many forms. They can be actuator faults, sensor faults, unexpected abrupt changes of some parameters or even unexpected structure changes [4, 5]. The purpose of detection is to generate an alarm to inform the operators that there is at least one fault in the system. This can be achieved from either the direct observation of the systems inputs and outputs or the use of certain types of redundant relations (i.e., model-based fault detection and diagnosis). However, compared with fault detection, fault diagnosis is not an easy task, as it requires that after the alarm has been set, an estimation of the location and the size of the fault should be made. Recently, some results for fault diagnosis have been obtained, for example, [6, 7, 8] based on adaptive or robust observers and [9] using learning approach. However, all those papers dealt with fault diagnosis for delay-free systems.

Time delay is commonly encountered in various engineering systems, such as chemical processes, long transmission lines in pneumatic, hydraulic and rolling mill systems, which usually results in unsatisfactory performances and is frequently a source of instability. Compared with rather rich literature on FDD of delay-free systems, there are very fewer research results on FDD of time-delay systems, as pointed out by Yang, H. in [10], but Yang, H. does not consider the uncertainty. In this paper, we have taken an approach similar to that of [8] for designing an observer for a class of linear time delay systems with uncertainties. At first, the systems are transformed into two different subsystems. The first subsystem is decoupled from actuator fault and the other is affected by the fault, but its states can be measured directly. As a generalization of the observer design approach in [8], a robust observer design is proposed for FDD. By using the estimation of states with bounded accuracy, we can approximate the fault by using the discretization of the other subsystems. When fault signal is greater than certain threshold, one can detect it or even diagnosis it.

Robust Observer Design

Consider a linear time delay systems with uncertainty,

$$\dot{\bar{x}} = \bar{A}\bar{x} + \sum_{i=1}^{N} \bar{A}_i \bar{x}(t-\tau_i) + \bar{B}u + \bar{E}f_a + \bar{D}\xi(x) \qquad (1a)$$

$$y = \bar{C}\bar{x} = [C_1 \ \ 0]\bar{x} \qquad (1b)$$

where $\bar{x} \in R^n$ is the state vector, $u \in R^q$ is the input vector, and the actuator fault is modeled as an additional input $f_a \in R^m$, $y \in R^p$ is the output vector, $p > m$, $\xi(x)$ with $\xi(0) = 0$ system model uncertainties, high-order nonlinearities or exogenous disturbance, $\tau_i \in [0, d](i = 1, \cdots N)$ are delays, d is a certain positive number, and C_1 is an $p \times p$ nonsingular matrix..

Assumption 1: $\rho(\bar{C}\bar{E}) = m$

Define a transformation $x = M^{-1}\bar{x}$, where

$$M = \begin{bmatrix} C_1 & 0 \\ 0 & I \end{bmatrix}$$

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Then the system described by (1) can be transformed into

$$\dot{x} = Ax + \sum_{i=1}^{N} A_i x(t-\tau_i) + Bu + Ef + D\xi$$

$$= \begin{bmatrix} A_1 \\ A_2 \\ A_3 \end{bmatrix} x + \sum_{i=1}^{N} \begin{bmatrix} A_1^{(i)} \\ A_2^{(i)} \\ A_3^{(i)} \end{bmatrix} x(t-\tau_i) + \begin{bmatrix} B_1 \\ B_2 \\ B_3 \end{bmatrix} u + \begin{bmatrix} E_1 \\ E_2 \\ E_3 \end{bmatrix} f_a + \begin{bmatrix} D_1 \\ D_2 \\ D_3 \end{bmatrix} \xi \tag{2a}$$

$$y = Cx = \begin{bmatrix} I_{(p-m)\times(p-m)} & 0 & 0 \\ 0 & I_m & 0 \end{bmatrix} x \tag{2b}$$

where

$A = M\bar{A}M^{-1}$, $A_i = M\bar{A}_i M^{-1}$, $B = M\bar{B}$, $E = M\bar{E}$, $D = M\bar{D}$

Partition x is given as

$$x = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} y_1 \\ y_2 \\ x_3 \end{bmatrix}$$

It can be seen that x_1, x_2 can be calculated from y_1, y_2, and $x_3 \in R^{n-p}$, whose estimate is required.

