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Filter-Based FDD Using PDFs for Stochastic Systems with Time Delays

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Abstract—**In this paper, a fault detection and diagnosis (FDD) scheme is presented for general stochastic dynamic systems with time delays. Originated from some practical processes, it is supposed that the measured information is the probability density function (PDF) of the system output rather than its actual value, which is different from the classical FDD problem. A B-spline expansion technique is applied so that the output PDF can be formulated in terms of the dynamic weights of the B-spline expansion. This leads to a dynamic model between the input and the weights where nonlinearities, uncertainties and time delays are included. As a result, the concerned FDD problem is transformed into a classic nonlinear FDD problem subject to an uncertain nonlinear system with time delays. Feasible criteria to detect the system fault are obtained and an adaptive fault diagnosis method is further presented to produce an estimate of the size of fault. Simple simulations are given to demonstrate the efficiency of the proposed approach.**

I. INTRODUCTION

TO improve reliability of control systems, the fault detection and diagnosis (FDD) has long been regarded as an important and integrated part in modern control systems (see [1,6,9,14] for surveys). For stochastic systems, in the past two decades many significant approaches have been presented and applied in practical processes successfully (see [1,9,12-14] and references therein). In general, the effective results can be classified into three groups. The first group is the filter or observer based approaches, where filters are used to generate residual signals to detect and estimate the fault. It is shown that in the (extended) Kalman filtering results, Gaussian variables are concerned where the mean and/or variance of the estimation error are minimized [2,12]. The second group is the identification based FDD scheme for unexpected changes, where the identification technique was applied to estimate the model parameter changes of systems [9]. The third group is the statistic approach for stochastic parameter changes and fault signals, where Bayesian theorem and likelihood methods can be used to evaluate the faulty parameters (see *e.g.* [1,10]). In [13], a feasible statistic approach was obtained to detect the unexpected changes in the mean value and variance where the static statistic behavior was concerned.

Among the above results, for dynamic stochastic systems, the filter-based FDD approach has been shown as an effective way where generally the variables are supposed to be Gaussian [2,11,12,14]. However, as shown in [7,19], in many processes of chemical and manufacture engineering, the involved stochastic variables are of the non-Gaussian type. Actually, even for stochastic systems with Gaussian inputs, nonlinearities in the system may lead to non-Gaussian outputs where mean and variance may be insufficient to characterize the probability distribution precisely. This means that the classical filtering theory for Gaussian systems may be incomplete in filter-based FDD approaches for general stochastic systems. Unfortunately, generally the formulation of the probability distributions for the non-Gaussian output requires the solutions of partial differential equations. The formulation of the corresponding control and optimization problem for the output probability density functions (PDFs) is difficult to proceed (see *e.g.* [7,19,20]). This is particularly true for system whose models link the input directly with the output PDFs. To simplify the representations of the distribution of dynamical stochastic systems, B-spline expansions were widely used to model the output PDFs ([7,19]). On the other hand, along with the developments of advanced instruments, data collections and imagination processing techniques, in practice the measurement for filter design can be stochastic distributions of the system output, rather than just the output values. This is also different from the traditional filtering or filter-based FDD results where some new methods may be developed to use the stochastic properties to construct the filter.

For such a new group of filter-based FDD design for non-Gaussian stochastic systems, in [20], a simple filter-based FDD research framework has been established. This work motivated from the retention system of paper making processes, where the residual of the filter is constructed using the measured output PDFs rather than the system output itself. However, in [20] the weight dynamical system was supposed to be a precise linear model and the design algorithm required the realizable and detectable

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conditions that were hard to verify. Practically uncertainties, nonlinearities and especially time delays exist in the above modeling procedures by using B-spline expansions. Actually, it is noted that a linear mapping between the input and output cannot change the shape of input PDFs ([7,8]). Recently, a systematic framework has been provided in [8] for the stochastic plants without time delays, where the stability and convergence of the error systems could be guaranteed.

