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Optimal Control of Discrete-Time Linear Systems with Network-Induced Varying Delay

Hiroyuki Hirano, Masakazu Mukai, Takehito Azuma, Masayuki Fujita

***Abstract*— This paper deals with an optimal control problem for networked control systems, that have network induced time delay in the communication networks. In our proposed approach, a linear quadratic controller for the systems with random time delays in delta domain is proposed. The optimal controller is derived in delta domain by using a dynamic programming approach. The derived optimal controller in delta domain can be represented as a linear delay-depending feedback from the state and the previous control signal. Finally a numerical example is shown to illustrate the effect of the proposed controller.**

I. INTRODUCTION

Recent technological advances have enabled control systems to be implemented via networks. Such a networked control system (NCS) [1], [2] consists of numerous physical and computational elements or agents, which have physical and informational interactions and dependencies, supported by overlapping network resources. In recent years, networked control systems have emerged as a topic of major interest. Furthermore, the experimental testbeds for networked control systems are presented in [3], [4]. Many real-time systems are implemented as distributed control systems, where the control loops are closed over a communication network or a field bus. The stability problems of networked control systems are considered in [2], [5], [6], [7], packet loss and droped out mesurement problems in [8], [9], [10], and limited communication channel and rare problems are studied in [11], [12], [13], [14], [15]. On the other hand, there will be network-induced varying delay in the communication networks. Some networks such as CAN and Ethernet involve with varying delays [16]. Linear quadratic control of systems with random network delays is studied in [17], [18]. The state estimation problems are also considered in [17], [18]. On the other hand, high-speed digital processing systems are of increasing importance in modern systems applications. However, most traditional digital control and algorithms are inherently ill-conditioned when applied to data taken at sampling periods that are high relative to the dynamics of the underlying continuous time processes being sampled. Furthermore, fast sampling period can cause numerical problems with poles aggregating near 1. The delta operator is known that it provides a practical solution to this problem [19], [20], [21].

This paper proposes optimal control of linear systems with long random time delays in delta domain. In this paper, networked control systems are discretized with delta operator. By using the delta operator representation, a dynamic programming approach is investigated and discussed to derive an optimal controller for the linear systems with quadratic cost. The methodology for random time delays is based on a stochastic description of the variations of the delays. It is assumed that the time delays are statistically mutually independent. The derived optimal controller in delta domain can be represented as a linear delay-depending feedback from the state and the previous control signal. The controller in [22] is extended to a controller with the delays that are longer than one sample period. Further, model predictive control with a terminal constraint can be considered to deal with stability as the extention of this work.

The reminder of this paper is organized as follows. Section II describes a system representation of networked control systems with delta operator and the optimal control problem is formulated. In Section III, we derive the optimal controller by using dynamic programming approach. Then the control algorithm is also considered. Section IV shows an illustrative example of the simulation result to illustrate the effect of the proposed controller. Section V describes a conclusion of our research.

II. PROBLEM FORMULATION

A. Networked Control Systems

In this paper, networked control Systems illustrated in Fig. 1 is considered.

The communication delay between sensor and controller is represented as τ_k^{sc}, between controller and actuator τ_k^{ca}. They are randomly varying. All time delays are independent over the full horizon and their probability distributions

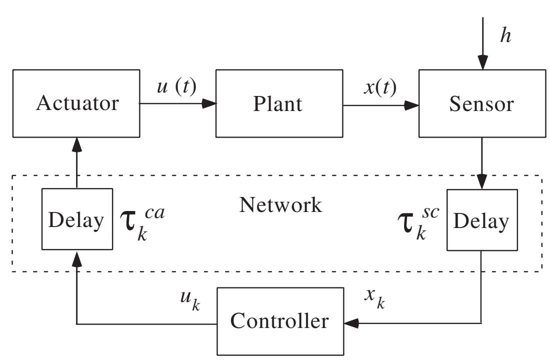

Fig. 1. Networked control systems with delays

H. Hirano, M. Mukai, T. Azuma, and M. Fujita are with Dep. of Electrical and Electronic Engineering, Kanazawa University, 2-40-20 Kodatsuno, Kanazawa 920-8667, Japan fujita@t.kanazawa-u.ac.jp

are known a priori [17], [18]. The sampling period that is positive value is denoted as h. The length of the past time delays are known to the controller. The plant to be controlled is represented as the form

$$\dot{x}(t) = Ax(t) + Bu(t) + B_w w(t), \quad (1)$$

where $x \in \mathbb{R}^m$ is the state, $u \in \mathbb{R}^p$ the input and w is white noise with unit incremental variance R_w.

