2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

A Framework for Biomimetic Robot Fish's Design and Its Realization

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***Abstract*— This paper is concerned with the design and motion control of a radio-controlled, multi-link and free-swimming biomimetic robot fish based on an improved kinematic propulsive model. The performance of the robot fish is determined by the fish's both morphological parameters and kinematic parameters. By ichthyologic theories of propulsion, a framework taking into consideration of both mechatronic constraints in physical realization and feasibility of control methods is presented, where multiple linked robot fish propelled by a flexible posterior body and an oscillating tail fin can be easily developed. The motion control of robot fish is decomposed into speed control, orientation control and submerging/ascending control. The speed of the swimming fish can be adjusted by changing oscillating frequency, oscillating amplitude and the length of oscillatory part, respectively, and its orientation is tuned by different joint's deflections. The up-down motion is realized by a pectoral mechanism. Our robot fish prototypes verify that the presented scheme is effective in design and implementation.**

I. INTRODUCTION

BIOMIMETICS, a new area of research involving in both biology and robotics, has been receiving more and more attention. Bio-mimicking systems ranging from meter-scale humanoid robots and nano-scale cell-biomimetry, provide important insights into the theories and applications of robotics. In the category of swimming robots, biomimetic robot fish is modelled after real fish in nature having the virtue of high speed, tremendous propulsive efficiency and excellent maneuverability [1]–[4]. Instead of the propeller-based propulsion used in ship or underwater vehicles, fishlike propulsion including undulation and oscillation generates the main thrust of the robot fish, which makes the fish more efficient, maneuverable and noiseless. These advantages are of great benefit to practical applications in marine and military fields such as undersea operation, military reconnaissance, leakage detection, aquatic life-form observation, and so on [5].

In recent years, most studies on robot fish focused on exploring the hydrodynamic mechanism of fishlike swimming, and constructing autonomous underwater vehicles with the efficiency and maneuverability of real fish. One famous prototype is MIT's RoboTuna developed in 1994, which initiated the research and development of robot fish. RoboTuna and subsequent RoboPike were used to study drag reduction in fishlike locomotion. Later, an improved version of RoboTuna: Vorticity Control Unmanned Undersea Vehicle (VCUUV), was constructed by Draper Laboratory of MIT [2]. The VCUUV was equipped with many different sensors, and could realize up-down motion and avoid obstacles, which allow it to be able to navigate autonomously in a 3-D workspace. In addition, California Institute of Technology, Northeastern University and University of California Berkeley carried out a series of fishlike propulsion projects in theoretic exploration and physical realization. In Japan, Tokai University examined the use of specific kinds of pectoral fin motions to maneuver a robotic Blackbass [6], Kagawa University developed a novel type of underwater micro biped robot with ionic conducting polymer film (ICPF) actuator [7], and the National Maritime Research Institute (NMRI) of Japan built a variety of robot fish prototypes designed to realize efficient propulsion and autonomous control [8].

However, in previous research, few small robot fish prototypes can realize up-down motion in 3-D workspace, and the corresponding motion control methods are even fewer. And yet, for future applications, autonomous motion control is surely necessary for robot fish. The objective of this paper is to design and build a radio-controlled, multi-link and free-swimming biomimetic robot fish, and to develop basic motion control methods for speed control and orientation control. The technical novelty of this paper lies in an improved approach to design a robot fish based on a simplified kinematics model in which its speed and orientation are separated, where geometric reduction is employed and complex hydrodynamic analysis is avoided. The rest of the paper is organized as follows. In section II, an overall design framework is described in detail. Several prototypes of robot fish are developed in section III. Basic motion control methods on speed control, orientation control and submerging/ascending control are presented

This work was supported by National Natural Science Foundation of China (10372002) and China Postdoctoral Science Foundation.

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respectively in section IV. Finally, section V summarizes the research progress and future work.

II. DESIGN FRAMEWORK BASED ON A IMPROVED PROPULSIVE MODEL

At present, biologically inspired artificial systems mimicking real fish mostly concentrate on fishlike motion mechanism. Before building such a biomimetic fish system, it is necessary to abstract and simplify the swimming motion of real fish. Combining morphological character with swimming feature, both morphological and kinematic parameters can then be qualitatively abstracted. Based on the above characteristics, a feasible mathematical model is finally established, which provides an engineering guide to design, implementation, control and optimization of the robot fish.