In this paper we develop an improved filter-based FDD approach for the stochastic processes with time delays. Firstly, different from the precise square root B-spline model (see [7,19,20]), the modeling error should be introduced with respect to the square root B-spine model, which leads to a coupled nonlinear relationship among the dependent and independent weights as well as the error (see also [8]). Secondly, since linear mappings will not change the shape of input PDFs, nonlinearity has to be formulated in the weighting systems. On the other hand, time delays as inherent characteristics of many physical systems (especially, chemical processes and paper making processes) should also be considered in such a model. After the B-spline approximation with modeling error is formulated, the weighting system can be established by a nonlinear system with modeling uncertainties and time delays. As such, the concerned problem for dynamic non-Gaussian systems can be transferred to a special nonlinear FDD problem for deterministic systems. Several feasible approaches can then be given in terms of linear matrix inequalities (LMIs) to detect and diagnose the faults by using of the measured output PDFs.

The remainder of this paper is organized as follows: In Section 2, B-spline expansions are adopted to model the output PDFs and an uncertain nonlinear time delayed system is applied to formulate the weighting dynamics. The fault detection problem is investigated in Section 3 where several criteria are obtained to construct the filter and the residual signal. Furthermore, an adaptive fault diagnosis filtering approach is provided in Section 4. Simulations are given in Section 5 to show the effectiveness of the proposed methods.

II. Formulation of the FDD problem

A. Output PDF model using B-spline expansions

Consider a dynamic stochastic system with the input $u(t) \in R^m$ and the output $y(t) \in [a,b]$. Denote F as the fault vector to be detected and diagnosed. The conditional output PDF is denoted by $\gamma(z,u(t),F)$. In this paper the square root B-spline model with an approximation error $\omega_0(z,u(t),F)$ will be adopted as follows (see also [8])

$$\sqrt{\gamma(z,u(t),F)} = B^T(z)V(t) + h(V(t))b_n(z) + \omega(z,u(t),F) \tag{1}$$

In (1), $b_i(z)(i=1,2,\cdots,n)$ are pre-specified basis functions on $[a,b]$, and $v_i(u(t),F)$ $(i=1,2,\cdots,n)$ are the corresponding weights of such an expansion. Other notations can be seen in [8]. It can be also supposed that $\left|\omega(z,u(t),F)\right| \le \delta$ for all $\{z,u(t),F\}$, where δ is a known positive constant. For the nonlinear function $h(V(t))$ denoted in (1), it is assumed that Lipschitz condition can be satisfied within its operation region, *i.e.* for any $V_1(t)$ and $V_2(t)$, there exists a known matrix U_1 satisfying

$$\left\|h(V_1(t)) - h(V_2(t))\right\| \le \left\|U_1(V_1(t) - V_2(t))\right\| \tag{2}$$

B. Nonlinear dynamic weight model with time delays

As shown in [8,19], in many practical cases the dynamic relationship between the input and the output PDFs can be established between the control input and the weights of the B-spline approximation to the output PDFs. Due to existence of time delays in many processes, we consider the following weighting model

$$\begin{cases} \dot{x}(t) &= Ax(t) + A_d x(t-d) + Gg(x(t)) \\ &\quad + Hu(t) + H_d u(t-d) + JF \\ V(t) &= Ex(t) \end{cases} \tag{3}$$

where $x(t) \in R^m$ is the state, F is the fault to be detected and diagnosed (i.e., the system is healthy when *F = 0.*), *A*, A_d, *G*, *H*, H_d , *J* and *E* represent the identified (known) parametric matrices of the weight system. Time delay represented by d is a positive constant. It is also assumed that for a known matrix U_2 , $g(x(t))$ is a nonlinear function satisfying the following norm condition

$$\left\|g(x_1(t)) - g(x_2(t))\right\| \le \left\|U_2(x_1(t) - x_2(t))\right\| \tag{4}$$

for any $x_2(t)$, $x_1(t)$. With (3), output equation (1) can be rewritten as

$$\sqrt{\gamma(z,u(t),F)} = B(z)Ex(t) + h(Ex(t))b_n(z) + \omega(z,u(t),F) \tag{5}$$

Equations (3) and (5) constitute a mathematical representation for the concerned stochastic systems. This model reveals how the output PDF is related to the control input $u(t)$ and the fault vector *F* in the system. The purpose of FDD is therefore to use the available input and the output PDF to detect and diagnosis the fault.

