A two degrees of freedom H₂ controller design methodology for multi-motors web handling system

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Abstract-In web transport systems, the main problem consists to control independently the web velocity and tensions, to prevent web breaks, folding, and damage. Some interesting results have been obtained using multivariable control strategies. However, the existing methodologies are often not systematic or have the drawback to handle the tracking and disturbance rejection problems as a whole. This paper presents a complete methodology in order to design a two degrees of freedom (DOF) controller. The feedforward part is composed of a reference model allowing to obtained desired tracking performances (in particular web tensions and velocity decoupling). The feedback part insuring robustness and disturbance rejection is designed from only two design high level tuning parameters thanks to the Standard State Control (SSC) methodology. The proposed controller is tested on a three motors web handling platform.

I. INTRODUCTION

Systems handling paper, metal, polymers, or fabric are very common in the industry. General trend for such system is to achieve high speed to increase the productivity. At the same time the web section is thinner and thinner on certain product. The main concern is to prevent web breaks, folding, and damage which may slow down or and even stop the production line. This requires decoupling between web tension and speed, so that a reference change on the speed does not affect the web tensions and conversely. Achieving robustness is also necessary, not only to provide a safe control of the web throughout the whole industrial process, but also to permit to use the same controller for different types of web.

Until now, most industrial web transport systems make use of decentralized PID controllers. Some research on web handling control propose alternative control laws using also fuzzy, neural [1], or H_{∞} approaches [2]. For instance, to take more concern about the speed and tension decoupling, multivariable control strategies have recently been proposed for industrial metal transport systems [3], and paper transport systems [4],[5]. Interesting results have been obtained on an experimental 3 motors platform (see fig. 1), with a H_{∞} multivariable control design [2],[6]. However, these methodologies have the drawback to treat the tracking and the disturbance rejection problems as a whole. To consider these problems separately may have a practical interest.

This paper presents a complete methodology in order to design a two degrees of freedom (DOF) controller, composed of a feedforward part, allowing to obtained desired tracking performances (response time and decoupling), and a feedback part, insuring robustness and disturbance rejection. The decoupling between web velocity and tensions will be achieved using an adequate serie / parallel reference model. To build the feedback action, a systematic control design strategy, namely the Standard State Control (SSC) methodology [7],[8], is used to obtain an H₂ estimated state feedback. The efficiency of the proposed controller is tested on the 3 motors platform.

The presentation of this paper is as follows; description of the 3 motors bench is first presented in section 2, as well as the control objectives. Section 3 introduces the structure of the proposed control law. Its subsections A and Brespectively introduce the design methodology used to obtain the feedforward and the feedback part of the 2 DOF controller. The results obtained with and without decoupling feedforward controller are presented in section 4. Based on it, some conclusions and perspectives are presented in section 5.

II. CONTROL OBJECTIVES FOR THE THREE MOTORS BENCHMARK WINDING SYSTEM

A. 3 motors bench

The system considered in this paper is an elastic web transport system including an unwinder, winder, and a

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Fig. 1. 3 motors benchmark

traction motor (see fig. 1). It is representative of some inherent difficulties of web transport systems.

Fig. 1 shows the different variables used in the model: the control inputs $u = [u_u, u_v, u_w]^T$ (the torque references of the three synchronous motors), and the outputs $y = [T_u, V, T_w]^T$ (the unwinding web tension T_u , the linear velocity V, and the winding web tension T_w) of the system G_o (defined by the dashed box). The web velocity is measured near the master traction motor and the tensions are derived from force sensors measuring the web tension between rolls in the webline.

The following state space representation of the linearized model will be used in the sequel to design the controller. The numerical values (operating point, physical parameters adjusted from experimental data by identification) have been taken from [9].

$$\begin{cases} E(t)\dot{X} = A(t)X + B(t)U\\ Y = CX + DU \end{cases}$$
(1)

where

$$C = \begin{bmatrix} 0 & 1/2 & 0 & 1/2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1/2 & 0 & 1/2 & 0 \end{bmatrix}, D = 0$$

 V_i , R_i , J_i , and f_i are the linear velocity, the radius, the inertia and the viscous friction coefficient of the roll *i* respectively. T_i and L_i are the web tension and the web length between the roll *i* and the roll *i*+1. K_u , K_t , K_w are the torque constants of each motor. V_o is the nominal linear web velocity. E_o is a parameter depending on elasticity modulus E, on web section section S, and on nominal tension T_o : $E_o = ES + T_o$. All parameters varying during the winding process are expressed as functions of time.

