2005 American Control Conference
June 8-10, 2005. Portland, OR, USA

Reduced Order Optimal Control Using Genetic Algorithms

VijaySekhar Chellaboina† and Mithun K. Ranga‡

Abstract— **In this paper, we present a case study of several examples involving optimal positive real control design for positive real plants. Specifically, we apply genetic algorithms to obtain low order optimal control designs with and without the strict positive real constraint on the compensator. This study shows that genetic search strategy is an effective approach for designing low order optimal controllers with complex constraints such as strict positive realness of the compensator. Furthermore, the examples studied also shed some new light on optimal positive real control problems.**

I. Introduction

The increasing complexity of modern control engineering problems and ever stringent performance constraints necessitates development of efficient algorithms for control design. Specifically, search for efficient algorithms to design fixed (low) order controllers that satisfy a given set of performance constraints remains open. One of the major breakthroughs in modern control is the development of linear-quadratic-Gaussian (LQG) theory which provides optimal dynamic compensators with respect to a given quadratic (H_2) performance [1]. However, the LQG theory is severely restricted by the fact that the optimal (dynamic) compensator thus obtained has dimension equal to that of the plant. In practice, it is desirable to obtain controllers that have significantly lower dimension than that of the plant. This has led to the development of optimal reduced order controllers (see [2], [3] and references therein). Specifically, [2] developed *optimal projection equations*, which are essentially necessary conditions for optimality for a fixed controller order. The optimal projection equations involve four highly coupled matrix equations and hence very difficult to solve. Thus the development of efficient algorithms to solve the optimal projection equations is central to low-order controller design techniques (see [4] and references therein). All these algorithms involve gradient based search techniques applied to the optimal projection equations. However, since the optimal projection equations are nonconvex and only necessary conditions for optimality, the convergence of these algorithms is not guaranteed (even to a local minimum). An alternate approach to optimal projection equations is formulating the fixed order controller problem as a set of (bilinear) matrix inequalities [3]. Once again these formulations also suffer from nonconvexity and hence the convergence of algorithms to a minimum depends on initialization of the algorithms.

Next, in addition to reducing order the controllers it is also desirable to restrict the dynamic compensator to be stable. Both LQG and the fixed order techniques guarantee that the closed-loop is stable but do not guarantee the stability of the optimal compensator. In applications involving large flexible space structures it is useful to further restrict the compensator to strictly positive real systems [5]. Specifically, since flexible space structures with collocated actuators and sensors are always positive real [6] strictly positive real compensators offer excellent robustness properties for controlling large flexible space structures. An optimal positive real control design problem for the full order case has been addressed in [5] involving certain equality conditions on the weighting matrices of the H_2 cost. Extensions to include H_∞ performance was also considered in [7]. Although the technique presented in [5], [7] does provide a way to design full order strictly positive real compensators, as indicated in [5], the resulting compensators are not necessarily optimal with respect to a given H_2 cost criterion. Furthermore, based on the optimal projection equation approach, a suboptimal fixed-order positive real control design framework has also been presented in [8]. Alternatively, we may design fixed order optimal positive real controllers by extending the approach based on matrix inequalities given in [3]. As in the case of fixed order optimal control problem, this extension also suffers from nonconvexity.

In this paper, we propose genetic search strategy [9–14] as a viable optimal control design technique to incorporate constraints such as fixed-order and strict positive realness of the compensator. Specifically, we study several examples involving optimal positive real control design for positive real plants. Here we implement genetic algorithms [9–14] to find optimal control designs with and without the strict positive real constraint on the compensator.

Genetic search strategies are population based probabilistic optimization techniques mimicking Darwin's idea of natural selection. Unlike traditional techniques such as gradient search algorithms, which require objective function as well as its gradient expression, genetic algorithms require minimal information about the objective function. Specifically, genetic algorithms require only an algorithm to compute the objective function and nothing more. This feature has been

This research was supported in part by the National Science Foundation under Grant ECS-0133038.