A. Ichthyologic Basis

In ichthyology, BCF (body and/or caudal fin) swimming movement is usually categorized into anguilliform, subcarangiform, carangiform and thunniform mode basically according to the wavelength and the amplitude envelope of the propulsive wave underlying fish's behavior [5]. Based on this observation, there are some alternative ways to design a robot fish. Recent studies on the robot fish, with a synthetical consideration of theoretic exploration and mechatronic implementation, primarily concentrate on the anguilliform swimming mode and the carangiform swimming mode. During the anguilliform locomotion, the whole body participates in large amplitude undulations, which is common in eel and lamprey. In the carangiform locomotion, the body's undulations are entirely confined to the last 1/3 part of the body, and thrust is produced by means of a rather stiff caudal fin. Compared to anguilliform swimmers, carangiform swimmers are generally faster, but with less agility due to the relative rigidity of their bodies. In this paper, only carangiform swimming is chosen as the model of robot fish.

Based on the biological information of carangoid, as shown in Fig. 1, a physical model of the carangiform motion can be divided into two parts: flexible body and oscillatory lunate caudal fin, where the flexible body is represented by a series of oscillatory hinge joints and the caudal fin by an oscillating foil. A swimming model for RoboTuna (typically carangiform) has been presented by Barrett *et al.* [9], whose undulatory motion is assumed to take the form of a traveling wave (1) originally suggested by Lighthill [10].

$$y_{body}(x,t) = [(c_1 x + c_2 x^2)][\sin(kx + \omega t)] \qquad (1)$$

where y_{body} represents the transverse displacement of the fish body, x denotes the displacement along the main axis, k indicates the body wave number ($k = 2\pi/\lambda$), λ is the body wave length, c_1 is the linear wave amplitude envelope, c_2 is the quadratic wave amplitude envelope, and ω is the body wave frequency ($\omega = 2\pi f = 2\pi/T$).

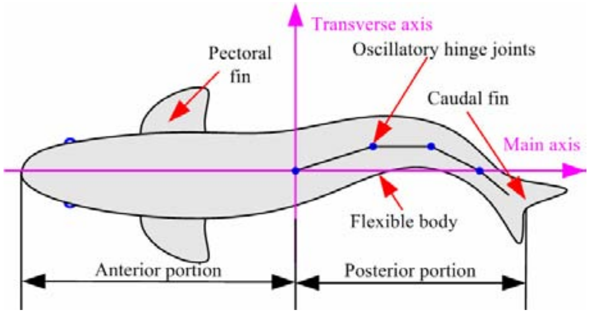


Fig. 1. Physical model of carangiform swimming.

In light of the biological, hydrodynamic, ichthyologic and engineering information collected in the literature [5], [9], [11], morphological parameters related to swimming motion mainly consist of:

1) The length ratio of the fish's oscillatory part to that of the fish-body (R_l). Depending on the different value of R_l, fish swimming can be classified as carangiform, anguilliform, thunniform and ostractiform. In a general way, with the decrease of R_l, efficiency and velocity of fish swimming remarkably increase, but maneuverability reduces to a certain extent.

2) The number of simplified joints in oscillatory part (N). The larger the value of N, in general, the better the mechanism's maneuverability and redundancy, but the worse the swimming efficiency. Yet N can't be too large in view of the construction and size constraints of mechatronic systems as well as large cumulative error in man-made mechanical approximation.

3) The length ratio of each link in oscillatory part ($l_1 : l_2 : \ldots : l_N$). In the part where l_i (i=1,2,…,N) is relatively short, density of joint is high, and flexibility of produced motion is large so that large-amplitude oscillation can be produed. The length ratio of each link in the direction from the nose to the tail of fish, as a general rule, is getting smaller and smaller. However, oscillatory amplitude increases gradually and reaches its maximum at tail peduncle of fish.