The transfer matrix associated for a particular operating point is denoted G(s),

$$G(s) = \left[C \left(sI - E^{-1}A \right)^{-1} E^{-1}B \right].$$
 (2)

Remark: Inertia J_i , radius R_i , and viscous friction coefficient f_i are time varying and may change substantially during the processing.

Remark 2: The order of the model (1) is n = 9. Taking into account the antialiasing filters leads to a model of order 12.

Remark 3: Note that the system is naturally stable.

B. Control requirements

The main concern is to prevent web breaks, folding, and damage which may slow down or and even stop the production line. In the same way, excessive or oscillating tension or velocity must be avoid to prevent deterioration. Therefore, transport control systems should meet the following requirements:

- Speed and tensions regulation with web tensions and speed decoupling so that a reference change on the speed does not affect the web tensions and conversely.
- Robustness with respect to variations in the web elasticity modulus due to temperature or moisture modification: achieving robustness not only provides a safe control of the web throughout the whole industrial process, but also permits to use the same controller for different types of web.
- Robustness to variations in roll diameter: the same performance should be maintained throughout web processing. A gain scheduling solution is not considered here [6]. An interpolation approach using the present work is under study. The process behavior will be considered here around the nominal operating point.

III. 2 DOF H_2 CONTROLLER

Different strategies are possible to design a two degrees of freedom controller. The feedback and the feedforward parts may be designed simultaneously from a simple



Fig. 2. 2 DOF control law

criterion as in [10]. Another way consists in conceiving independently [11] or sequentially [12] the feedback and feedforward parts. An important problem however consists to design a reference model pertinent to the system and the control objectives. Particularly, the decoupling objectives have to be appropriately translated.

The control scheme of fig. 2 will be used in the following. The feedforward part $K_{ff}(s)$ is in fact a serie / parallel reference model allowing to define a reference trajectory (u_r, x_r) for the system. The feedback part $K_{fb}(s)$ is based on an estimated state feedback, which has to drive the system around the reference trajectory despite model uncertainties and disturbances. Some links may be found with the Internal Model Control (IMC) structure [13]. The whole scheme has obvious practical interest in that it can easily take into accounts saturations on the control input. At last, notice that both feedforward and feedback parts are design independently; the design of the feedforward part is studied in subsection A, whereas the feedback part is considered in subsection B.

A. Feedforward $K_{ff}(s)$: serie / parallel reference model under decoupling constraints



Fig. 3. Reference model in the feedforward

A way to build a reference model compatible with the structural properties of the system is to derive it from the model of the system itself. As the main problem is speed and tensions decoupling, the row-by-row decoupling method [14] was used as part of the methodology. Generally speaking, The row-by-row decoupling problem amounts to find a control law

$$u_r = F(s)x_r + L(s)y_{ref}$$
(3)

for the system G(s) (2) such that the closed-loop transfer function matrix $G_r(s)$ between the reference signal y_{ref} and the output y_r (see fig. 3) is an internally stable diagonal matrix:

$$G_r(s) = \frac{y_r}{y_{ref}} = diag\{b_i(s)/a_i(s)\}, \ i = 1, 2, ..., p.$$
(4)

Appealing aspect of the algorithm proposed in [14] is that it permits to solve the row-by-row decoupling problem without *a priori* structural information on the system. More precisely, the method goes through the determination of a precompensator P(s), the deconnector (see fig. 3). Its role is to rub out in a systematic way the problems due to the presence of interconnection infinite and/or finite zeros, in order to obtain a precompensated system which is decouplable by static state feedback with internal stability. For the 3 motors platform, the deconnector P(s) is useless. A static state feedback (i.e. P(s) and F_2 are null) is sufficient to insure decoupling. It comes from the two following classical results.

Theorem 1: If G(s) is square, it has as many zeros as poles (including those at infinity).