According to assumption 1, $\rho(CE) = m$, from the structure of matrix C, it is easy to show that

$$\rho \begin{bmatrix} E_1 \\ E_2 \end{bmatrix} = m \tag{3}$$

From (3), without loss of any generality, it can be assumed that E_2 is nonsingular.

Assumption 2: There exist positive constants α, β and γ such that

$$\left\|(D_1 - E_1E_2^{-1}D_2)\xi\right\| \le \alpha, \left\|D_2\xi\right\| \le \beta, \left\|(D_3 - E_3E_2^{-1}D_2)\xi\right\| \le \gamma$$

Remark 1: Assumption 2 implies the system uncertainty in the transformed system (2) is bounded. This improves existing results in [8, 11, 12], in which a perfect decoupling of system uncertainty is required.

Let

$$S = \begin{bmatrix} I & -E_1E_2^{-1} & 0 \\ 0 & I & 0 \\ 0 & -E_3E_2^{-1} & I \end{bmatrix} \tag{4}$$

By premultiplying (4) into (2a), we have that

$$\begin{bmatrix} \dot{x}_1 - E_1E_2^{-1}\dot{x}_2 \\ \dot{x}_2 \\ \dot{x}_3 - E_3E_2^{-1}\dot{x}_2 \end{bmatrix} = \begin{bmatrix} A_1 - E_1E_2^{-1}A_2 \\ A_2 \\ A_3 - E_3E_2^{-1}A_2 \end{bmatrix} x + \sum_{i=1}^{N} \begin{bmatrix} A_1^{(i)} - E_1E_2^{-1}A_2^{(i)} \\ A_2^{(i)} \\ A_3^{(i)} - E_3E_2^{-1}A_2^{(i)} \end{bmatrix}$$

$$x(t-\tau_i) + \begin{bmatrix} B_1 - E_1E_2^{-1}B_2 \\ B_2 \\ B_3 - E_3E_2^{-1}B_2 \end{bmatrix} u + \begin{bmatrix} 0 \\ E_2 \\ 0 \end{bmatrix} f_a + \begin{bmatrix} D_1 - E_1E_2^{-1}D_2 \\ D_2 \\ D_3 - E_3E_2^{-1}D_2 \end{bmatrix} \xi \tag{5}$$

It is clear that in (5) the actuator faults enter only through the second block row, whereas the other two block rows are not affected by any faults. Define

$G_j = A_j - E_jE_2^{-1}A_2 \quad H_j = A_j^{(i)} - E_jE_2^{-1}A_2^{(i)}$

$J_j = B_j - E_jE_2^{-1}B_2 \qquad j = 1,3$

Then the first and the third block rows of (5) can be rewritten as

$$\dot{x}_1 - E_1E_2^{-1}\dot{x}_2 = G_1x + \sum_{i=1}^{N} H_1 x(t-\tau_i) + J_1u + (D_1 - E_1E_2^{-1}D_2)\xi \tag{6}$$

$$\dot{x}_3 - E_3E_2^{-1}\dot{x}_2 = G_3x + \sum_{i=1}^{N} H_3 x(t-\tau_i) + J_3u + (D_3 - E_3E_2^{-1}D_2)\xi \tag{7}$$

Partitioning G_j, H_j as

$G_j = \lfloor G_{j1} \; G_{j2} \quad G_{j3} \rfloor \quad H_j = \lfloor H_{j1} \quad H_{j2} \quad H_{j3} \rfloor \quad j = 1,3$

Then we can rewrite (6) and (7) as

$$\dot{x}_3 = G_{33}x_3 + \sum_{i=1}^{N} H_{33}x_3(t-\tau_i) + s + (D_3 - E_3E_2^{-1}D_2)\xi \tag{8}$$

$$v = G_{13}x_3 + \sum_{i=1}^{N} H_{13}x_3(t-\tau_i) + (D_1 - E_1E_2^{-1}D_2)\xi \tag{9}$$

where

$$s = G_{31}x_1 + G_{32}x_2 + \sum_{i=1}^{N} H_{31}x_1(t-\tau_i) + \sum_{i=1}^{N} H_{32}x_2(t-\tau_i) + E_3E_2^{-1}\dot{x}_2 + J_3u \tag{10}$$

$$v = \dot{x}_1 - E_1E_2^{-1}\dot{x}_2 - G_{11}x_1 - G_{12}x_2 - \sum_{i=1}^{N} H_{11}x_1(t-\tau_i) - \sum_{i=1}^{N} H_{12}x_2(t-\tau_i) - J_1u \tag{11}$$