B. Discretization

The timing of signals in the control system is illustrated in Fig. 2. Fig. 2 shows delays τ_k^{sc}, τ_k^{ca}. Time step is denoted by t_k and discretized state is denoted by x_k.

It is assumed that the total time delay satisfies the following condition (2),

$$\tau_k^{sc} + \tau_k^{ca} < Zh, \quad (2)$$

where Z is a positive integer [18]. Using this assumption, the total time delay that is longer than the sampling period can be considered.

The linear process with the zero-order-hold actuator on the input, periodically sampled

$$x_{k+1} = \tilde{A}x_k + \tilde{\Gamma}(\tau_k)\begin{bmatrix} u_{k-Z} \\ \vdots \\ u_k \end{bmatrix} + \tilde{B}_w w_k \quad (3)$$

where

$$\begin{aligned} \tilde{A} &= e^{Ah}, \ \ \tilde{B}_w = \int_0^h e^{As} ds B_w, & (4) \\ \tau_k &= \{\, \tau_{k-Z}^{sc} \quad \tau_{k-Z}^{ca} \quad \cdots \quad \tau_k^{sc} \quad \tau_k^{ca} \,\} & (5) \\ \tilde{\Gamma}(\tau_k) &= \sum_{i=0}^{q_k} \Phi(t_{k+1} - t_k^{i+1}, t_{k+1} - t_k^i, I_i + Z - k) & (6) \end{aligned}$$

[18]. If the integer Z is set as 1, the system is equivalent to the system in [17].

C. Delta Operator Systems

In this section, a system representation with delta operator [20] for the discrete time system (3) is proposed. By using the delta operator representation the system is calculatable even if the sampling period h is small. From the relation of delta operator, the system (3) can be transformed into

$$\begin{aligned} \frac{\Delta x_k}{\Delta h} &= \frac{x_{k+1} - x_k}{h} \\ &= A^* x_k + \Gamma(\tau_k)\begin{bmatrix} u_{k-Z} \\ \vdots \\ u_k \end{bmatrix} + B_w^* w_k, \quad (7) \end{aligned}$$

where

$$A^* = \frac{1}{h}(\tilde{A} - I), \ \Gamma(\tau_k) = \frac{1}{h}\tilde{\Gamma}(\tau_k), \ B_w^* = \frac{1}{h}\tilde{B}_w. \quad (8)$$

Introducing a vector ν_k

$$\nu_k = \left[\, u_{k-Z+1}^T \quad \cdots \quad u_{k-1}^T \,\right]^T, \quad (9)$$

the system (7) is written as follows

$$\begin{aligned} \frac{\Delta x_k}{\Delta h} &= [\, A^* \quad \Gamma_Z(\tau_k) \quad \Gamma_a(\tau_k) \,] \begin{bmatrix} x_k \\ u_{k-Z} \\ \nu_k \end{bmatrix} \\ &\quad + \Gamma_0(\tau_k) u_k + B_w^* w_k. \quad (10) \end{aligned}$$

Here

$$\Gamma_Z \in \mathbb{R}^{m \times p}, \ \Gamma_a \in \mathbb{R}^{m \times (Z-1)p}, \ \Gamma_0 \in \mathbb{R}^{m \times p}. \quad (11)$$

Notice that the matrices $[\, A^* \ \ \Gamma_Z(\tau_k) \ \ \Gamma_a(\tau_k) \,]$ and $\Gamma_0(\tau_k)$ depend on the time delay τ_k. If the integer Z is set as 1, a system representation with deltor operator of the system in [17] can be obtained [22].

When the sampling period h is very small, the matrix (4) may be a unit matrix. While by using delta operator numerical calculation becomes possible that implies (8) is not a unit matrix. A numerical example is considered to show the dependence of matrices to the delay and information that the controller can use.