4) Characteristics of caudal fin. The aspect ratio (AR) of the caudal fin plays an important role in propulsive efficiency, which is defined as the fin span b squared, divided by the projected fin area s_c, i.e., $AR=b^2/s_c$. High AR caudal fin results in improved efficiency in that it induces less drag per unit of lift or thrust produced. At the same time, the shape of the caudal fin makes a great difference to fish's propulsion. A crescent or forked caudal fin will usually lend itself to high-speed swimming.

The kinematic parameters of fish swimming are those fundamental physical quantities describing swimming performance and propulsive efficiency, which primarily include [1], [12]:

1) The form of traveling body wave equation $y_{body}(x,t)$, which represents the transient movement of body-spline, determining the form of fish swimming.

2) Transverse oscillatory amplitude of each joint in body. In nature, many fish typically utilize large-amplitude and high-frequency tail oscillation for starting, accelerating and turning, while using small-amplitude and low-frequency oscillation for long-distance cruising or foraging. For carangoid, maximum transverse amplitude of caudal oscillation is generally not more than one tenth of its body length.

3) Angle of attack in caudal fin. Amount of dynamic lift during swimming is determined by angle of attack and speed of propulsion.

After briefly reviewing the key elements of fish swimming, the next step we will take is designing a bio-mimetic fish in a comprehensive way.

B. Design Framework of Robot Fish

A schematic diagram describing the design of a robot fish is shown in Fig. 2. Construction/structure determines its functions. As surveys conducted by zoologists show, it is common for high-speed aquatic animal to be in the shape that the maximum width of the body is about one fourth of its body length, which can furthest reduce the form drag. A central issue in the systematic design is the comprehensive consideration of the following two questions: (a) Does the current design meet the constraints of the mechatronic system? (b) Is the adopted control method feasible? In addition, geometric configuration and mass distribution should be taken into account as well. Based on the prior preparations, through some rounds of systematic emulations and physical tests, a final robot fish design scheme that balances the stability and propulsive efficiency can be achieved. The detailed steps in robot fish's design are then illustrated as follows:

1) Abstracting morphological parameters and kinematic parameters from the geometric characteristics and swimming properties of biological fish respectively. In this step, a theoretical model of robot fish can be derived from the biological model.

2) Following the steps of "selecting hydrodynamic shape and its drag coefficients → determining thrust in light of the requirements of drag, acceleration and maneuverability → determining the shape of caudal fin and its area according to the calculated thrust" to select the appropriate kinematic parameters.

3) Modifying and resetting each parameter conditioned by both constraints of the mechatronic system and feasibility of the used control method, and return to step *2)* to design again until good hydrodynamic shape and stability are achieved.

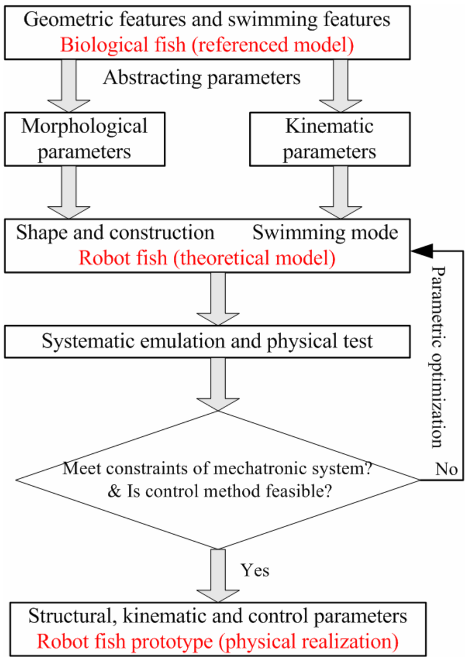


Fig. 2. An overall framework of robot fish design.

4) Conducting the geometric space configuration and mass distribution of robot fish, in terms of the requirements of hydrodynamic shape, stability and control algorithm, and developing the prototype of robot fish.

5) Optimizing the kinematic parameters based on hydrodynamic experiments on the prototype. In this case, intelligent control method can be applied to the robot's motion control so as to improve its maneuverability under complex environments. Therefore, the comprehensive performance in locomotion and agility of the fish reaches a relatively optimal level.