†Mechanical, Aerospace, and Biomedical Engineering, University of Tennessee, Knoxville, TN 37996-2210

‡Mechanical and Aerospace Engineering, University of Missouri, Columbia, MO 65211

0-7803-9098-9/05/$25.00 ©2005 AACC

extremely useful in the ease of implementing genetic algorithms for a variety of complex optimization problems [9–14]. Although genetic algorithms do not guarantee convergence to an optimum solution the high point of genetic algorithms is that they provide near *global* optima for many complex problems in practice. The algorithms have been used to solve a range of complex problems from combinatorial problems such as the travelling salesman problem [9] to many problems in optimal control [15–22]. The success of genetic algorithms for solving a particular problem depends on primarily on variable representation (real, binary, combinatorial etc.) and its corresponding genetic *recombination* (also known as *crossover*) and *mutation* operators. In this paper, we first transform the fixed order optimal control problem to an unconstrained optimization problem involving *nonsmooth* objective function. Next, we apply genetic algorithms to solve three optimal control design problems using a real variable representation with real mutation operator [23] and two different recombination operators namely; *discrete* recombination operator and *line* recombination operator.

The manuscript is organized as follows. Section 2 contains several definitions and key results used in this paper. Section 3 presents the fixed order optimal control and fixed order optimal positive real control problems. Section 4 presents several numerical examples involving reduced order control design for positive real plants using genetic algorithms. Finally, Section 5 presents the conclusion.

II. Mathematical Preliminaries

In this section we introduce notation, definitions, and two key lemmas used in this paper. Specifically, $\mathbb{R}$ denotes the reals and $\mathbb{R}^n$ is an n-dimensional linear vector space over the reals with Euclidean norm $\|\cdot\|$. Furthermore, for $M \in \mathbb{R}^{m\times n}$ (resp., $M \in \mathbb{C}^{m\times n}$), we write M^{T} (resp., M^*) to denote the transpose (resp., complex conjugate transpose) of M and $M \geq 0$ (resp., $M > 0$) to denote the fact that the symmetric matrix M is nonnegative (resp., positive) definite. For Let $G(s) \sim \left[\begin{array}{c|c} A & B \\ \hline C & D \end{array}\right]$ denote a state space realization of a transfer function $G(s)$; that is, $G(s) = C(sI - A)^{-1}B + D$. The notation "$\overset{\min}{\sim}$" is used to denote a minimal realization. Finally, we write I_n to denote the $n \times n$ identity matrix.

Definition 2.1: A square transfer function $G(s)$ is called *positive real* if $i)$ all elements of $G(s)$ are analytic in $\mathrm{Re}[s] > 0$ and $ii)$ $G(s) + G^*(s) \geq 0$, $\mathrm{Re}[s] > 0$. A square transfer function $G(s)$ is *strictly positive real* if there exists $\varepsilon > 0$ such that $G(s-\varepsilon)$ is positive real.

Next, we state the well-known positive real lemma [6].

Lemma 2.1: Let $G(s) \overset{\min}{\sim} \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$ be a strictly proper transfer function where $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, and $C \in \mathbb{R}^{m\times n}$. Then $G(s)$ is strictly positive real (resp., positive real) if and only if there exist a matrix $P \in \mathbb{R}^{n\times n}$ and a scalar $\varepsilon > 0$ (resp., $\varepsilon = 0$) such that $P > 0$ and

$$\begin{bmatrix} A^{\mathrm{T}}P + PA + \varepsilon P & PB - C^{\mathrm{T}} \\ B^{\mathrm{T}}P - C & 0 \end{bmatrix} \leq 0. \quad (1)$$

Definition 2.2: Let $G(s) \overset{\min}{\sim} \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$ be a positive real transfer function. Then $\left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$ is a *self-dual realization* of $G(s)$ if $A + A^{\mathrm{T}} \leq 0$ and $B = C^{\mathrm{T}}$.

The next result due to [6] shows that positive real transfer functions always have self-dual realizations.

Lemma 2.2: Let $G(s) \overset{\min}{\sim} \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$ be a positive real transfer function and let $P > 0$ satisfy (1). Then $\left[\begin{array}{c|c} P^{1/2}AP^{-1/2} & P^{1/2}B \\ \hline CP^{-1/2} & 0 \end{array}\right]$ is a self-dual realization of $G(s)$.

Remark 2.1: It follows from Lemmas 2.1 and 2.2 that $G(s)$ is positive real (resp., strictly positive real) if and only if there exist A, B, and C such that $G(s) \overset{\min}{\sim} \left[\begin{array}{c|c} A & B \\ \hline C & 0 \end{array}\right]$ and $A + A^{\mathrm{T}} \leq 0$ (resp., $A + A^{\mathrm{T}} < 0$) and $B = C^{\mathrm{T}}$.