Example 1: The system is represented as follows (12)

$$\dot{x} = \begin{bmatrix} 0 & 1 \\ 0.2 & -0.1 \end{bmatrix} x + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u + \begin{bmatrix} 0 \\ 10 \end{bmatrix} w, \quad (12)$$

where w is white noise with average is zero. Set the sampling period $h = 0.5$ s and the delays satisfies the condition $\tau^{sc} + \tau^{ca} < 2h \ \ (Z = 2)$. Then the system (12) can be represented as

$$\begin{aligned} \frac{\Delta x_k}{\Delta h} &= [\, A^* \quad \Gamma_2(\tau_k) \quad \Gamma_a(\tau_k) \,] \begin{bmatrix} x_k \\ u_{k-2} \\ u_{k-1} \end{bmatrix} \\ &\quad + \Gamma_0(\tau_k) u_k + B_w^* w_k. \quad (13) \end{aligned}$$

First, the case of the timing of signal shown in Fig. 2 is considered. In time interval $[t_k, t_{k+1}]$ we can set $\{t_k^0, t_k^1, t_k^2\} = \{t_k, t_k^{act}, t_{k+1}\}$, and A^*, B_w^* is calculated as follows

$$A^* = \begin{bmatrix} 0.0494 & 0.9836 \\ 0.1967 & -0.0490 \end{bmatrix}, \ B_w^* = \begin{bmatrix} 2.4691 \\ -90.1644 \end{bmatrix}.$$

Further, Γ_2, Γ_a, and Γ_0 can be calculated as follows

$$\Gamma_2 = 0, \ \ \Gamma_a(\tau_k) = \frac{1}{h}\int_{t_{k+1} - t_k^1}^{t_{k+1} - t_k^0} e^{As} ds B,$$

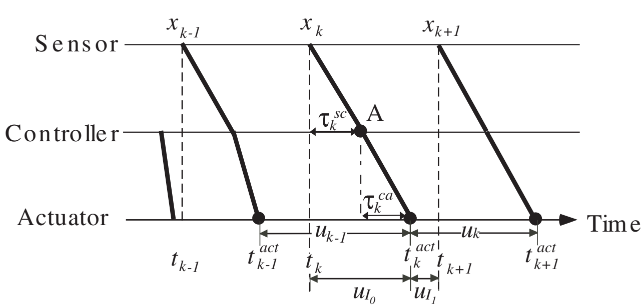

Fig. 2. A diagram of signals

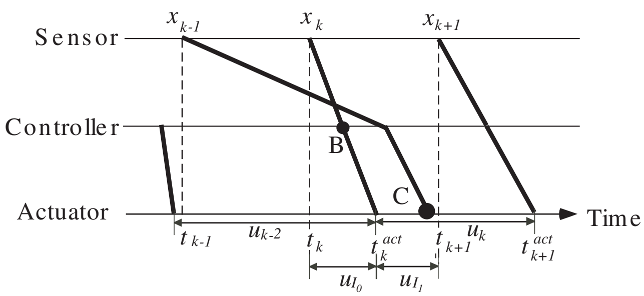

Fig. 3. A diagram of signals (irregular case)

$$\Gamma_0(\tau_k) = \frac{1}{h}\int_{t_{k+1}-t_k^2}^{t_{k+1}-t_k^1} e^{As} ds B. \tag{14}$$

From this example, it is clear that these matrices depend on the time delay τ_k. Further, in Fig. 2 at the point A the controller can use the current state x_k, past input sequence $u_0, \cdots, u_{k-1}$, and delays $\tau_0^{sc}, \cdots, \tau_k^{sc}$.

Example 2: Here the timing of signals shown in Fig. 3 is considered. In this case, the order of the signals is replaced, which is caused by the assumption (2). In Fig. 3, the replace is happen at the point C and the signal that arrived later is ignored [18]. For this case, the signal u_{k-1} is ignored at the point C. Matrices A^* and B_w^* are same as (14), since they do not depend on the delays. Matrices Γ_2, Γ_a and Γ_0 can be clculated as follows:

$$\begin{aligned}\Gamma_2(\tau_k) &= \frac{1}{h}\int_{t_{k+1}-t_k^1}^{t_{k+1}-t_k^0} e^{As} ds B, \quad \Gamma_a = 0\\ \Gamma_0(\tau_k) &= \frac{1}{h}\int_{t_{k+1}-t_k^2}^{t_{k+1}-t_k^1} e^{As} ds B.\end{aligned} \tag{15}$$