C. Mechanical Design Based on an Improved Carangiform Propulsive Model

The oscillatory part of a fish is composed of many rotating hinge joints, as shown in Fig. 3. In robot fish's realization, it can be modeled as a planar serial (N) chain of links in an interval of 0 to $R_l \times 2\pi$ along the axial body displacement, where R_l is defined as the length ratio of wavelength

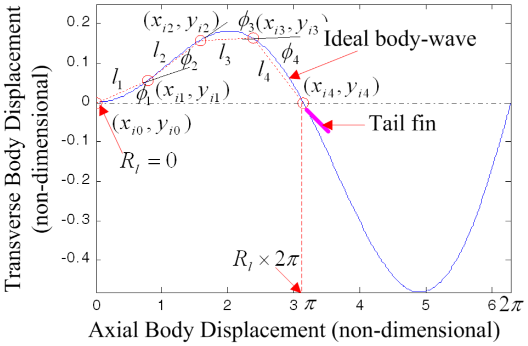


Fig. 3. Link-based body-wave fitting. Note that a 4-link mechanism is used as the model of body-wave fitting.

exhibited by the fish's oscillatory part to that of a whole sinusoid wave.

Before conducting link-based body-wave fitting, for simplicity, a discrete planar spline curve parameterized as sinusoid is employed, i.e., time variable t is separated from the body-wave function $y_{body}(x,t)$. Namely, the traveling body-wave is decomposed into two parts: (a) the time-independent spline curve sequences $y_{body}(x,i)$ (i =0,1,…, M–1) in an oscillation period, which is described by (2), and (b) the time-dependent oscillating frequency f, which is described as the times of recurring oscillations in an unit time interval.

$$y_{body}(x,i) = [(c_1 x + c_2 x^2)][\sin(kx \pm \tfrac{2\pi}{M} i)] \qquad (2)$$

where i denotes the i-th variable of the spline curve sequence $y_{body}(x,i)$, M is defined as body-wave resolution that represents the discrete degree of the overall traveling wave, which is restricted by the maximum oscillating frequency of the actuators. It should be noticed that the "+" sign or the "–" one has the same effect on the sequence $y_{body}(x,i)$ in an overall oscillation period, but their initial moving direction is different due to different initial values.

Since the wavelength of a whole propulsive wave is non-dimensionally viewed as 2π at k=1 ($\lambda = 2\pi / k = 2\pi / 1 = 2\pi$), the wavelength of the oscillatory part at R_l is then $R_l \times 2\pi$. Let the length of each link be l_j (j=1,2,…,N), the corresponding end-point coordinate pairs are (x_{j-1},y_{j-1}) and (x_j,y_j), the ratio of the links' length is $l_1 : l_2 : \ldots : l_N = m[l'_1 : l'_2 : \ldots : l'_N]$, where m denotes length factor, l'_j indicates the normalized length of the j-th link, and especially l'_1 equals 1. Then once the amplitude coefficients (i.e., c_1 and c_2) and k are determined, the shape of the propulsive wave at some time will be determined. Mathematically, the j-th link's joint angle ϕ_{ij} at the i-th (i=0,1,…,M–1) time can be calculated by fitting the current curve. The next task is to search the appropriate joint angle ϕ_{ij} to meet the requirement that the end-point of link l_j falls into the wavy curve and the x-coordinate of the last link's endpoint (x_{iN},y_{iN}) just equals $R_l \times 2\pi$. That is, it must satisfy the following constraint:

$$\begin{cases} (x_{i,j} - x_{i,j-1})^2 + (y_{i,j} - y_{i,j-1})^2 = {l'_j}^2 \\ y_{i,j} = (c_1 x_{ij} + c_2 {x_{ij}}^2) \sin(k x_{ij} - \frac{2\pi}{M} i) \end{cases} \qquad (3)$$

where the subscript i indicates the i-th time of the oscillating sequence, and j denotes the j-th link. Through a series of analytical iterative operations, the end-point coordinate pair $(x_{i,j}, y_{i,j})$ can be calculated. Then the slope of each link l'_j at arbitrary i-th time can be computed. Finally, as illustrated in (4), a two-dimensional rectangular array OscData[M][N] for the joint angle ϕ_{ij} is obtained, which will be used as the primitive oscillating data of the robot fish. Based on this oscillatory array, the fish body's shape can geometrically be changed by adding different deflections $\Delta\phi_i$ to each joint, the corresponding oscillatory array OscData'[M][N] is shown in (5). Especially, some deflections may be zero as necessary in practice.