Since the dynamical system represented by (8) and (9) is driven by known input s and uncertain term $(D_3 - E_3E_2^{-1}D_2)\xi$, its state can be estimated with a robust observer:

$$\dot{\hat{x}}_3 = G_{33}\hat{x}_3 + \sum_{i=1}^{N} H_{33}\hat{x}_3(t-\tau_i) + s + K(v - G_{13}\hat{x}_3 - \sum_{i=1}^{N} H_{13}\hat{x}_3(t-\tau_i)) \tag{12}$$

where K is the observer's gain. Substituting v and s of (10) and (11) into (12) yields

$$\dot{\hat{x}}_3 = (G_{33} - KG_{13})\hat{x}_3 + \sum_{i=1}^{N}(H_{33} - KH_{13})\hat{x}_3(t-\tau_i)$$
$$+ (G_{31} - KG_{11})x_1 + (G_{32} - KG_{12})x_2$$
$$+ \sum_{i=1}^{N}(H_{31} - KH_{11})x_1(t-\tau_i) + \sum_{i=1}^{N}(H_{32} - KH_{12})x_2(t-\tau_i)$$
$$+ (E_3E_2^{-1} - KE_1E_2^{-1})\dot{x}_2 + K\dot{x}_1 + (J_3 - KJ_1)u \quad (13)$$

Note that the above observer uses the derivative of the outputs that is not available for direct measurement. To solve this problem a new variable w is defined as follows, in order to eliminate the need for differentiating the output:

$$w = \hat{x}_3 - [(E_3E_2^{-1} - KE_1E_2^{-1})x_2 + Kx_1] \quad (14)$$

then (13) can be expressed in the following form:

$$\dot{w} = (G_{33} - KG_{13})w + \sum_{i=1}^{N}(H_{33} - KH_{13})\hat{w}(t-\tau_i)$$
$$+ (J_3 - KJ_1)u + [G_{31} - KG_{11} + K(G_{33} - KG_{13})]x_1$$
$$+ [G_{32} - KG_{12} + (G_{33} - KG_{13})(E_3E_2^{-1} - KE_1E_2^{-1})]x_2$$
$$+ \sum_{i=1}^{N}[H_{31} - KH_{11} + K(H_{33} - KH_{13})]x_1(t-\tau_i)$$
$$+ \sum_{i=1}^{N}\begin{bmatrix} H_{32} - KH_{12} \\ + (H_{33} - KH_{13})(E_3E_2^{-1} - KE_1E_2^{-1})\end{bmatrix} x_2(t-\tau_i) \quad (15)$$

In which case the error dynamics of $e_3 = x_3 - \hat{x}_3$ is

$$\dot{e}_3 = (G_{33} - KG_{13})e_3 + \sum_{i=1}^{N}(H_{33} - KH_{13})e_3(t-\tau_i)$$
$$+ (D_3 - E_3E_2^{-1}D_2)\xi + K(D_1 - E_1E_2^{-1}D_2)\xi \quad (16)$$

Assumption 3: The matrix H_{13} is of full row rank.

It is note that Assumption 3 does not lose any generality from the structure of (5).

The following theorem establishes conditions under which the observer's error dynamics (16) converges to a bounded set.