Remark 1: The filter-based FDD problem for nonlinear dynamical deterministic systems has been shown to be a complicated research subject [4,11,14,22]. Recently, some robust FDD approaches have been developed where the solutions can be reduced to Riccati equation or LMI-formulations [3,4,15-18,21]. In order to deal with the new FDD problems for the above transformed systems, in the following several new criteria will be provided in terms of LMIs.

III. FILTER-BASED FAULT DETECTION BY USING OF OUTPUT PDFS

Since the measured information is the output probability distribution, in order to detect the fault based on the changes of output PDFs, the following nonlinear filter is considered

$$\begin{cases} \dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-d) + Gg\left(\hat{x}(t)\right) \\ +Hu(t) + H_d u(t-d) + L\varepsilon(t) \\ \varepsilon(t) = \int_a^b \sigma(z)\left(\sqrt{\gamma(z,u(t),F)} - \sqrt{\hat{\gamma}(z,u(t))}\right)dz \\ \sqrt{\hat{\gamma}(z,u(t))} = B(z)E\hat{x}(t) + h\left(E\hat{x}(t)\right)b_n(z) \end{cases} \tag{6}$$

where $\hat{x}(t)$ is the estimation of the state, $L\in R^{m\times p}$ is the filter gain to be determined. Different from the classical filtering methods, residual $\varepsilon(t)$ is formulated as an integral of the difference between the measured PDFs and the estimated ones, where $\sigma(z)\in R^{p\times 1}$ is a pre-specified weighting vector defined on $[a,b]$. In fact, $\varepsilon(t)$ can be regarded as a generalized distance or difference of two PDFs.

Define $\tilde{x}(t) = x(t) - \hat{x}(t)$, the estimation error system can be described as

$$\begin{aligned} \dot{\tilde{x}}(t) &= \left(A - L\Gamma_1\right)\tilde{x}(t) + A_d\tilde{x}(t-d) \\ &\quad +G\left[g\left(x(t)\right) - g\left(\hat{x}(t)\right)\right] \\ &\quad -L\Gamma_2\left[h\left(Ex(t)\right) - h\left(E\hat{x}(t)\right)\right] + JF - L\Delta(t) \end{aligned} \tag{7}$$

where

$$\Gamma_1 = \int_a^b \sigma(z)B(z)Edz\,,\ \Gamma_2 = \int_a^b \sigma(z)b_n(z)dz\,,$$

$$\Delta(t) = \int_a^b \sigma(z)\omega\left(z,u(t),F\right)dz \tag{8}$$

It can be seen that

$$\varepsilon(t) = \Gamma_1\tilde{x}(t) + \Gamma_2\left(h\left(Ex(t)\right) - h\left(E\hat{x}(t)\right)\right) + \Delta(t) \tag{9}$$

Since $\left|\omega\left(z,u(t),F\right)\right| \le \delta$, we have

$$\left\|\Delta(t)\right\| \le \tilde{\delta}\,,\ \tilde{\delta} = \delta\int_a^b \sigma(z)\,dz \tag{10}$$

The following result provides a design criterion to detect the fault by using the output PDFs.

Theorem 1: For the parameters $\lambda_i > 0\,(i=1,2)$, if there exist matrices $P>0$, $Q>0$, R and constant $\eta>0$ satisfying

$$\Pi = \begin{bmatrix} \Pi_1 + \eta I & PA_d & \Pi_2 \\ A_d^T P & -Q+\eta I & 0 \\ \Pi_2^T & 0 & -I \end{bmatrix} < 0 \tag{11}$$

where

$$\Pi_1 = \left(PA - R\Gamma_1\right) + \left(PA - R\Gamma_1\right)^T + Q\,,$$

$$+\frac{1}{\lambda_1^2}E^T U_1^T U_1 E + \frac{1}{\lambda_2^2}U_2^T U_2$$

and $\Pi_2 = \left[\lambda_1 R\Gamma_2 \quad \lambda_2 PG\right]$, then in the absence of F, error system (7) with gain $L = P^{-1}R$ is stable and the error satisfies

$$\left\|\tilde{x}(t)\right\| \le \alpha = \max\left\{\max_{-d\le t\le 0}\left\|\tilde{x}(t)\right\|, \eta^{-1}\tilde{\delta}\left\|R\right\|\right\} \tag{12}$$

for all $t\in[-d,\infty)$, where $\tilde{\delta}$ is denoted by (10).