III. Fixed Order Positive Real Optimal Control

In this section we present the *fixed order optimal control problem* and the *fixed order optimal positive real control problem*.

Fixed Order Optimal Control Problem: Given a stabilizable and detectable plant

$$\dot{x}(t) = Ax(t) + Bu(t) + D_1w(t), \; x(0) = x_0, \quad (2)$$
$$y(t) = Cx(t) + D_2w(t), \quad (3)$$

where $t \geq 0$, $A \in \mathbb{R}^{n\times n}$, $B \in \mathbb{R}^{n\times m}$, $C \in \mathbb{R}^{m\times n}$, $D_1 \in \mathbb{R}^{n\times p}$, $y(t) \in \mathbb{R}^m$ is the measured output, $w(t) \in \mathbb{R}^p$ is a disturbance input, determine a n_{c}th-order dynamic compensator $G_{\mathrm{c}}(s) \sim \left[\begin{array}{c|c} A_{\mathrm{c}} & B_{\mathrm{c}} \\ \hline C_{\mathrm{c}} & 0 \end{array}\right]$ of the form

$$\dot{x}_{\mathrm{c}}(t) = A_{\mathrm{c}}x_{\mathrm{c}}(t) + B_{\mathrm{c}}y(t), \quad (4)$$
$$u(t) = -C_{\mathrm{c}}x_{\mathrm{c}}(t), \quad (5)$$

that satisfies the following design criteria.

i) the undisturbed (that is, $w(t) \equiv 0$) closed-loop system (2)–(5) given by $\tilde{A} \triangleq \left[\begin{array}{c|c} A & -BC_{\mathrm{c}} \\ \hline B_{\mathrm{c}}C & A_{\mathrm{c}} \end{array}\right]$ is Hurwitz;

ii) the H_2 performance measure

$$J(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}}) \triangleq \lim_{t\to\infty} \frac{1}{t}\int_0^t [x^{\mathrm{T}}(s)R_1x(s) + u^{\mathrm{T}}R_2u(s)]\mathrm{d}s \quad (6)$$

is minimized, where $R_1 \triangleq E_1^{\mathrm{T}} E_1$, $R_2 \triangleq E_2^{\mathrm{T}} E_2 > 0$ and $E_1^{\mathrm{T}} E_2 = 0$.

Fixed Order Optimal Positive Real Control Problem: Given the minimal positive real plant (2), (3) determine an n_{c}th-order compensator $G_{\mathrm{c}}(s) \sim \left[\begin{array}{c|c} A_{\mathrm{c}} & B_{\mathrm{c}} \\ \hline C_{\mathrm{c}} & 0 \end{array}\right]$ that satisfies the design criteria ii) with the additional property that $G_{\mathrm{c}}(s)$ is strictly positive real.

Remark 3.1: Note that in the case of the fixed order optimal positive real control problem since the plant is positive real and the compensator is positive real, criterion i) of the fixed order optimal control problem is trivially satisfied [6]. Hence, the feasible space for the fixed order optimal positive real control problem is a subset to that of the fixed order optimal control problem. Hence, the minimal cost obtained in the fixed order optimal control problem is always less than or equal to that of the fixed order optimal positive real control problem.

For convenience of notation, define $V_1 \triangleq D_1 D_1^{\mathrm{T}}$ and $V_2 \triangleq D_2 D_2^{\mathrm{T}}$ and assume $D_1 D_2^{\mathrm{T}} = 0$. Now, recall that, given a dynamic compensator $G_{\mathrm{c}}(s)$ such that the closed-loop is asymptotically stable, the H_2 performance measure $J(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$ is given by [2]

$$J(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}}) = \mathrm{tr}\ \tilde{P}\tilde{V}, \tag{7}$$

where $\tilde{V} \triangleq \begin{bmatrix} V_1 & 0 \\ 0 & B_{\mathrm{c}} V_2 B_{\mathrm{c}}^{\mathrm{T}} \end{bmatrix}$ and $\tilde{P} \in \mathbb{R}^{(n+n_{\mathrm{c}})\times(n+n_{\mathrm{c}})}$ satisfies

$$0 = \tilde{A}^{\mathrm{T}}\tilde{P} + \tilde{P}\tilde{A} + \tilde{R}, \tag{8}$$