Theorem 1: If the pair (G_{33}, G_{13}) is observable and Assumptions 1 and 2 hold, then system (8) can be estimated by (14) and (15). Furthermore,

$$\|e_3\| \le \frac{2(\|K\|\alpha + \gamma)[\lambda_{\max}(P)]^2}{\lambda_{\min}(Q)\lambda_{\min}(P)} = \delta \quad (17)$$

for all $t \ge 0$, where $\lambda_{\max}(\cdot)$ and $\lambda_{\min}(\cdot)$ denote the maximum and minimum eigenvalues of a symmetric matrix respectively, and P is the positive definite solution of

$$(G_{33} - KG_{13})^T P + P(G_{33} - KG_{13}) + \varepsilon I_n$$
$$+ P\left[\sum_{i=1}^{N}\frac{1}{\varepsilon_i}(H_{33} - KH_{13})(H_{33} - KH_{13})^T\right]P = -Q \quad (18)$$

where Q is positive definite matrix, $\varepsilon_i (i = 1, \cdots, N)$ are positive constants and $\varepsilon = \sum_{i=1}^{N}\varepsilon_i$. Moreover, the observer's gain:

$$K = P^{-1}(G_{13}^T + P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{13}^T)(\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{13}H_{13}^T)^{-1} \quad (19)$$

In fact, according Assumption 3, $(H_{13}H_{13}^T)^{-1}$ exists, spreading (18), and squaring interrelated K items, it can be obtained,

$$[PK - (G_{13}^T + P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{13}^T)(\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{13}H_{13}^T)^{-1}]$$
$$(\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{13}H_{13}^T)[PK - (G_{13}^T + P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{13}^T)(\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{13}H_{13}^T)^{-1}]^T$$
$$+ G_{33}^T P + PG_{33} + P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{33}^T P + \varepsilon I_n - (G_{13}^T +$$
$$P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{13}^T)(\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{13}H_{13}^T)^{-1}(G_{13}^T + P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{13}^T)^T = -Q \quad (20)$$

By choosing K as in (19), it can be obtained:

$$G_{33}^T P + PG_{33} + P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{33}^T P + \varepsilon I_n - (G_{13}^T +$$
$$P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{13}^T)(\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{13}H_{13}^T)^{-1}(G_{13}^T + P\sum_{i=1}^{N}\frac{1}{\varepsilon_i}H_{33}H_{13}^T)^T = -Q \quad (21)$$

For a given positive definite matrix Q, by solving (21), we can obtain the positive definite solution P, and then K is determined.

Proof: Consider the Lyapunov function

$$V = e_3^T P e_3 + \sum_{i=1}^{N}\varepsilon_i \int_{-\tau_i}^{t} e_3(s)^T e_3(s)ds \quad (22)$$

From error dynamics (16), we have that

$$\dot{V} = e_3^T[(G_{33} - KG_{13})^T P + P(G_{33} - KG_{13})]e_3$$
$$+ 2\sum_{i=1}^{N} e_3^T P(H_{33} - KH_{13})e_3(t-\tau_i)$$
$$+ 2e_3 P[K(D_1 - E_1E_2^{-1}D_2)\xi + (D_3 - E_3E_2^{-1}D_2)\xi]$$
$$+ \sum_{i=1}^{N}\varepsilon_i e_3^T e_3 - \sum_{i=1}^{N}\varepsilon_i e_3(t-\tau_i)^T e_3(t-\tau_i) \quad (23)$$

It is easy to show that

$$2\sum_{i=1}^{N} e_3^T P(H_{33} - KH_{13})e_3(t-\tau_i) \le$$
$$\sum_{i=1}^{N}\left[\begin{matrix}\frac{1}{\varepsilon_i}e_3^T P(H_{33} - KH_{13})(H_{33} - KH_{13})^T Pe_3 \\ + \varepsilon_i e_3(t-\tau_i)^T e_3(t-\tau_i)\end{matrix}\right] \quad (24)$$

According to (18) and Assumption 2, it can be obtained that

$$\dot{V} \le -e_3^T Q e_3 + 2(\|K\|\alpha + \gamma)e_3 P \le -\lambda_{\min}(Q)\|e_3\|^2 + 2\alpha\|P\|\|e_3\| \quad (25)$$

It can be seen that V have the maximum value when

$$\|e_3\| = \frac{2(\|K\|\alpha + \gamma)\|P\|}{\lambda_{\min}(Q)}$$

On the other hand, define $\Xi = \frac{1}{2}\sum_{i=1}^{N}\varepsilon_i \int_{-\tau_i} e_3(s)^T e_3(s)ds$, from (22), we have that

$$\lambda_{\min}(P)\|e_3\|^2 + \Xi \le V \le \lambda_{\max}(P)\|e_3\|^2 + \Xi \tag{26}$$

We can further obtain that

$$\lambda_{\min}(P)\|e_3\|^2 + \Xi \le V \le \frac{4(\|K\|\alpha + \gamma)^2[\lambda_{\max}(P)]^4}{\lambda_{\min}(P)[\lambda_{\min}(Q)]^2} + \Xi \tag{27}$$

Thus relation (17) holds. This completes the proof.