$$\text{OscData}[M][N] = \begin{pmatrix} \phi_{01} & \phi_{02} & \cdots & \phi_{0N} \\ \phi_{11} & \phi_{12} & \cdots & \phi_{1N} \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{M-1,1} & \phi_{M-1,2} & \cdots & \phi_{M-1,N} \end{pmatrix} \qquad (4)$$

$$\text{OscData}'[M][N] = \begin{pmatrix} \phi_{01} + \Delta\phi_1 & \phi_{02} + \Delta\phi_2 & \cdots & \phi_{0,N} + \Delta\phi_N \\ \phi_{11} + \Delta\phi_1 & \phi_{12} + \Delta\phi_2 & \cdots & \phi_{1,N} + \Delta\phi_N \\ \cdots & \cdots & \cdots & \cdots \\ \phi_{M-1,1} + \Delta\phi_1 & \phi_{M-1,2} + \Delta\phi_2 & \cdots & \phi_{M-1,N} + \Delta\phi_N \end{pmatrix} \qquad (5)$$

To mimic fish's body wave more effectively and expand the range of different swimming as well as applicability of the curve, based on its intrinsic characteristics, an improved body wave is given in (6), where the quadratic gain of body wave number k_2 relating to axial displacement x is introduced. According to results from numerical simulation, by adjusting the value of k_1 and k_2, more flexibility and maneuverability can be achieved in orientation control of fish swimming. Moreover, the swimming performance may be partly optimized by tuning the body wave number k_1 and k_2.

$$y_{body}(x,t) = [(c_1 x + c_2 x^2)][\sin((k_1 + k_2 x)x + \omega t)] \qquad (6)$$

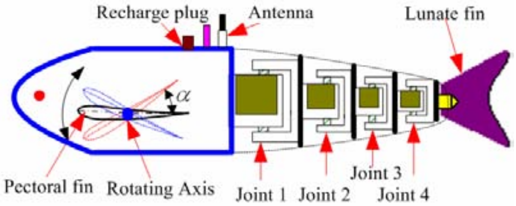
Observing that the body of a fish may not be moving in the direction that the head is pointing, we next assume that the controllability of the fish relies on the internal shape (the joint angle ϕ_{ij}) for maneuverability and the oscillating frequency f of the tail for speed. As mentioned before, the eventual results for the propulsive wave fitting according to the given parameters are a 2-D $M \times N$ rectangular array of joint angles and the oscillating frequency, which are independent of the fish's dimension and shape. Therefore, a parameters set $\{\phi_{i1}, \phi_{i2}, \ldots, \phi_{iN}, f\}$ is chosen to control the fish's motion.

III. DEVELOPMENT OF ROBOT FISH

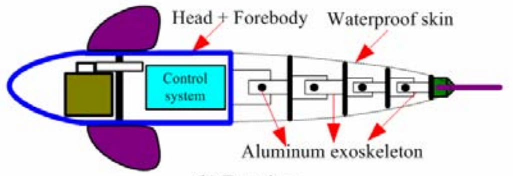
Based on the above simplified kinematic model of propulsive mechanism, a radio-controlled, multi-link and free-swimming biomimetic robot fish mimicking carangiform-like locomotion has been designed. The mechanical configuration of an up-down motioned robot fish is shown in Fig. 4 and its drive and control architecture in Fig. 5. Fig. 6 shows that it swims downwards in a swimming pool. It can swim realistically like a fish in the water. Fig. 7 exhibits five types of robot fish prototypes developed in our laboratory. These robot fishes primarily consist of six parts:

- Control unit (microprocessor + peripherals)
- Communication unit (wireless receiver)

- Sensor unit (infrared, visual, ultrasonic, etc.)
- Support unit (aluminum exoskeleton + head + forebody)
- Actuator unit (DC servomotors)
- Accessories (battery, waterproofed skin, tail fin, etc.)