and where $\tilde{R} \triangleq \begin{bmatrix} R_1 & 0 \\ 0 & C_{\mathrm{c}}^{\mathrm{T}} R_2 C_{\mathrm{c}} \end{bmatrix}$. Hence, the fixed order optimal control problem may be solved by minimizing the objective function tr $\tilde{P}\tilde{V}$ subject to the equality constraint (8) where $A_{\mathrm{c}} \in \mathbb{R}^{n_{\mathrm{c}}\times n_{\mathrm{c}}}$, $B_{\mathrm{c}} \in \mathbb{R}^{n_{\mathrm{c}}\times m}$ and $C_{\mathrm{c}} \in \mathbb{R}^{m\times n_{\mathrm{c}}}$. In the case where $n = n_{\mathrm{c}}$ the solution to this constrained minimization is given by the LQG controller [1], [2] in terms of solutions to two independent algebraic Riccati equations. However, if $n_{\mathrm{c}} < n$ then this problem is extremely complex and leads to four highly coupled matrix equations (necessary conditions for optimality) known as *optimal projection equations* [2]. The typical approach to solve these equations are gradient based methods such as quasi Newton methods [4]. It should be noted that the optimal projection equations are nonconvex and only necessary conditions for optimality and hence the convergence of gradient based algorithms is not guaranteed (even to a local minimum). Alternatively, the H_2 performance measure $J(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$ can be obtained from the minimization

$$\begin{aligned} J(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}}) &= \min\{\lambda :\ \mathrm{tr}\ PV \le \lambda, \\ &\qquad \tilde{A}^{\mathrm{T}}\tilde{P} + \tilde{P}\tilde{A} + \tilde{R} \le 0\}, \end{aligned} \tag{9}$$

which may be solved using a *dual linear matrix inequality iteration* method [3]. Once again, since this problem is nonconvex the convergence of this algorithm is extremely sensitive to the initial conditions. Finally, it follows from Lemmas 2.1 and 2.2 that the fixed order optimal positive real control may be solved using the dual LMI iteration method with additional constraints $A_{\mathrm{c}} + A_{\mathrm{c}}^{\mathrm{T}} < 0$ and $B_{\mathrm{c}} = C_{\mathrm{c}}^{\mathrm{T}}$. However, as above, the convergence of this algorithm is extremely sensitive to the initial conditions.

IV. Fixed Order Optimal Control using Genetic Algorithms

In this section, we use genetic algorithms to solve both the fixed order optimal control and the fixed order optimal positive real control problems for two given positive real plants. Since both these problems involve complex equality and/or inequality constraints we, first, reformulate both problems as unconstrained optimization problems. Specifically, we introduce a new objective function $F(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$ given by

$$F(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}}) = \begin{cases} \frac{J}{J+1}, & \text{if } (A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}}) \in \mathcal{F}, \\ 1, & \text{otherwise}, \end{cases} \tag{10}$$

where $\mathcal{F}$ denotes the feasible space. For the fixed order optimal control problem

$$\mathcal{F} \triangleq \{(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}}) :\ \text{such that } \tilde{A} \text{ is Hurwitz}\},$$

and for the fixed order optimal positive real control problem

$$\begin{aligned} \mathcal{F} \ \triangleq\ \{(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}}) :\ &\text{such that } A_{\mathrm{c}} + A_{\mathrm{c}}^{\mathrm{T}} < 0, \\ & B_{\mathrm{c}} = C_{\mathrm{c}}^{\mathrm{T}}\}. \end{aligned}$$

Note that minimizing $F(\cdot,\cdot,\cdot)$ is equivalent to minimizing $J(\cdot,\cdot,\cdot)$. Hence, the fixed order optimal control problem (resp., fixed order optimal positive real control problem) can be solved by minimizing $F(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$ where $A_{\mathrm{c}} \in \mathbb{R}^{n_{\mathrm{c}}\times n_{\mathrm{c}}}$, $B_{\mathrm{c}} \in \mathbb{R}^{n_{\mathrm{c}}\times m}$, and $C_{\mathrm{c}} \in \mathbb{R}^{m\times n_{\mathrm{c}}}$. Finally, since genetic algorithms require a bounded search space we further restrict A_{c}, B_{c}, and C_{c} such that their entries are within the interval $[-1000, 1000]$.[1] It is important to note that the objective function $F(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$ is nonsmooth and hence the traditional gradient based approaches cannot be used to minimize $F(\cdot,\cdot,\cdot)$. Furthermore, given a triple $(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$ it is easy to compute $F(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$ although there does not exist a closed-form expression for $F(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$. This, however, does not present a problem for implementing genetic algorithms as they require only a method to compute $F(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$. Finally, since effort for computing $F(A_{\mathrm{c}}, B_{\mathrm{c}}, C_{\mathrm{c}})$ is practically the same for both problems the computational time for optimal controller design with or