Proof: See [15]. *Theorem 2*: Let consider the system (*A*,*B*,*C*,*D*) and the row subsystems (*A*,*B*,*C_i*,*D_i*), i = 1,...,3. Let's denote $n_{z_{inf}}$ the number of zeros at infinity for (*A*,*B*,*C*,*D*), and $n_{z_{inf}}^{i}$ the number of zeros at infinity for (*A*,*B*,*C_i*,*D_i*). Then, a square invertible system is decouplable by static state feedback if

and only if
$$n_{z_{inf}} = \sum_{i=1}^{p} n_{z_{inf}}^{i}$$
.
Proof: See [14].

In the present case, G(s) is square of dimension $p \times p$, with p = 3, and furthermore, it presents $n_{z_f} = 4$ finite transmission zeros $(-2.1525 \pm 97.85i, -3.7665 \pm 82.439i)$; thus, after theorem 1, the whole system has $n_{z_{inf}} = n - n_{z_f} = 8$ infinite zeros. The first row subsystem associated to the first output T_u has $n_{z_{inf}}^1 = 3$, the second one $n_{z_{inf}}^2 = 2$, and the last one $n_{z_{inf}}^3 = 3$ infinite zeros. Therefore decoupling by static state feedback is possible.

Finally, the procedure leads to the closed-loop transfer function $G_r(s) = diag\{b_i(s)/a_i(s)\}$ with

$$\frac{b_i(s)}{a_i(s)} = \frac{k_i}{(s - \alpha_{i1})(s - \alpha_{i2})(s - \alpha_{i3})}, \text{ for } i=1,3$$
(5)

and

$$\frac{b_2(s)}{a_2(s)} = \frac{k_2}{(s - \alpha_{21})(s - \alpha_{22})}.$$
(6)

The k_i are chosen so as to insure the static gain $G_r(0)=I$.

The poles α_{ij} have been chosen in accordance with the control objective in terms of response time.

B. Feedback $K_{fb}(s)$: the H_2 -based SSC methodology

Let consider now the design methodology for the feedback part. We used the (SSC) methodology [7],[8]. We describe here its different steps. The main idea consists, starting from the problem specification, to build an H_2 optimization problem which is well-posed, parameterized by the minimum number of high level design parameters allowing to manage basic compromises between performances and robustness.

The SSC proceeds as follows.

1. Define the conceptual model from the identification of the significant signals of the plant considered: the disturbance input d, the control input u, the controlled output y_c (to which a reference signal r will be associated) and the observed output (available for feedback) y_o . This leads to the model (M_l) :

$$\begin{pmatrix} \dot{\mathbf{x}}_{M_1} \\ \overline{\mathbf{y}}_c \\ \mathbf{y}_o \end{pmatrix} = \begin{bmatrix} \underline{A}_p & B_d & B_u \\ \overline{C}_{y_c} & D_{y_cd} & D_{y_cu} \\ C_{y_o} & D_{y_cd} & 0 \end{bmatrix} \begin{bmatrix} \mathbf{x}_{M_1} \\ \overline{d} \\ u \end{bmatrix}$$
(7)

2. Insert the additional input, w_x , which will make possible hereafter to take into account exogenous inputs (such as noises) entering to the state equation. Then obtain model (M_2):

$$\begin{pmatrix} \dot{x}_{M_2} \\ y_c \\ y_o \end{pmatrix} = \begin{bmatrix} A_p & B_d & [0 & I] & B_u \\ C_{y_c} & D_{y_cd} & 0 & D_{y_cu} \\ C_{y_o} & D_{y_od} & 0 & 0 \end{bmatrix} \begin{bmatrix} x_{M_2} \\ d \\ w_x \\ u \end{bmatrix}.$$
(8)

Associate predictor models (M_p) to the disturbance and reference input. The more classical model associated to a signal v(·) whose best prediction is v(t + τ) = v(t) is given by: v(t) = 0. Once again, a state noise whose intensity will be fixed according to the control objectives is inserted.

$$\begin{pmatrix} \dot{x}_{\xi} \\ r \\ d \end{pmatrix} = \begin{pmatrix} A_{\xi} & [I & 0] \\ C_{\xi_1} & 0 \\ C_{\xi_2} & 0 \end{pmatrix} \begin{pmatrix} x_{\xi} \\ w_x \end{pmatrix}$$
(9)