(a) Front view



(b) Top view

Fig. 4. Mechanical configuration of an up-down-motioned robot fish.

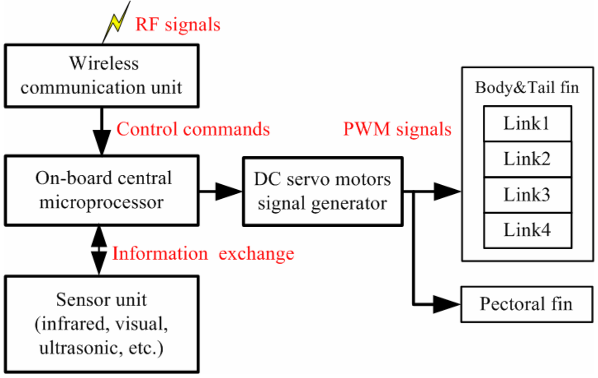


Fig. 5. The architecture of actuator and control system.

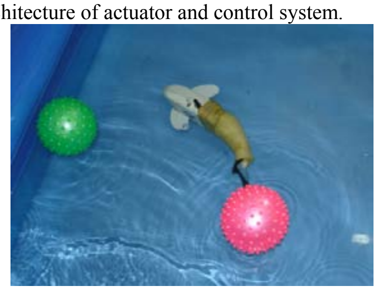


Fig. 6. Photo of swimming downwards in a swimming pool.

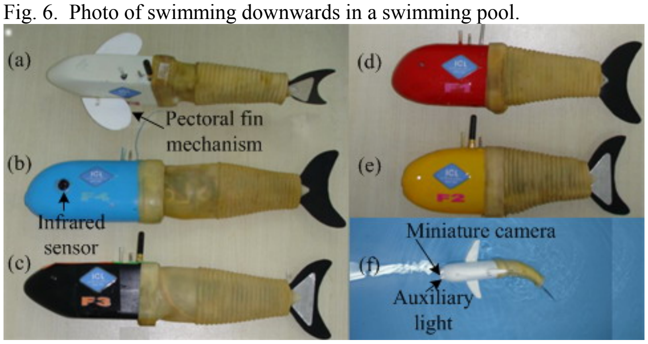


Fig. 7. Prototypes of different robot fishes. Note: (a) Up-down motioned, 3-link robot fish, 380mm long; (b) 4-link robot fish with infrared sensors, 450mm long; (c) 4-link, 2-D motioned robot fish, 400mm long; (d) 3-link, 2-D motioned robot fish, 380mm long; (e) 2-link robot fish with infrared sensors, 280mm long; (f) 3-link robot fish with miniature wireless camera, 400 mm long.

In the robot fish's construction, based on a hydrodynamic analysis, hollow, streamlined, rigid head and forebody are molded using fiberglass, which allows for larger space to house electrical and communication components. In order to swim in a small experimental water pond, the built fish has to be as compact as possible. The onboard microprocessor, sensors, additional peripherals, wireless receiver and power supply are hence put in the cell of the fish's forebody. DC servomotors acted as the actuator of joints are linked with aluminum exoskeleton. A lunate foil is attached to the last link, which serves as the tail fin. The installation position of the joints and the size of the tail are designed with consideration of the shape of a specific biological fish "model". In the meantime, some steel balance weights are located in the forebody and lower side of the exoskeleton to adjust the balance of gravitational forces and buoyant forces.

In the fish's control unit, the servomotors are controlled by an onboard microprocessor and a Pulse-Width Modulation (PWM) signal generator. The speed of fish's straight motion is adjusted by modulating the joint's oscillating frequency, and its orientation is tuned by different joint's deflection. Adding various deflections, plus or minus, to the front joint angles (e.g., ϕ_{i1} and ϕ_{i2}) in each oscillation period, different motion direction is achieved. By the manual mode (remote controller) or the automatic control mode, the robot fish can swim forward, accelerate, decelerate, turn right or left, or even submerge or ascend like a real fish. The detailed motion control method will be illustrated in section IV.