[1] Note that this range is chosen *sufficiently* large to include the optimal controller. However, it should be emphasized that there is no rigorous method to choose this range to guarantee existence of the optimal controller exists within the range.

without strict positive real constraint. It should be noted that inclusion of other constraints such as H_∞ performance constraints is relatively straightforward.

Example 4.1: This example adopted from [4] involves a simply supported beam of length 2 with two collocated sensor/ actuator pairs located at $a = \frac{55}{172}$ and $b = \frac{46}{43}$. In this case, a continuous time model retaining the first five modes is given by (2) and (3) where

$$A = \text{block–diag}\left(\begin{bmatrix} 0 & 1 \\ -\omega_i^2 & -2\zeta\omega_i \end{bmatrix}\right), \quad \omega_i = i^2,\ i = 1, \ldots, 5,\ \zeta = 0.005,$$

$$B = \begin{bmatrix} 0 & 0 \\ -0.2174 & -0.8439 \\ 0 & 0 \\ 0.4245 & -0.9054 \\ 0 & 0 \\ -0.6112 & -0.1275 \\ 0 & 0 \\ 0.7686 & 0.7686 \\ 0 & 0 \\ -0.8893 & 0.9522 \end{bmatrix},$$

and $C = B^{\mathrm{T}}$. The noise intensities are $V_1 \triangleq D_1 D_1^{\mathrm{T}} = 0.1 I_{10}$, $V_2 \triangleq D_2 D_2^{\mathrm{T}} = 0.1 I_2$, $R_2 = 0.1 I_2$, and $R_1 = I_{10}$ and it is assumed that $V_{12} \triangleq D_1 D_2^{\mathrm{T}} = 0$. For this example, the optimal cost for the full order (LQG) controller is 29.0798. Next, we designed a second order optimal dynamic compensator whose cost was found to be 29.5717. For this design, we used both discrete and line recombination operators. Figure 1 shows the best cost (in a generation) versus the generation number for the two operators. As can be seen from the figure the line recombination operator has a faster convergence rate than the discrete recombination operator. That has been the common trend for the all the examples considered for this paper. Furthermore, the resulting dynamic compensator satisfies the optimal projection equations (necessary conditions for optimality) and hence we conjecture that this is most probably the global minimum. Finally, it is interesting to note that the optimal second order controller thus obtained is also strictly positive real. Hence, the solution to the fixed order optimal positive real control problem should result in the same controller (modulo a similarity transformation). This has been verified by constraining the compensator to be a self dual strictly positive real compensator and performing the genetic algorithm with both the genetic recombination operators. Figure 2 shows the best cost (in a generation) versus the generation number for the two operators.

Example 4.2: This example involves a positive real plant given by (2) and (3) where

$$A = \text{block–diag}\left(\begin{bmatrix} 0 & 1 \\ -i^2 & -0.02i \end{bmatrix}\right), \quad i = 1, \ldots, 5,$$

$$B = C^{\mathrm{T}} = \begin{bmatrix} 0 \\ 0.9877 \\ 0 \\ -0.0309 \\ 0 \\ -0.891 \\ 0 \\ 0.5878 \\ 0 \\ 0.7071 \end{bmatrix}.$$

The noise intensities are $V_1 \triangleq D_1 D_1^{\mathrm{T}} = 0.1 I_{10}$, $V_2 \triangleq D_2 D_2^{\mathrm{T}} = 3.61$, $R_2 = 3.61$, and $R_1 = I_{10}$ and it is assumed that $V_{12} \triangleq D_1 D_2^{\mathrm{T}} = 0$. For this example, the optimal cost for the full order (LQG) controller is 26.8607. Using the genetic algorithm a second order dynamic compensator has been designed for the two genetic operators as in the first example. The *optimal* costs are 27.4172 and 26.0636 for discrete and linear recombination genetic operators, respectively. Figure 3 shows the best cost (in a generation) versus the generation number. As in the first example, the line recombination operator outperforms the discrete recombination operator. Although it seems that we obtain two different optimal costs with two different operators, it should be noted that the genetic algorithm with the discrete recombination operator continues to search for designs with lower costs (albeit slowly). Furthermore, the optimal compensator obtained from the line recombination operator satisfies the optimal projection equations.