Proof: Denote $\Pi_0 = \begin{bmatrix} \Pi_1 + \Pi_2\Pi_2^T & PA_d \\ A_d^T P & -Q \end{bmatrix}$. Using Schur complement, it can be shown that $\Pi < 0 \Leftrightarrow \Pi_0 + \eta I < 0$. Define a Lyapunov candidate as follows

$$\begin{aligned} &\Phi\left(t,\tilde{x}(t),x(t),\hat{x}(t)\right) \\ &= \tilde{x}^T(t)P\tilde{x}(t) + \int_{t-d}^t \tilde{x}^T(\tau)Q\tilde{x}(\tau)d\tau \\ &+\frac{1}{\lambda_1^2}\int_0^t\left[\left\|U_1E\tilde{x}(\tau)\right\|^2 - \left\|h\left(Ex(\tau)\right) - h\left(E\hat{x}(\tau)\right)\right\|^2\right]d\tau \\ &+\frac{1}{\lambda_2^2}\int_0^t\left[\left\|U_2\tilde{x}(\tau)\right\|^2 - \left\|g\left(x(\tau)\right) - g\left(\hat{x}(\tau)\right)\right\|^2\right]d\tau \end{aligned} \tag{13}$$

then in the absence of F, it can be verified that

$$\begin{aligned} &\dot{\Phi}\left(t,\tilde{x}(t),\tilde{x}(t-d),x(t),\hat{x}(t)\right) \\ &\le \begin{pmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{pmatrix}^T \Pi_0 \begin{pmatrix} \tilde{x}(t) \\ \tilde{x}(t-d) \end{pmatrix} - 2\tilde{x}^T(t)PL\Delta(t) \\ &< -\eta\left(\left\|\tilde{x}(t)\right\|^2 + \left\|\tilde{x}(t-d)\right\|^2\right) + 2\left\|\tilde{x}(t)\right\|\left\|R\right\|\tilde{\delta} := \varphi(t) \end{aligned}$$

where $R = PL$. When $\left\|\tilde{x}(t)\right\| \le \left\|\tilde{x}(t-d)\right\|$, we have $\varphi(t) \le -2\eta\left\|\tilde{x}(t)\right\|\left(\left\|\tilde{x}(t)\right\| - \eta^{-1}\tilde{\delta}\left\|R\right\|\right)$, which means that $\varphi(t) \le 0$ if $\left\|\tilde{x}(t)\right\| \ge \eta^{-1}\tilde{\delta}\left\|R\right\|$. Similarly when $\left\|\tilde{x}(t)\right\| \ge \left\|\tilde{x}(t-d)\right\|$, it can be seen that

$$\varphi(t) \le -2\eta\left\|\tilde{x}(t)\right\|\left(\left\|\tilde{x}(t-d)\right\| - \eta^{-1}\tilde{\delta}\left\|R\right\|\right)$$

and $\varphi(t) \le 0$ holds if $\left\|\tilde{x}(t-d)\right\| \ge \eta^{-1}\tilde{\delta}\left\|R\right\|$. Thus, $\dot{\Phi} < 0$ holds if $\left\|\tilde{x}(t)\right\| \ge \eta^{-1}\tilde{\delta}\left\|R\right\|$ occurs. Therefore, it can be seen that (12) always holds and the estimation error system is stable. Q.E.D

From the proof of Theorem 1, it can be seen that under the condition of Theorem 1, it can be guaranteed that system (7) is asymptotically stable in the absence of F and $\Delta(t)$. This can be summarized in the following result. This can be summarized in the following result.

Corollary 1: For the parameters $\lambda_i > 0\,(i = 1,2)$, if there exist matrices $P > 0$, $Q > 0$, R and constant $\eta > 0$ satisfying (14), then in the absence of F and $\Delta(t)$, error system (10) with gain $L = P^{-1}R$ is stable and satisfies $\left\|\tilde{x}(t)\right\| \le \max\limits_{-d \le t \le 0}\left\{\left\|\tilde{x}(t)\right\|\right\}$.