Lemma1 (theorem 2.4 in [13]): Let $A < 0$ and $Q > 0$, then the minimal and maximal eigenvalues of the solution matrix of (28), satisfy:

$$\frac{\lambda_{\min}(Q)}{|\lambda_{\min}(A + A^T)|} \le \lambda_{\min}(P) \le \frac{\lambda_{\max}(Q)}{2|\min\{\mathrm{Re}\{\lambda(A)\}\}|}$$

$$\frac{\lambda_{\min}(Q)}{2|\max\{\mathrm{Re}\{\lambda(A)\}\}|} \le \lambda_{\max}(P) \le \frac{\lambda_{\max}(Q)}{|\lambda_{\max}(A + A^T)|}$$

valid when $\lambda_{\max}(A + A^T) < 0$

$$A^T P + PA = -Q \tag{28}$$

Remark 2: Let

$$G = G_{33} - KG_{13}$$

$$\overline{Q} = \varepsilon I_n + P\left[\sum_{i=1}^{N}\frac{1}{\varepsilon_i}(H_{33} - KH_{13})(H_{33} - KH_{13})^T\right]P + Q$$

from (17) and (18) and lemma 1, we have

$$\|e_3\| \le \frac{2(\|K\|\alpha + \gamma)}{|\lambda_{\max}(G + G^T)|}\left\{\frac{[\lambda_{\max}(\overline{Q})]^2}{\lambda_{\min}(\overline{Q})\lambda_{\max}(Q)}\right\}$$

Because the pair (G_{33}, G_{13}) is observable, $|\lambda_{\max}(G + G^T)|$ can in theory be made arbitrarily large if an appropriate observer gain K is chosen. Hence, the above estimate error e_3 can be made arbitrarily small. Of course, because of sensor noise in practice, this observer gain K cannot be too large either.

II. Fault Detection and Isolation

In this section, a simple approach for detecting and isolating the actuator fault is proposed to estimate the magnitude of the actuator fault f_a.

Theorem 2: Suppose that the magnitude of the actuator fault signal satisfies

$$\|f_a\| > \left\|E_2^{-1}A_{23} + E_2^{-1}\sum_{i=1}^{N}A_{23}^{(i)}\right\|\delta + \|E_2^{-1}\|\beta \tag{29}$$

where δ is as in relation (17), α, β, γ is as in Assumption 2, and A_{23} is the third block column of A_2. Then actuator faults can be detected.

Proof: The second row within matrix (5) is written as

$$\begin{aligned}\dot{x}_2 &= A_{21}x_1 + A_{22}x_2 + A_{23}x_3 + B_2u + E_2f_a + D_2\xi \\ &+ \sum_{i=1}^{N}A_{21}^{(i)}x_1(t-\tau_i) + \sum_{i=1}^{N}A_{22}^{(i)}x_2(t-\tau_i) + \sum_{i=1}^{N}A_{23}^{(i)}x_3(t-\tau_i)\end{aligned} \tag{30}$$