Unlike the first example, the optimal second order controller (obtained using the line recombination operator) is *not* strictly positive real. This may be easily verified by observing the phase of $G_{\mathrm{c}}(s)$ (see Figure 6). Next, we have performed optimal positive real compensator design with the positive real constraint on the compensatorwhich resulted in an optimal cost 27.4172. Figure 4 shows the best cost (in a generation) versus the generation number. In addition, Figure 5 shows cost $J(A_{\mathrm{c}}(\alpha), B_{\mathrm{c}}(\alpha), C_{\mathrm{c}}(\alpha))$ versus $\alpha \in [0, 1]$ where $A_{\mathrm{c}}(\alpha)$, $B_{\mathrm{c}}(\alpha)$, and $C_{\mathrm{c}}(\alpha)$ are convex combination of $A_{\mathrm{c}}(0)$, $B_{\mathrm{c}}(0)$, and $C_{\mathrm{c}}(0)$ and $A_{\mathrm{c}}(1)$, $B_{\mathrm{c}}(1)$, and $C_{\mathrm{c}}(1)$ where $A_{\mathrm{c}}(0)$, $B_{\mathrm{c}}(0)$, and $C_{\mathrm{c}}(0)$ is the solution to the optimal control problem and $A_{\mathrm{c}}(1)$, $B_{\mathrm{c}}(1)$, and $C_{\mathrm{c}}(1)$ is the solution to the optimal positive real control problem. It is clear from Figure 5 that there is a continuous degradation of the cost as we move from the optimal controller to optimal positive real controller. Similarly, Figure 6 shows a continuous transition from a non-positive-real compensator to a positive real compensator as α changes from 0 to 1.

Figures 5 and 6 confirm the necessity of the positive real constraint for the compensator and also the fact that there can be a degradation in the cost due to the additional constraint. Finally, it is interesting to note that the genetic algorithm with the discrete recombination operator (and without the positive real constraint) converges to a cost close to that of the optimal positive real compensator. At this time we have no reason to conclude that this anything but a coincidence. Further study of this interesting coincidence will be pursued in a later work.

V. Conclusion

In this paper, we studied two examples involving optimal positive real control design for positive real plants. Specifically, we applied genetic algorithms to obtain optimal control designs with and without the strict positive real constraint on the compensator. This study shows that genetic search strategy is an effective approach for designing low order optimal controllers with highly complex constraints such as strict positive realness of the compensator. The advantage of the genetic algorithms is that adding constraints such as the positive realness of the compensator does not alter the efficiency of the algorithm.

The two examples considered in this paper further emphasize the importance of the fixed-order control problem. As seen in both the examples the degradation due to reducing the order from a 10th order LQG controller to a 2nd order controller is small, establishing the importance of optimal reduced order controller design techniques for obtaining high performance low complex controllers.

The two examples also provide comparison of two different recombination operators. It has been observed that the line recombination operator has consistently performed better than the discrete recombination operator.

References

[1] B. D. O. Anderson and J. B. Moore, *Optimal Control, Linear Quadratic Methods*. Englewood Cliffs, NJ: Prentice-Hall, 1991.

[2] D. C. Hyland and D. S. Bernstein, "The optimal projection equations for fixed order dynamic compensation," *IEEE Trans. Autom. Contr.*, vol. 29, pp. 1034–1037, 1984.

[3] R. E. Skelton, T. Iwasaki, and K. Grigoriadis, *A Unified Algebraic to Linear Control Design*. London, UK: Taylor and Francis, 1997.

[4] E. G. Collins, W. M. Haddad, and S. S. Ying, "Nearly non-minimal linear-quadratic-gaussian compensators for reduced-order control design initialization," *AIAA J. Guid. Contr. Dyn.*, vol. 19, pp. 259–261, 1996.

[5] R. Lozano-Leal and S. M. Joshi, "On the design of dissipative LQG-type controllers," in *Proc. IEEE Conf. Dec. Contr.*, Austin, TX, 1988.

[6] B. D. O. Anderson, "A system theory criterion for positive real matrices," *SIAM J. Contr. Optim.*, vol. 5, pp. 171–182, 1967.