4. Define the reference error $e = r - y_c$ and an asymptotic trajectory for the plant such that e = 0. Such a trajectory $(u_a(\cdot), x_a(\cdot))$ with $x_a = T_a x_{\xi}$ and $u_a = K_a x_{\xi}$ if (T_a, K_a) is solution of the Sylvester equation [16]:

$$\begin{cases} T_a A_{\xi} - A_p T_a - B_u K_a + B_d C_{\xi_2} = 0 \\ C_{y_c} T_a + C_{\xi_1} = 0 \end{cases}$$
(10)



Fig. 4. Augmented model (M_3)

5. Define $e_u = u - u_a$ the control input deviation from the asymptotic trajectory and $y = \begin{bmatrix} r^T & y_c^T \end{bmatrix}^T + w_y$ the vector of signals available for feedback. Then build a minimal realization of the augmented model (M_3) from (M_2) and (M_p) as shown in fig. 4. Associate the transfer matrix G(s) to the model (M_3) :

$$\begin{pmatrix} e_u \\ e \\ y \end{pmatrix} = \begin{pmatrix} G_{11}(s) & G_{12}(s) \\ G_{21}(s) & G_{22}(s) \end{pmatrix} \begin{pmatrix} w_y \\ w_x \\ u \end{pmatrix}$$
(11)

6. The problem is now to define pertinent weighting matrices $Q_c = Q_c {}^{\frac{1}{2}^T} Q_c {}^{\frac{1}{2}}$, $R_c = R_c {}^{\frac{1}{2}^T} R_c {}^{\frac{1}{2}}$ and $Q_o = Q_o {}^{\frac{1}{2}} Q_o {}^{\frac{1}{2}^T}$, $R_o = R_o {}^{\frac{1}{2}} R_o {}^{\frac{1}{2}^T}$ from high-level design parameters (see fig. 5). Two design parameters will be considered from now on: the time horizon for control T_c and filtering or observation T_o . It is wished that T_c allows to tune the time response to a reference change and T_o the time response for disturbance rejection. Following [7], a powerful way to do this consists to derive the weighting matrices from these two parameters, by using energetic considerations, thanks to gramians:

- The matrices Q_c and R_c are linked to the control horizon T_c as:

$$\begin{cases} R_{c} = T_{c} \int_{0}^{T_{c}} (C_{y_{c}} e^{A_{p'}} B_{u})^{T} (C_{y_{c}} e^{A_{p'}} B_{u}) dt = T_{c} B_{u}^{T} G_{ol} (T_{c}) B_{u} \\ Q_{c} = I \end{cases}$$
(12)

- The matrices Q_o and R_o are linked to the filtering horizon T_o :

$$\begin{cases} Q_o = [T_o \int_0^{T_o} e^{A_p^T t} C_{y_o}^{T} C_{y_o} e^{A_o t} dt]^{-1} = [T_o G_{o2}(T_o)]^{-1} \\ R_o = I \end{cases}$$
(13)

The problem now consists to find a dynamical feedback u = K(s)y in order to minimize the H₂ norm of some weighted closed-loop transfer matrix, as defined on fig.
 Despite the non-stabilizability of G_{H2} ((M_p) is usually unstable), thanks to the building process used,

the generalized H₂ problem consisting to find K(s) such that $||F_i(G_{H_2}(s), K(s))||_2$ is minimized admits a solution [8]. Furthermore, the separation principle can still be applied. Thus, denoting the standard model associated to G_{H_2} as follows,

$$G_{H_2} \coloneqq \begin{bmatrix} A & B_1 & B_2 \\ C_1 & 0 & D_{12} \\ C_2 & D_{21} & 0 \end{bmatrix},$$
 (14)

the dynamic optimal controller K(s) is given by,

$$K(s) \coloneqq \left[\frac{A + B_2 K + LC_2}{K} \mid \frac{L}{0}\right]$$
(15)

where K and L are respectively the optimal feedback gain and the optimal observer gain.