Discretization of this system yields

$$\begin{aligned}\delta E_2 f_a(k) &= x_2(k+1) - x_2(k) - \delta[A_{21}x_1(k) + A_{22}x_2(k) \\ &+ A_{23}x_3(k) + \sum_{i=1}^{N}A_{21}^{(i)}x_1(k-\Theta_i) + \sum_{i=1}^{N}A_{22}^{(i)}x_2(k-\Theta_i) \\ &+ \sum_{i=1}^{N}A_{23}^{(i)}x_3(k-\Theta_i) + B_2u(k)] + \delta D_2\xi\end{aligned} \tag{31}$$

where k represents the kth time step, and δ is the sampling period satisfying $\tau_i = \Theta_i\delta$, Θ_i is positive integer, $i = 1, \cdots N$. Assuming that no fault occurs during the initial transient of the observer and using the estimation $\hat{x}_3(k)$ for $x_3(k)$, we can approximate the actuator fault as

$$\begin{aligned}\hat{f}_a(k) &= E_2^{-1}\{\frac{x_2(k+1) - x_2(k)}{\delta} - [A_{21}x_1(k) + A_{22}x_2(k) \\ &+ A_{23}\hat{x}_3(k) + \sum_{i=1}^{N}A_{21}^{(i)}x_1(k-\Theta_i) + \sum_{i=1}^{N}A_{22}^{(i)}x_2(k-\Theta_i) \\ &+ \sum_{i=1}^{N}A_{23}^{(i)}\hat{x}_3(k-\Theta_i) + B_2u(k)]\}\end{aligned} \tag{32}$$

with no uncertainty, the actuator fault estimate $\hat{f}_a$ would have a zero norm of there is no actuator fault and a nonzero norm if an actuator fault occurs, where x_1, x_2 are linear combination of y, and thus are known signals.The estimate error of the actuator fault is given by

$$\begin{aligned}\tilde{f}_a(k) &= \hat{f}_a(k) - f_a(k) \\ &= -E_2^{-1}A_{23}e_3(k) - E_2^{-1}\sum_{i=1}^{N}A_{23}^{(i)}\hat{e}_3(k-\Theta_i) + E_2^{-1}D_2\xi\end{aligned} \tag{33}$$

which, according to (17) and Assumption 2, is bounded by

$$\|\tilde{f}_a(k)\| \le \left\|E_2^{-1}A_{23} + E_2^{-1}\sum_{i=1}^{N}A_{23}^{(i)}\right\|\delta + \|E_2^{-1}\|\beta \tag{34}$$

If this norm bound of the fault estimate error is taken as the threshold, actuator fault detection can be achieved if condition (29) holds. In addition, the fault can be isolate by checking the nonzero entry or orientation of $\hat{f}_a$.

This completes the proof.

A similar approach as above can be employed for detection and isolation for sensor faults. To account for the effect of sensor failures, consider writing the output equation in system (1b) as

$$y = \overline{C}\overline{x} + E_s f_s \tag{35a}$$

where matrix $E_s \in R^{p\times m_s}$, and $f_s \in R^{m_s}$ represents the vector of sensor failures which is completely unknown.

Consider now representing the sensor fault vector f_s as the output of the following dynamical system [14]:

$$\dot{f}_s = A_s f_s + u_s \quad (35b)$$

where u_s is unknown and A_s is Hurwitz. Augmenting (1) and (35) we get

$$\begin{bmatrix} \dot{\overline{x}} \\ \dot{f}_s \end{bmatrix} = \underline{A}\begin{bmatrix} \overline{x} \\ f_s \end{bmatrix} + \sum_{i=1}^{N} \underline{A_i}\begin{bmatrix} \overline{x}(t-\tau_i) \\ f_s(t-\tau_i) \end{bmatrix} + \underline{B}u + \underline{E}\begin{bmatrix} f_a \\ u_s \end{bmatrix} + \overline{D}\xi \quad (36a)$$

$$y = \underline{C}\begin{bmatrix} \overline{x} \\ f_s \end{bmatrix} \quad (36b)$$

where

$$\underline{A} = \begin{bmatrix} \overline{A} & 0 \\ 0 & A_s \end{bmatrix} \quad \underline{A_i} = \begin{bmatrix} \overline{A}_i & 0 \\ 0 & 0 \end{bmatrix}$$

$$\underline{B} = \begin{bmatrix} \overline{B} \\ 0 \end{bmatrix} \quad \underline{E} = \begin{bmatrix} \overline{E} & 0 \\ 0 & I \end{bmatrix} \quad \underline{C} = \begin{bmatrix} \overline{C} & E_s \end{bmatrix}$$

Note that the above system is again in the form of (1). In the design of the observer for system in (1), it was assumed that $\rho(\overline{CE}) = m$, In terms of the system in (36), this condition would translate into the requirement that $\rho[\overline{CE} \quad E_s] = m + m_s$, on the other hand, if $\rho[\overline{CE} \quad E_s] \neq m + m_s$, then the observer for this system cannot be designed.