[7] W. M. Haddad, D. S. Bernstein, and Y. W. Wang, "Dissipative H_2/H_∞ controller synthesis," *IEEE Trans. Autom. Contr.*, vol. 39, pp. 827–831, 1994.

[8] C. A. P. D. M.Jacobus, M. Jamshidi and D.Bernstein, "Design of strictly positive real, fixed order dynamic compensators," in *Proc. of Conference of Decision and Control*, Honolulu, Hawaii, 1990.

[9] D. E. Goldberg, *Genetic Algorithms in Search Optimization and Machine Learning*. Reading, Mass.: Addison - Wesley Pub. Co., 1989.

[10] Y. Davidor, *Genetic Algorithms and Robotics: A Heuristic Strategy for Optimization*. Singapore: World Scientific, 1991.

[11] J. R. Koza, *Genetic Programming : On the Programming of computer by means of natural selection*. Cambridge, Mass.: MIT Press, 1992.

[12] Z. Michalewicz, *Genetic Algorithms + Data Structures = Evolution Programs*. Berlin: Springer- Verlag, 1994.

[13] K. F. Man, K. S. Tang, and S. Kwong, *Genetic Algorithms :Concepts and Designs*. New York: Springer, 1999.

[14] M. Gen and R. Cheng, *Genetic Algorithms and Engineering Optimization*. New York: Wiley, 2000.

[15] C. T. Voth, "Genetic algorithms for control systems design and analysis," in *Advances in the Astronautical Sciences*, 1994.

[16] C. I. Marrison and R. F. Stengel, "Robust control system design using random search and genetic algorithms," *IEEE Trans. Autom. Contr.*, vol. 42, pp. 835–839, 1997.

[17] K. F. Man, K. S. Tang, S. Kwong, and W. A. Halang, *Genetic Algorithms for Control and Signal Processing*. New York: Springer, 1997.

[18] I. Kitsios, T. Pimenides, and P. Groumpos, "A genetic algorithm for designing H_∞ structured specified controllers," in *Proc. IEEE Conf. Contr. Appl.*, 2001, pp. 1196–1201.

[19] F. Le Mauff and G. Duc, "Design of fixed structure controllers with respect to various specifications using robust control and genetic algorithm," in *Proc. Amer. Contr. Conf.*, 2002, pp. 4994–4999.

[20] H. Du and X. Shi, "Low-order H_∞ controller design using LMI and genetic algorithm," in *Proc. Amer. Contr. Conf.*, Anchorage, AK, 2002.

[21] J. Ouyang and W.-D. Qu, "Robust pole placement using genetic algorithms," in *Intl. Conf. on Machine Learning and Cybernetics*, Beijing, 2002.

[22] Q. Wang, P. Spronck, and T. Rudolf, "An overview of genetic algorithms applied to control engineering problems," in *Intl. Conf. on Machine Learning and Cybernetics*, 2003.

[23] H. Pohlheim, *Genetic and Evolutionary Algorithm Toolbox for use with MATLAB*. http://www.geatbx.com: toolbox v 3.3, February 2003.