IV. BASIC MOTION CONTROL OF ROBOT FISH

Since the controllability of the fish relies on the internal shape (the joint angle ϕ_{ij}) for maneuverability and the oscillating frequency f of the tail for speed, as mentioned in section II, the robot fish's motion control in a 2-D plane is then decomposed into the speed control and the orientation control. For an up-down-motioned robot fish, in particular, submerging/ascending control has to be implemented in a 3-D workspace.

A. Speed Control

It is observed that fish in nature uses a combination of frequency and amplitude for speed control. Oscillating frequency f, oscillatory amplitude of the posterior body and length of the oscillatory part, are utilized in our method to achieve different swimming speeds.

1) Speed control based on oscillating frequency f

Substituted ω for $2\pi f$, another form of body-wave equation is easily derived from (1):

$$y_{body}(x,t)=[(c_1x+c_2x^2)][\sin(kx+2\pi ft)] \qquad (7)$$

As a general tendency, the swimming speed increases with the oscillating f, and f will approximate a constant when the desired speed is achieved.

2) Speed control based on oscillatory amplitude

A second order amplitude envelop ($y_{amplitudeenvelop} = [(c_1 x + c_2 x^2)]$) is chosen to produce different body-waves by different values of c_1 and c_2. In practice, oscillatory-amplitude-based speed control method adjusts the transverse movement at a constant oscillating frequency. Thrust is hence changed and so did the swimming speed.

3) Speed control based on length of the oscillatory part

Since the length ratio of the fish's oscillatory part to that of the fish body R_l is an important morphological parameter, and not all the body or the oscillatory portion takes part in thrust production at all time, it may be a feasible way to employ different length of the oscillatory part at various speeds. For multi-link robot fish, this method can be easily implemented by locking or unlocking some links.

B. Orientation Control

As shown in (5), the internal shape of propulsive mechanism can geometrically be changed by adding different deflections to each joint, and the robot fish therefore changes dynamically its heading due to different reactions of surrounding water to dynamic body shape. Accordingly, different turning modes are able to be implemented by adding various deflections in each oscillation period, plus or minus, small or large, to part of or all links. In particular, some transient motions such as rapid starting, turning, and stopping, can be realized by adding some specific deflections to the links.

C. Submerging/Ascending Control

In an up-down-motioned robot fish using pectoral fin mechanism shown in Fig. 4, the magnitude and direction of the lift can be adjusted by changing the attack angle of the pectoral fins α , so 3-D locomotion can be realized by controlling body & caudal fin and pectoral fins movement harmoniously. Similar to the DC servomotors used in link's actuator unit, a discrete control method is employed to actuate the pectoral fin. As shown in (8), a pectoral-data array including different rotating angles is served as the basis of up-down control.

$$Pec\_Data[2K+1] = [\alpha_{dk} \quad \cdots \quad \alpha_{d1} \quad \alpha_m \quad \alpha_{u1} \quad \cdots \quad \alpha_{uk}] \tag{8}$$

where α_m is the absolute rotating angle of the actuator at median position, and α_{di} (i=1,2,…,k) is the absolute rotating angle of the actuator when the pectoral fins move downwards. Analogously, α_{ui} (i=1,2,…,k) corresponds to upward motion. The relationship between absolute rotating angle of the actuator α and the speed of submerge can be experimentally determined.

V. CONCLUSIONS AND FUTURE WORK

This paper described an overall design procedure for a radio-controlled, multi-link and free-swimming biomimetic robot fish based on an improved kinematic propulsive model. Within a systematic framework taking account of both mechatronic constraints and hydrodynamic characteristics, the detailed design method was proposed. The basic motion control laws for speed control, orientation control and submerging/ascending control were then presented. Several kinds of robot fish prototypes with different functions were designed and built to validate the presented method.

Future research should be focused on self-contained robot fish's design and optimization combining kinematics and hydrodynamics. In the meantime, human-machine interaction-based autonomous robot fish using advanced sensor technology and intelligent control techniques will also be developed.

ACKNOWLEDGMENTS

The authors would like to express appreciation to Mr. Yimin Fang, Mr. Ruifeng Fan, and Mr. Yonghui Hu for their daily technical assistance in our laboratory.

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