Theorem 3: Consider system (36), with E_s in (35), if $\rho[\overline{CE} \quad E_s] = m + m_s$, system (36) can be transformed to the form of (2). In addition, for the newly transformed system, if Assumption 2 and conditions in theorem 1 are satisfied, then we can design observer for system (36) and detection and isolate both the actuator and sensor faults.

Proof: similar to the proof of theorem 1and theorem 2, omit it.

III. EXAMPLE

In this section, we will illustrate the applicability of the aforementioned techniques on a couple of numerical examples. First example illustrates how actuator fault detection and diagnosis can be accomplished in a time delay systems, whereas the second example illustrates that it is possible to perform both actuator and sensor fault detection and diagnosis with slight modifications as described in the above.

***Example 1**:* For simplicity, consider a three-order system:

$$\dot{x} = \begin{bmatrix} -2 & 1.5 & 1 \\ -0.5 & -1 & 1 \\ 3 & 0.5 & -3 \end{bmatrix} x + \begin{bmatrix} -1 & 2 & 0.2 \\ 0 & -1 & 0 \\ 0 & 0.5 & -0.6 \end{bmatrix} x(t-\tau)$$

$$+ \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0.5 \\ 1 \\ -0.6 \end{bmatrix} f_a + \begin{bmatrix} 0.7 \\ 0.5 \\ 0.6 \end{bmatrix} \xi$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \end{bmatrix} x$$

$\xi = 0.1 \times randN(0,\ 1)$ where $randN(0,\ 1)$ denotes the gaussian white noise with zero mean value and unity standard deviation.

By selecting

$$M = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

We have that

$$A = M\overline{A}M^{-1} = \begin{bmatrix} -2 & 6 & 1 \\ -0.125 & -1 & 0.25 \\ 3 & 2 & -3 \end{bmatrix}$$

$$A_i = M\overline{A}_i M^{-1} = \begin{bmatrix} -1 & 8 & 0.2 \\ 0 & -1 & 0 \\ 0 & 2 & -0.6 \end{bmatrix}$$

$$B = M\overline{B} = [1 \quad 0.25 \quad 1]^T$$

$$D = M\overline{D} = [0.7 \quad 0.125 \quad 0.6]^T \quad E = M\overline{E} = [0.5 \quad 0.25 \quad -0.6]^T$$

$$G_1 = A_1 - E_1 E_2^{-1} A_2 = [-1.75 \quad 8 \quad 0.5]$$

$$G_3 = A_3 - E_1 E_2^{-1} A_2 = [2.7 \quad -0.4 \quad -2.4]$$

$$H_1 = A_1^{(1)} - E_1 E_2^{-1} A_2^{(1)} = [-1 \quad 10 \quad 0.2]$$

$$H_3 = A_3^{(1)} - E_1 E_2^{-1} A_2^{(1)} = [0 \quad -0.4 \quad -0.6]$$

$$\left\|(D_1 - E_1 E_2^{-1} D_2)\xi\right\| \le \alpha, \quad \left\|D_2 \xi\right\| \le \beta, \quad \left\|(D_3 - E_3 E_2^{-1} D_2)\xi\right\| \le \gamma$$

$$\left\|(D_1 - E_1 E_2^{-1} D_2)\xi\right\| \le \alpha = 0.045 \quad \left\|D_2 \xi\right\| \le \beta = 0.0125$$

$$\left\|(D_3 - E_3 E_2^{-1} D_2)\xi\right\| \le \gamma = 0.09$$

Thus (G_{33}, G_{13}) is observerable, and all the assumptions in theorem 1 hold, according theorem 1, Select $Q = 10.4, \varepsilon = 1$, by solving (21), we obtain $P = 3$, and according (19), $K = 2$, it can be further obtained that:

$$\|e_3\| \le \frac{2(\|K\|\alpha + \gamma)[\lambda_{\max}(P)]^2}{\lambda_{\min}(Q)\lambda_{\min}(P)} = \delta = 0.10$$

$$\|f\| > \left\| E_2^{-1} A_{23} + E_2^{-1} \sum_{i=1}^{N} A_{23}^{(i)} \right\| \delta + \left\| E_2^{-1} \right\| \beta = 0.15$$