It is clear that the matrices Γ_2, Γ_a and Γ_0 depend on the delays τ_k. Further the matrix that corresponds to the ignored signal u_{k-1} becomes zero matrix i.e. $\Gamma_a = 0$. On the other hands, in Fig. 3 at the point B the controller can use state x_k, past input sequence $u_0, \cdots, u_{k-2}$ and delays $\tau_0^{sc}, \cdots, \tau_k^{sc}$.

D. Problem Formulation

The control problem setup for the system (10) by the cost function

$$J = \mathrm{E}\left\{h\sum_{k=0}^{N}\begin{bmatrix}x_k\\u_k\end{bmatrix}^T W \begin{bmatrix}x_k\\u_k\end{bmatrix} + x_{N+1}^T Q_f x_{N+1}\right\}, \tag{16}$$

where W is positive semi-definite and denoted as

$$W = \begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}. \tag{17}$$

Here it is assumed that R is positive definite and Q_f is positive semi-definite.

III. Linear Quadratic Control

In this section, an optimal control for the delta operator systems (10) with the object function (16) is proposed. First a theorem is derived, whose proof is done by using dynamic programming. Then the control algorithm to compute the optimal control is presented.

A. Optimal Controller

The controller in [22] is extended to a controller with the delays that are longer than one sample period. The following theorem is obtained by extending the result in [22].

Theorem: Assume that the delays satisfy the condition (2). The optimal control for the system (10) with the objective function (16) is given by

$$u_k = K_k'(\tau_k^{sc}, \cdots, \tau_{k-N+1}^{sc})\begin{bmatrix}x_k\\ hu_{k-N}\\ h\nu_k\end{bmatrix}, \tag{18}$$

where

$$\begin{aligned}&K_k'(\tau_k^{sc}, \cdots, \tau_{k-N+1}^{sc})\\ &= -(R + \tilde{P}_{k+1}'^{22})^{-1}\left(\begin{bmatrix}S^T & 0 & 0\end{bmatrix} + \tilde{P}_{k+1}'^{21}\right)\end{aligned} \tag{19}$$

$$\begin{aligned}&\tilde{P}_{k+1}'(\tau_k^{sc}, \cdots, \tau_{k-N+1}^{sc})\\ &= \frac{1}{h}\mathop{\mathrm{E}}_{\tau_k^{ca},\cdots,\tau_{k-N+1}^{ca}}\Big\{G'^T(\tau_k)P_{k+1}'G'(\tau_k)\\ &\qquad\qquad |\tau_k^{sc}, \cdots, \tau_{k-N+1}^{sc}\Big\}\end{aligned} \tag{20}$$

$$G'(\tau_k) = \begin{bmatrix}hA^* + I & \Gamma_N & \Gamma_a & h\Gamma_0\\ 0 & 0 & I & 0\\ 0 & 0 & 0 & hI\end{bmatrix} \tag{21}$$

$$\begin{aligned}&P_k'(\tau_{k-1}^{sc}, \cdots, \tau_{k-N+1}^{sc})\\ &= h\mathop{\mathrm{E}}_{\tau_k^{sc}}\left\{\tilde{Q}' + \tilde{P}_{k+1}'^{11} - K_k'(R + \tilde{P}_{k+1}'^{22})K_k'\right\}\end{aligned} \tag{22}$$

$$\tilde{Q}' = \begin{bmatrix}Q & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, P_{N+1}' = \begin{bmatrix}Q_f & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\end{bmatrix}, \tag{23}$$

P_k' is positive semi-definite.