Fig. 5. H₂-problem associated to the SSC methodology

Remark: The SSC Methodology leads to the design of a basic reference prefilter, which time response is T_o . Obviously for our practical application, this prefilter is omitted and replaced by the feedforward $K_f(s)$ defined in section 3.

Remark 2: Practically, T_o is chosen in order to obtain admissible performances in regulation without having a control input too much sensible to noise. It is proved in [8] that the Kalman filter obtained with this choice is independent of the realization used to describe the plant. The robustness issue is addressed following *LTR* considerations and is obtained with T_c much smaller than T_o (practically by a factor of 3 to 10).

Remark 3: Following the line of the SSC-methodology, under some assumptions [8] easily satisfied, the references and disturbances predictor model is included in the resulting controller, as an internal model [16]. For example, all the controllers designed in this paper include an integral action guarantying robust disturbance rejection.

Remark 4: It is conjectures in [7] that, following the SSC-methodology, the observer poles are left from $-1/T_o$ and the other closed-loop poles are left from $-1/T_c$.

IV. RESULTS



Fig. 6. Step changes with and without decoupling feedforward



Fig. 7. Influence of the decoupling feedforward $K_{ff}(s)$

The design parameters T_o and T_c of the SSC methodology were chosen in order to manage the robustness-performance trade-off; the following tuning was considered $T_o = 1, 1s$ and $T_c = T_o/4$.

The control law is tested on a 12th order non-linear model of the 3 motors web transport benchmark, following the scenarios of figure 6. The simulations have been performed around the operating point used to defined the design model for control. The scenarios consist first in a step on the reference for the tension T_u , and second in a reference step for the web velocity. In both cases, the decoupling feedforward control improves the results when compared to the case where the feedforward part is only a prefilter.

The decoupling effect of the feedforward part $K_{ff}(s)$ is also demonstrated in the bode diagram of the figure 7. Decoupling between reference inputs y_{ref_i} and outputs to be controlled y_j , $i \neq j$, is obtained at all frequency in a more important way than with only the feedback part $K_{fb}(s)$.

The robustness properties are analyzed from the following indicators (Table I), still around the operating point: the input and output gain-phase margins $M_{i/o}^{gp}$ and complementary margins $M_{i/o}^{gpc}$, defined respectively by $1/||S_{i/o}||_{\infty}$ and $1/||T_{i/o}||_{\infty}$, where $S_{i/o}$ and $T_{i/o}$ are the input or output sensitivity and complementary sensitivity functions. The input / output multivariable delay margins $M_{i/o}^{d}$ are calculated following [17]. As expected the SSC methodology permits to reach good output but also input robustness properties.

At last, let recall that robust disturbance rejection is insure, thanks to the presence of integrators in the feedback $K_{fb}(s)$, resulting from the SSC methodology.

V. CONCLUSION AND PERSPECTIVES

This paper proposes a complete methodology leading to a two degrees of freedom controller. The conception of the feedforward and feedback parts are done independently. The feedforward part is in fact a serie / parallel reference model allowing to define a reference trajectory (u_r, x_r) for the system. The feedback part is based on an estimated state feedback, which has to drive the system around the reference trajectory despite model uncertainties and disturbances. The reference model is derived from the model of the system itself, using the row-by-row decoupling method [14]. It has therefore by construction structural properties compatible with the system and the control objectives. The SSC methodology [7],[8] was used to design the feedback part. Simulations around the operating point demonstrate benefits of the decoupling feedforward.

The perspectives of this work will concern, at primary goal, the improvement of the robustness to variations in the rolls diameter. To insure the same level of performance throughout the web processing, a Linear Parameter Variant (LPV) controller is required. Such a controller will be designed by interpolation using the methodology presented in this paper (to design the frozen LTI controllers). This LPV controller will be then experimented on the high speed 3 motors platform located at Strasbourg (http://ertenroulement.u-strasbg.fr).

TABLE I										
ROBUSTNESS PROPERTIES OF THE FEEDBACK PART										
Man input	Manc input	Man output	Manc output	+/- Md	input +/- Md output					

Mgp Input	Nigpe input	Mgp Output	Nigpe output	(ms)	(ms)	
0,7	0,84	0,74	0,88	447	473	

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