Theorem 1 provides a set of necessary conditions for fault detection. Using Theorem 1, it can be seen that when $\left\|\tilde{x}(t)\right\| > \alpha$ holds, a fault F occurs. In order to detect F, we select $\varepsilon(t)$ as the residual signal and propose the following result to determine the threshold.

Theorem 2: For the parameters $\lambda_i > 0\,(i = 1,2)$, if there exist matrices $P > 0$, $Q > 0$, R and constant $\eta > 0$ satisfying (13), then fault F can be detected by the following criterion

$$\left\|\varepsilon(t)\right\| > \beta = \alpha\left(\left\|\Gamma_1\right\| + \left\|\Gamma_2\right\|\left\|U_1\right\|\left\|E\right\|\right) + \tilde{\delta} \tag{14}$$

where α is determined by (12).

Proof: From (12) and based on Theorem 1, when $F = 0$, it is easy to see that

$$\begin{aligned}\left\|\varepsilon(t)\right\| &\le \left\|\Gamma_1\tilde{x}(t)\right\| + \left\|\Gamma_2\right\|\left\|h(Ex(t)) - h(E\hat{x}(t))\right\| + \left\|\Delta(t)\right\| \\ &\le \left\|\Gamma_1\right\|\left\|\tilde{x}(t)\right\| + \left\|\Gamma_2\right\|\left\|U_1\right\|\left\|E\right\|\left\|\tilde{x}(t)\right\| + \left\|\Delta(t)\right\| \\ &\le \alpha\left(\left\|\Gamma_1\right\| + \left\|\Gamma_2\right\|\left\|U_1\right\|\left\|E\right\|\right) + \tilde{\delta} = \beta\end{aligned}$$

Hence, it is reasonable to conclude that fault occurs when $\left\|\varepsilon(t)\right\| > \beta$. Q.E.D

IV. FILTER-BASED FAULT DIAGNOSIS BY USING OF OUTPUT PDFS

Once the fault is detected, its diagnosis should follow where the input and the output PDFs are to be used to give a good estimation of the fault. Denote $\hat{F}(t)$ is the estimation of F. $\Upsilon_1(>0)$ and Υ_2 are learning operators to be determined by the proposed diagnostic method. For this purpose, the following adaptive observer is constructed

$$\begin{cases}\dot{\hat{x}}(t) = A\hat{x}(t) + A_d\hat{x}(t-d) + Gg\left(\hat{x}(t)\right) \\ +Hu(t) + H_du(t-d) + L\varepsilon(t) + J\hat{F}(t) \\ \dot{\hat{F}}(t) = -\Upsilon_1\hat{F}(t) + \Upsilon_2\varepsilon(t) \\ \varepsilon(t) = \int_a^b \sigma(z)\left(\sqrt{\gamma(z,u(t),F)} - \sqrt{\hat{\gamma}(z,u(t))}\right)dz \\ \sqrt{\hat{\gamma}(z,u(t))} = B(z)E\hat{x}(t) + h\left(E\hat{x}(t)\right)b_n(z)\end{cases} \tag{15}$$

By using $\tilde{x}(t) = x(t) - \hat{x}(t)$ and $\tilde{F}(t) = F - \hat{F}(t)$, the estimation error system can be formulated to give

$$\begin{aligned}\dot{\tilde{x}}(t) &= \left(A - L\Gamma_1\right)\tilde{x}(t) + A_d\tilde{x}(t-d) \\ &+G\left[g\left(x(t)\right) - g\left(\hat{x}(t)\right)\right] + J\tilde{F}(t) - L\Delta(t) \\ &-L\Gamma_2\left[h\left(Ex(t)\right) - h\left(E\hat{x}(t)\right)\right]\end{aligned} \tag{16}$$

For simplified notations, we suppose that $x(0)$ is known and $\tilde{x}(0)$ can be assumed to be 0 or sufficiently small. Similarly to some previous FDD approaches [20], it is assumed that $\left\|F(t)\right\| \le \frac{M}{2}$, $\left\|\hat{F}(t)\right\| \le \frac{M}{2}$ and consequently $\left\|\tilde{F}(t)\right\| \le M$. Thus, the following result provides a new design method for the fault diagnosis problem based on LMIs.