So the threshold is set to 0.15. Fig. 1 illustrate the estimates of actuator faults 1 as follow:

$$f_a = \begin{cases} 0 & t \le 2 \\ 0.6 & t > 2 \end{cases}$$

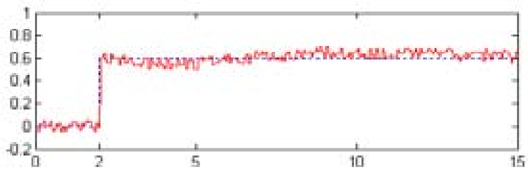

Fig. 1 f_a fault detection and diagnosis

It can be seen that actuator fault can be detected and isolated in a desired manner.

***Example 2**:* In this example, we consider a time delay system where it is desired to detect its actuator fault as well as one of its sensor faults. The system is described by

$$\dot{x} = \begin{bmatrix} -2 & 1.5 & 1 \\ -0.5 & -1 & 1 \\ 3 & 0.5 & -3 \end{bmatrix} x + \begin{bmatrix} -1 & 2 & 0.2 \\ 0 & -1 & 0 \\ 0 & 0.5 & -0.6 \end{bmatrix} x(t-\tau) + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0.5 \\ 1 \\ -0.6 \end{bmatrix} f_a + \begin{bmatrix} 0.2 \\ 0.5 \\ 0.1 \end{bmatrix} \xi$$

$$y = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 0.25 & 0 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} f_s$$

The value of A_s in (32) was set to zero in this example; the augmented system was assembled and is given by

$$\begin{bmatrix} \dot{x} \\ \dot{f}_s \end{bmatrix} = \begin{bmatrix} -2 & 1.5 & 1 & 0 \\ -0.5 & -1 & 1 & 0 \\ 3 & 0.5 & -3 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x \\ f_s \end{bmatrix} + \begin{bmatrix} -1 & 2 & 0.2 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0.5 & -0.6 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x(t-\tau) \\ f_s(t-\tau) \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix} u + \begin{bmatrix} 0.5 & 0 \\ 1 & 0 \\ -0.6 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} f_a \\ u_s \end{bmatrix} + \begin{bmatrix} 0.2 \\ 0.5 \\ 0.1 \\ 0 \end{bmatrix} \xi$$

$$y = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0.25 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ f_s \end{bmatrix}$$

The detail calculation similar to example 1, omit it. It was assumed that the sensor fails at $t = 2$ and this is followed by the actuator failure that occurs at $t = 12$. Simulation results in Fig. 2 clearly indicate that an accurate estimate of both failures was obtained, and therefore the faults can be detected with no difficulty.

$$f_s = \begin{cases} 0 & t \le 2 \\ 0.8 + 0.1\sin(0.4t) & 2 < t \le 12 \end{cases}, \quad f_a = \begin{cases} 0 & t \le 12 \\ 1.2 & t > 12 \end{cases}$$

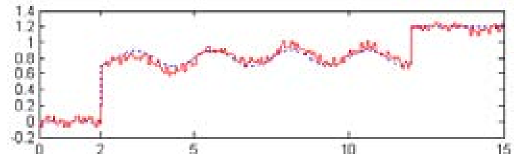

Fig. 2 sensor and actuator fault detection and diagnosis

IV. CONCLUSION

This paper presents a novel approach for actuator fault and sensor fault diagnosis of time-delay systems. A simple approach for designing observer for time-delay systems has been presented. The observer design approach was extended for fault detection and diagnosis of both sensors and actuators. The fault detection and isolation approach that was proposed in this paper uses a robust observer to detect and identify actuator faults and sensors faults with certain accuracy under bounded uncertainties. Simulated examples are included to demonstrate the applicability of the proposed method and encouraging results are obtained

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