Proof: To prove the theorem, dynamic programming is used. Introduce the cost functions J_a and J_b, which are given by (25) and (26) respectively.

$$J = J_a + J_b \tag{24}$$

$$J_a = \mathrm{E}\left\{h\sum_{k=0}^{N-1}\begin{bmatrix}x_k\\u_k\end{bmatrix}^T\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}\begin{bmatrix}x_k\\u_k\end{bmatrix}\right\} \tag{25}$$

$$\begin{aligned}J_b &= \mathrm{E}\left\{h\begin{bmatrix}x_N\\u_N\end{bmatrix}^T\begin{bmatrix}Q & S\\ S^T & R\end{bmatrix}\begin{bmatrix}x_N\\u_N\end{bmatrix}\right.\\ &\qquad\qquad\left. + x_{N+1}^T Q_f x_{N+1}\right\}\\ &=: V_N(z_N, \tau_N^{sc})\end{aligned} \tag{26}$$

V_N can be interpreted as the cost from k to N and is a function of the state x_k at time k. Define the vector

$$z_k' = \begin{bmatrix}x_k\\ Tu_{k-N}\\ T\nu_k\end{bmatrix}. \tag{27}$$

Using P'_N (23) gives

$$
\begin{aligned}
&V_N^*(z'_N, \tau_{N-1}^{sc}, \cdots, \tau_{N-N+1}^{sc}) \\
&= \min_{u_N} \mathrm{E}\left\{ T \begin{bmatrix} x_N \\ u_N \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} x_N \\ u_N \end{bmatrix} \right. \\
&\left. + {z'}_{N+1}^T P'_{N+1} z'_{N+1} \right\}. \qquad (28)
\end{aligned}
$$

The system representation with delta operator (10) can be rewritten as follows

$$
\begin{aligned}
\frac{\Delta x_k}{\Delta} &= [\, TA^* + I \quad T\Gamma_N(\tau_k) \quad T\Gamma_a(\tau_k) \,] \begin{bmatrix} x_k \\ u_{k-N} \\ \nu_k \end{bmatrix} \\
&\quad + T\Gamma_0(\tau_k) u_k + B_w^* w_k. \qquad (29)
\end{aligned}
$$

By using (29), the system can be represented as

$$
z'_{k+1} = \begin{bmatrix} x_{k+1} \\ Tu_{k-N+1} \\ \vdots \\ Tu_k \end{bmatrix} = G'(\tau_N) \begin{bmatrix} z_k \\ u_k \end{bmatrix} + \begin{bmatrix} B_w^* \\ 0 \end{bmatrix} w_k \qquad (30)
$$

where

$$
G'(\tau_N) = \begin{bmatrix} TA^* + I & \Gamma_N & \Gamma_a & T\Gamma_0 \\ 0 & 0 & I & 0 \\ 0 & 0 & 0 & TI \end{bmatrix}. \qquad (31)
$$

Since at time $N-1$ the controller can use τ_N^{sc}, w_k is white noise, the delays τ_k^{sc}, τ_k^{sc} are independent, it follows

$$
\begin{aligned}
&V_N^*(z'_N, \tau_{N-1}^{sc}, \cdots, \tau_{N-N+1}^{sc}) \\
&= T \mathop{\mathrm{E}}_{\tau_N^{sc}} \min_{u_N} \left\{ \begin{bmatrix} x_N \\ u_N \end{bmatrix}^T \begin{bmatrix} Q & S \\ S^T & R \end{bmatrix} \begin{bmatrix} x_N \\ u_N \end{bmatrix} \right. \\
&\left. + \begin{bmatrix} z'_N \\ u_N \end{bmatrix}^T \tilde{P}'_{N+1} \begin{bmatrix} z'_N \\ u_N \end{bmatrix} \right\} + \mathrm{tr}\, B_w^T {P'}_{N+1}^{11} B_w R_1 \qquad (32)
\end{aligned}
$$

$$
\begin{aligned}
&\tilde{P}'_{N+1}(\tau_N^{sc}, \cdots, \tau_{N-N+1}^{sc}) \\
&= \frac{1}{T} \mathop{\mathrm{E}}_{\tau_N^{ca}, \cdots, \tau_{N-N+1}^{ca}} \\
&\left\{ G'^T(\tau_N) P'_{N+1} G'(\tau_N) \,\middle|\, \tau_N^{sc}, \cdots, \tau_{N-N+1}^{sc} \right\}. \qquad (33)
\end{aligned}
$$