Theorem 3: For the parameters $\lambda_i > 0\,(i = 1,2)$, if there exist matrices $P > 0$, $Q > 0$, R and constants $\mu > 0, \kappa > 0, \theta > 0$ satisfying

$$\Psi = \begin{bmatrix}\Psi_{11} + \mu I & \Psi_{21}^T & \Psi_{31}^T \\ \Psi_{21} & -2\Upsilon_1 + \kappa I & 0 \\ \Psi_{31} & 0 & -I\end{bmatrix} < 0 \tag{17}$$

where

$$\Psi_{11} := \begin{bmatrix} \Pi_1 & PA_d & -R\Gamma_2 & PG \\ A_d^T P & -Q & 0 & 0 \\ -\Gamma_2^T R^T & 0 & -\lambda_1^{-2} I & 0 \\ G^T P & 0 & 0 & -\lambda_2^{-2} I \end{bmatrix}$$

$$\Psi_{21} = \begin{bmatrix} J^T P - \Upsilon_2 \Gamma_1 & 0 & -\Upsilon_2 \Gamma_2 & 0 \end{bmatrix}$$

$$\Psi_{31} = \begin{bmatrix} \theta R^T & 0 & 0 & 0 \end{bmatrix}$$

then under diagnostic filter (15) with gain $L = P^{-1}R$, error system (16) is stable and satisfies

$$\min\{\mu, \kappa\} \left\| \begin{bmatrix} \tilde{x}(t) & \tilde{F}(t) \end{bmatrix} \right\|^2 \leq \theta^{-2} \tilde{\delta}^2 + 2\|M\| \|\Upsilon_2\| \tilde{\delta} + \|\Upsilon_1\| M^2 \quad (18)$$

for all $t \in [0, \infty)$, where $\tilde{\delta}$ is denoted by (10).

Proof: Define the following Lyapunov function $\Theta = \Phi + \tilde{F}^T(t) \tilde{F}(t)$, where $\Phi\left(t, \tilde{x}(t), x(t), \hat{x}(t)\right)$ is denoted by (13). Along with the trajectories of (16), it can be obtained that

$$\dot{\Theta} \leq -\mu \|\tilde{x}(t)\|^2 - \kappa \|\tilde{F}(t)\|^2 + \theta^{-2} \tilde{\delta}^2 + 2M \Upsilon_2 \tilde{\delta} + \Upsilon_1 M^2$$

Thus, when $\tilde{x}(0)$ is sufficiently small, $\dot{\Theta} < 0$ holds if

$$\min\{\mu, \kappa\} \left\| \begin{bmatrix} \tilde{x}^T(t) & \tilde{F}^T(t) \end{bmatrix} \right\|^2 \geq \theta^{-2} \tilde{\delta}^2 + 2M \Upsilon_2 \tilde{\delta} + \Upsilon_1 M^2$$

holds. This completes the required proof. Q.E.D

Remark 2: From Theorem 3, it is shown that under the proposed diagnosis filter, the estimation error for F is bounded by

$$\|\tilde{F}(t)\|^2 \leq \kappa^{-1} \left(\theta^{-2} \tilde{\delta}^2 + 2M \Upsilon_2 \tilde{\delta} + \Upsilon_1 M^2 \right) \quad (19)$$

Several parameters can be also selected such that the bound is small enough. Moreover, when modeling error is neglected, inequality (19) turns to be $\|\tilde{F}(t)\|^2 \leq \kappa^{-1} \|\Upsilon_1\| M^2$.

V. Simulation Examples

Suppose these output PDFs can be approximated using square root B-spline models described by (1) with $n = 3$, $z \in [0, 1.5]$ and ($i = 1, 2, 3$)

$$b_i(z) = \begin{cases} |\sin 2\pi z|, z \in [0.5(i-1); 0.5i] \\ 0, \text{others} \end{cases} \quad (23)$$

It is assumed the identified weighting system is formulated by (6) with the following coefficient matrices

$$A = \begin{bmatrix} -2.2 & 0 \\ 0 & -2 \end{bmatrix}, \; D = \begin{bmatrix} -0.1 & 0 \\ 0 & -0.5 \end{bmatrix}, \; H = \begin{bmatrix} 1 & 0 \\ 0 & -2 \end{bmatrix},$$

$$G = \begin{bmatrix} 0 & 1 \\ 0 & 1 \end{bmatrix}, \; E = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}, \; J = \begin{bmatrix} 0.9 & 0.9 \end{bmatrix}^T.$$