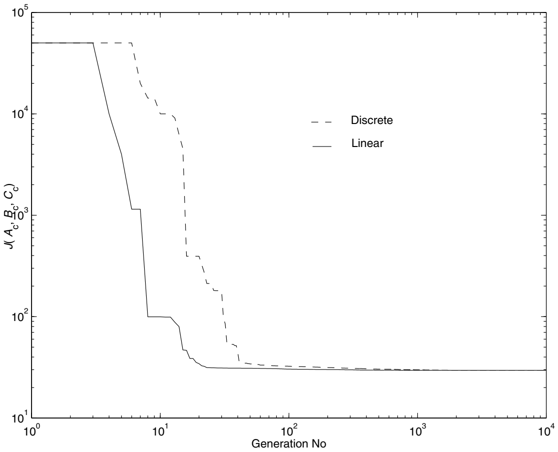

Fig. 1. Best cost versus generation: Example 4.1

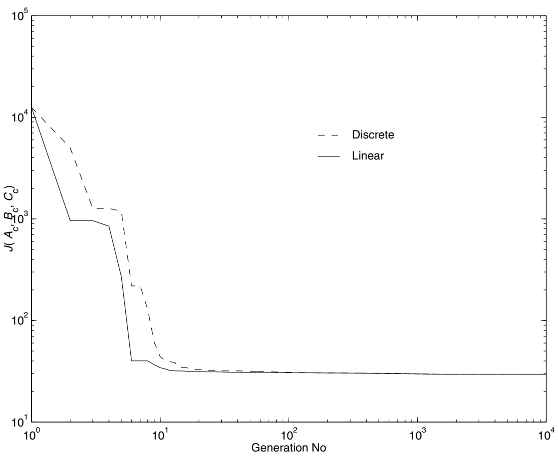

Fig. 2. Best cost versus generation: Example 4.1 with SPR constraint

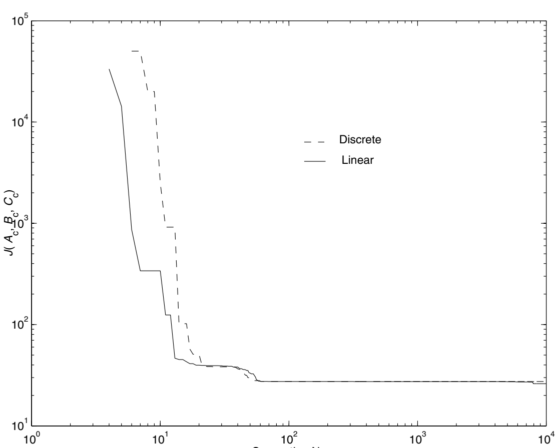

Fig. 3. Best cost versus generation: Example 4.2

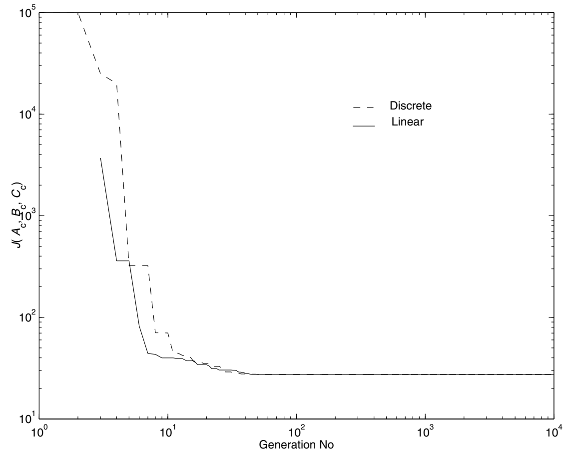

Fig. 4. Best cost versus generation: Example 4.2 with SPR constraint

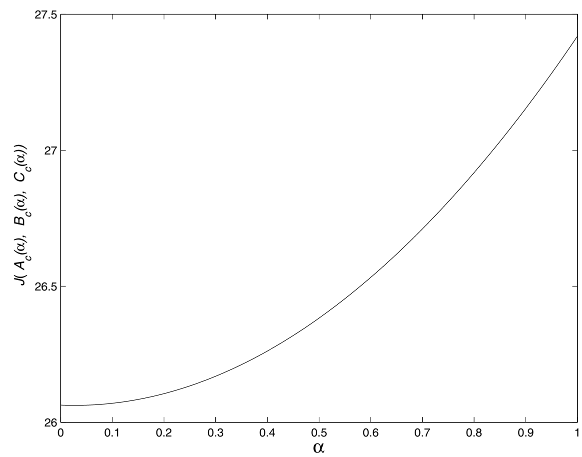

Fig. 5. Cost variation from optimal controller to optimal positive real controller: Example 4.2

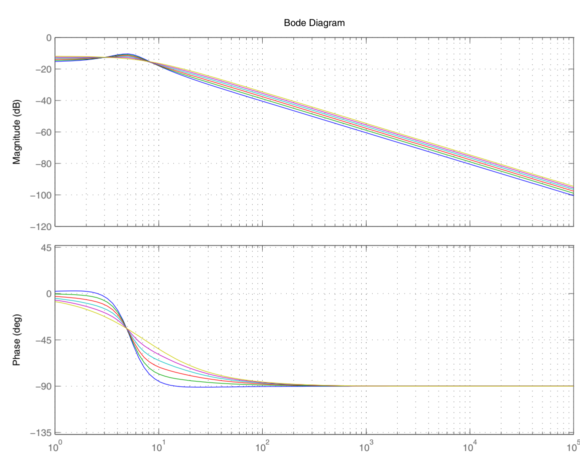

Fig. 6. Bode plot of the optimal 2nd order compensator varying from optimal to optimal positive real: Example 4.2