Rewrite V_N^* by using the completing square form. Then we can represent V_N^* as

$$
\begin{aligned}
V_N^* &= T \mathop{\mathrm{E}}_{\tau_N^{sc}} \min_{u_N} \left\{ \left\{ u_N + (R + \tilde{P}'^{22}_{N+1})^{-1} \right. \right. \\
&\qquad \left. \times \left([\, S^T \quad 0 \quad 0 \,] + \tilde{P}'^{21}_{N+1} \right) z'_N \right\}^T \\
&\quad \times (R + \tilde{P}'^{22}_{N+1}) \left\{ u_N + (R + \tilde{P}'^{22}_{N+1})^{-1} \right. \\
&\qquad \left. \left. \times \left([\, S^T \quad 0 \quad 0 \,] + \tilde{P}'^{21}_{N+1} \right) z_N \right\} \right\} \\
&\quad + z_N^T P'_N z_N + \mathrm{tr}\, B_w^T {P'}_{N+1}^{11} B_w R_1. \qquad (34)
\end{aligned}
$$

It is clear that P'_{N+1} is symmetric. It implies that the matrix $\tilde{P}'_{N+1}$ is symmetric. Since $R > 0$ and $P'_{N+1} \geq 0$, the matrix $(TR + \tilde{P}'^{22}_{N+1})$ is positive definite. Hence, the optimal control u_N^* is given by

$$
u_N^* = K'_N(\tau_N^{sc}, \cdots, \tau_{N-N+1}^{sc}) z'_N, \qquad (35)
$$

where

$$
\begin{aligned}
&K'_N(\tau_N^{sc}, \cdots, \tau_{N-N+1}^{sc}) \\
&= -(R + \tilde{P}'^{22}_{N+1})^{-1} \left([\, S^T \quad 0 \quad 0 \,] + \tilde{P}'^{21}_{N+1} \right). \qquad (36)
\end{aligned}
$$

Repeating the above procedure for the times $N-1, N-2, \ldots, 0$ gives the optimal control u_k^*

$$
u_k^* = K'_k(\tau_k^{sc}, \cdots, \tau_{k-N+1}^{sc}) z_k, \qquad (37)
$$

where

$$
\begin{aligned}
&K'_k(\tau_k^{sc}, \cdots, \tau_{k-N+1}^{sc}) \\
&= -(R + \tilde{P}'^{22}_{k+1})^{-1} \left([\, S^T \quad 0 \quad 0 \,] + \tilde{P}'^{21}_{k+1} \right). \qquad (38)
\end{aligned}
$$

This proves (18). □

Here, Riccati equation is given by

$$
\begin{aligned}
&-\frac{\Delta P_k}{\Delta h} \\
&= \mathop{\mathrm{E}}_{\tau_k^{sc}} \left\{ \left[\begin{gathered}
Q + hA^{*T} P_{k+1}^{11} A^* + A^{*T} P_{k+1}^{11} + P_{k+1}^{11} A^* \\
\tfrac{1}{h} \Gamma_N^T P_{k+1}^{11} (hA^* + I) - \tfrac{1}{h} P_{k+1}^{21} \\
\tfrac{1}{h} (\Gamma_a^T P_{k+1}^{11} + P_{k+1}^{21})(hA^* + I) - \tfrac{1}{h} P_{k+1}^{31} \\
\tfrac{1}{h} (hA^* + I)^T P_{k+1}^{11} \Gamma_N - \tfrac{1}{h} P_{k+1}^{12} \\
\tfrac{1}{h} \Gamma_N^T P_{k+1}^{11} \Gamma_N - \tfrac{1}{h} P_{k+1}^{22} - \tfrac{1}{h} P_{k+1}^{22} \\
\tfrac{1}{h} (\Gamma_a^T P_{k+1}^{11} + P_{k+1}^{21}) \Gamma_N - \tfrac{1}{h} P_{k+1}^{32} \\
\tfrac{1}{h} (hA^* + I)^T (P_{k+1}^{11} \Gamma_a + P_{k+1}^{12}) - \tfrac{1}{h} P_{k+1}^{13} \\
\tfrac{1}{h} \Gamma_N^T (P_{k+1}^{11} \Gamma_a + P_{k+1}^{12}) - \tfrac{1}{h} P_{k+1}^{23} \\
\tfrac{1}{h} \{ (\Gamma_a^T P_{k+1}^{11} + P_{k+1}^{21}) \Gamma_a + \Gamma_a^T P_{k+1}^{12} + P_{k+1}^{22} \} - \tfrac{1}{h} P_{k+1}^{33}
\end{gathered} \right] \right. \\
&\left. - K_k^T (R + \tilde{P}_{k+1}^{22}) K_k \right\}. \qquad (39)
\end{aligned}
$$