The bound of uncertainties is denoted by $U_1 = diag\{0.1, 0.1\}$ and $U_2 = \begin{bmatrix} 0 & 0 \\ 0 & 0.5 \end{bmatrix}$. In the simulations, it is selected that $x(0) = \begin{bmatrix} 0.1 + \exp(t-5) & -0.1 + \exp(t-5) \end{bmatrix}^T$ $\left(t \in [-0.5, 0]\right)$, $\hat{x}(0) = \begin{bmatrix} 0 & 0 \end{bmatrix}^T$ $\left(t \in [-0.5, 0]\right)$, and $\lambda_1 = \lambda_2 = 1$. $\Delta(t)$ is denoted by (10), where $\omega\left(z, u(t), F\right)$ is selected as a random input within $[-0.008, 0.008]$. Fault is supposed as $F = 0.32$ when $t = 10s$. Using Theorems 1 and 2, the residual signal should satisfy $|\varepsilon(t)| > \beta = 0.1043$ where β is the threshold determined by (14). With the obtained detection filter, Figure 1 shows that the threshold and the norm of residual. Based on Theorem 3, a diagnosis filter can be obtained. Figure 2 shows that the fault can be well tracked by using the proposed fault diagnosis filter.

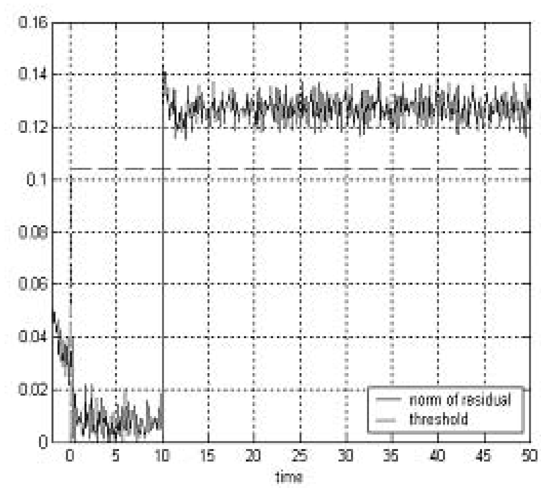

Figure 1: Threshold and the norm of residual

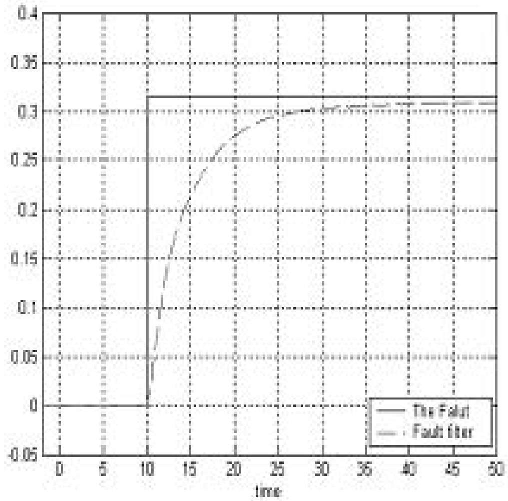

Figure 2: Responses of the fault and the its estimation

VI. Conclusion

Following the results in [8], in this paper a new FDD framework is further investigated for stochastic dynamic systems with time delays. Different from the classical FDD problem, the on-line measurement output is the stochastic behaviors (represented by distribution of system output) rather than the value of output signals and non-Gaussian stochastic variables are concerned. By using the square root B-spline approximations and weighting modeling, the concerned FDD problem can be transformed into a nonlinear FDD problem subject to the nonlinear time delayed system, where both the state equation and the measurement equation are nonlinear, and are subjected to modeling errors and time delays. Based on LMI techniques and by using the augmented auxiliary vectors, some new criteria are obtained to detect the system fault with a threshold. Furthermore, an adaptive fault diagnosis method is provided to estimate the size of the fault.

VII. Acknowledgment

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References

[1] Basseville M. Nikiforov I. *Detection of Abrupt Changes*: *Theory & Applications*. Prentice-Hall: Englewood Cli!s, NJ, 1993.