The theorem gives the optimal control for the networked control system (29) with the objective function (16). The optimal control gain K depends on τ^{sc} and the optimal control is represented as a state feedback form. This controller is extended the controller in [22] in terms of the delays that are longer than one sample period. In Theorem the system is the delta operator representation of the system in [18]. Especially the system representation in [18] can be obtained by setting sampling period $h = 1$. Hence, it can be said that the proposed control is an extended controller of one in [18] in the sense of the system representation.

B. Control Algorithm

In this section, a control algorithm that constructs the optimal control is presented. The optimal control is calculated off-line as same as the standard finite Linear quadratic control for discrete time systems. Since the optimal controller depends on the delay τ^{sc}, the gain that is function of τ^{sc} is calculated as first step. Then, substituting the delay τ^{sc} into the gain gives the optimal control that is applied to the system.

First, for the given terminal weighting matrix Q_f using (23) derives P_{N+1}. Then from (20), (21) $\tilde{P}_{N+1}$ is obtained.

In this calculation for $\tilde{P}_{N+1}$, the delay $\tau_N^{sc}, \cdots, \tau_{N-M+1}^{sc}$ are considered as variables. By using $\tilde{P}_{N+1}$, the gain $K_N(\tau_N^{sc}, \cdots, \tau_{N-M+1}^{sc})$ can be obtained from (19). Further P_N is calculated from (22). At this time, the calculation of expectation for τ_N^{sc} makes τ_N^{sc} no variable. Using P_N, (20) and (21) gives $\tilde{P}_N$ (then τ_{N-M}^{sc} becomes a new variable). The gain K_{N-1} is also calculated similar to K_N from (19). Repeat these calculations the controller gain $K_0, \cdots, K_N$ can be obtained. The algorithm can be summarized as follows:

1) Calculate P_{N+1}.
2) Set $k = N + 1$.
3) Repeat following steps (a)-(d) until $k = 0$.
   a) Decrement k.
   b) From (20) and (21) calculate $\tilde{P}_{k+1}$.
   c) By (19) calculate the gain K_k.
   d) By using (22) calculate P_k.

In this algorithm the backward calculation is repeated $N+1$ times.

IV. EXAMPLE

In this section, consider the optimal control with the objective function (16) for the system (12). This system is discretized with sampling period $h = 0.5$. Then we have

$$\dot{x} = \begin{bmatrix} 1.025 & 0.492 \\ 0.098 & 0.976 \end{bmatrix} x + \begin{bmatrix} 0.124 \\ 0.492 \end{bmatrix} u + \begin{bmatrix} 1.235 \\ 4.918 \end{bmatrix} w.$$

The horizon is $N = 30$ and the weighting matrices are set as

$$Q = \begin{bmatrix} 10 & 0 \\ 0 & 10 \end{bmatrix}, \ R = 1, \ S = 0. \tag{40}$$

The time delay τ^{sc} is assumed to be uniformly distributed on the interval $[0, \alpha h]$ $(0 \leq \alpha < 2)$. In this example the time delay τ^{ca} is assumed to be zero for the sake of simplicity. The closed loop systems are simulated with the optimal controller proposed in this paper and a linear quadratic controller neglecting the time delays.

The proposed optimal controller is calculated by using the algorithm proposed in the above section. In this simulation, MATLAB Symbolic Math Toolbox is used to obtain the optimal controller. The resulting cost in the case of initial state response with $x_0 = [1\ 0]^T$ is plotted in Fig. 4. The time histories of the state and the input are also shown in Fig. 5 and 6.

In Fig. 4 for the region $\alpha < 1.2$, the cost with the proposed optimal controller is higher than the cost with the linear quadratic controller. Since the linear quadratic controller neglecting the time delays gives the optimal controller for no delay case, only in the small delay case the performance can be better than the proposed optimal controller. However, for the large delay case the proposed optimal controller performs better than the linear quadratic controller. This means that the proposed optimal controller has effectiveness for the long time delay. It causes that the sampling period can be made short (fast sampling systems).

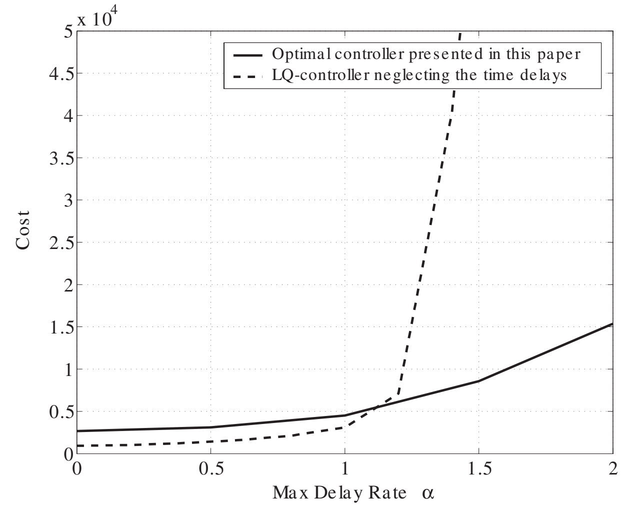

Fig. 4. Costs for difference maximum delays in the case of initial state response with $x_0 = [1\ 0]^T$, The real line shows the result of the proposed optimal controller, and the dashed line shows the linear quadratic controller's case.

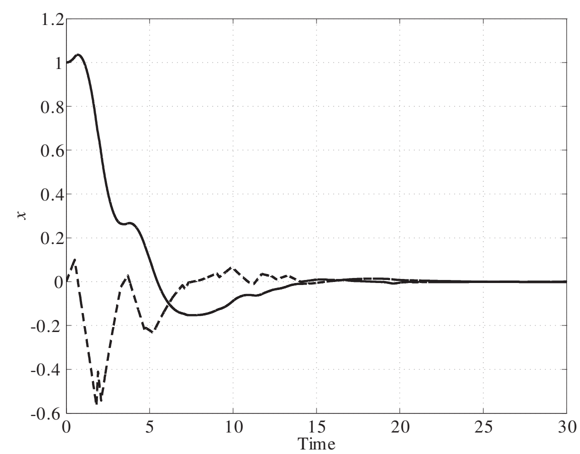

Fig. 5. The time histories of the state, The real line shows the result of x_1 and the dashed line shows x_2.

From Fig. 5 and 6, we can see that the state and the input convarge to 0.

V. CONCLUSION AND FUTURE WORKS

A. Conclusion

This paper has proposed the optimal control of linear systems in delta domain with long random time delays which are longer than one sampling interval. With the delta operator representation, the dynamic programming approach has been investigated and discussed to derive the optimal controller with quadratic cost for the delta operator system representation. We have shown that the optimal controller with full-state information in delta domain can be represented by a linear delay-depending feedback from

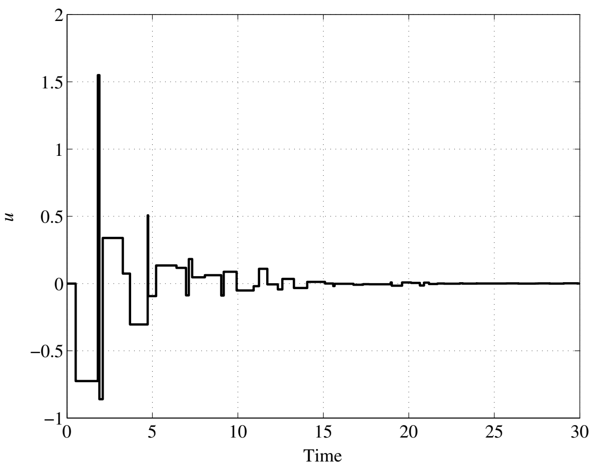

Fig. 6. The time history of the input

the state and the previous control signal. The derived optimal controller has had the numerical property in very fast sampling period. Theresults have been illustrated with numerical examples.

B. Futurework

In this paper, the full-state information was assumed. State estimation problem is one of the extentions for this paper. Moreover, model predictive control will be considered to guarantee stability.

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