[2] Chen R. H., Mingori D. L., and Speyer J. L., Optimal stochastic fault detection filter. *Automatica*, 39:377–390, 2003.

[3] Chen W., Saif M., Fault Detection and accommodation in nonlinear time-delay sytems. *Proceedings of the ACC*, Denver, Colorado, 4255-4260, 2003.

[4] Ding S. X., Zhang P. and Frank P. M., Threshold calculation using LMI-technique and its integration in the design of fault detection systems. *Proceedings of the 42nd IEEE CDC,* Manui, Hawaii USA, 469-474, 2003.

[5] Ding S. X., Jeinsch T., Frank P. M., Ding E. L., A unified approach to the optimization of fault detection systems. *International Journal of Adaptive Control and Signal Processing*, Vol. 14: 725-745, 2000.

[6] Frank P. M., Ding S. X., Survey of robust residual generation and evaluation methods in observer-based fault detection systems. *Journal of Process Control,* 7(6): 403-424,1997.

[7] Guo L., Wang H., Applying Constrained Nonlinear Generalized PI Strategy to PDF Tracking Control through Square Root B-Spline Models. *Int. J . Control*, Vol. 77(17):1481-1492, 2004

[8] Guo L. and Wang H,. Fault Detection and Diagnosis for General Stochastic Systems Using B-Spline Expansions and Nonlinear Observers. in *Proceedings Of 43rd IEEE CDC 2004*, USA, 2004

[9] Isermann R., Balle P., Trends in the application of model based fault detection and diagnosis of technical process. in *Proceedings of the IFAC World Congress*, N, 1-12, 1996.

[10] Kárný M., Nagy I, Novovicová J., Mixed-data multi-modelling for fault detection and isolation. *International Journal of Adaptive Control and Signal Processing*, Vol.16: 61-83, 2002.

[11] Liu J., Wang J. L. and Yang G. H. Reliable Guaranteed Variance Filtering Against Sensor Failures. *IEEE Trans. on Signal Processing*, Vol 51(5): 1403-1411. 2003.

[12] Mufeed M., Jiang J., Zhang Y. M., Stochastic stability analysis of fault–tolerant control systems in the presence of noise. *IEEE Trans. on Automatic Control,* 46(11): 1810-1813, 2001.

[13] Nikiforv I., Staroswiecki M., and Vozel B., Duality of analytical redundancy and statistical approach in fault diagnosis. *Proceedings of the IFAC World Congress*, N, 19-24, 1996

[14] Patton R. J., Chen J. *Control and dynamic systems: Robust fault detection and isolation (FDI) systems*. London: Academic Press, 1996, 74: 171-224.

[15] Rambeaux F., Hamelin F. and Sauter D., Optimal thresholding for robust fault detection of uncertain systems *Int. J. Robust Nonlinear Control,* 10:1155-1173, 2000.

[16] Stoorvogel, A. A., H. H. Niemann, A. Saberi and P. Sannuti, Optimal fault signal estimation, *Int. J. Robust Nonlinear Control*, 12: 697-727, 2002.

[17] Stoustrup J. and H. H. Niemann, Fault estimation-a standard problem approach, *Int. J. Robust Nonlinear Control,* 12:649-673, 2002

[18] Wang H. B., Wang J. L., Liu J., and Lam J., Iterative LMI approach for robust fault detection observer design. *Proceedings of the 42nd IEEE CDC*, Manui, Hawaii USA, 1794-1799, 2003.

[19] Wang H., *Bounded Dynamic Stochastic Systems: Modelling and Control*. Springer-Verlag, London, 2000.

[20] Wang H., Lin W., Applying observer based FDI techniques to detect faults in dynamic and bounded stochastic distributions. *Int. J. Control,* 73(15): 1424-1436, 2000.

[21] Zhong M.Y., Ding S. X., J. Lam, and Zhang C.H., Fault detection filter design for LTI system with time-delay. *Proceedings of the 42nd IEEE CDC*, Manui, Hawaii, USA, 1467-1472, 2003.

[22] Zhang X., Polycarpou X. X., and Parisini T., A Robust Detection and Isolation Scheme for Abrupt and Incipient Faults in Nonlinear Systems. *IEEE Trans. on Automatic Control*, Vol. 47, 4:576-594, 2002.